NONLINEAR INTERACTIONS OF WAVES IN A WEAKLY TURBULENT PLASMA WITH THE PARTICIPATION OF ION SOUND

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Nonlinear processes are considered, involving ion-sound waves which are described as a gas of quasiparticles. It is shown that within the limits of applicability of the method the results for three-plasmon processes and scattering of sound by particles are identical with the expressions obtained by the regular method. Expressions are obtained for turbulent shift of the sound frequency and equations describing scattering of sound are derived.

VEDENOV and Rudakov[1] proposed the following method for describing a weakly-turbulent plasma: The plasma oscillations are described by a kinetic equation that is averaged over a time interval much longer than the characteristic period of the high-frequency waves. In this equation, the adiabatic interaction of the particles with the waves is taken into account by introducing the plasmon-gas pressure gradient. This method yielded, in particular, equations describing the time variation of the number of Langmuir plasmons produced by plasmon-electron and plasmon-plasmon scattering.

In this paper we derive by an analogous method equations describing the time variation of the number of ion-sound plasmons as a result of scattering by plasma particles and scattering of sound by sound. The same method is used to consider spontaneous emission of ion sound by Langmuir plasmons.

1. "SOUND-PARTICLE" AND "SOUND-SOUND" INTERACTIONS

Let us consider the system of equations[1]

$$\frac{\partial f^\alpha}{\partial t}+\mathbf{v}\nabla f^\alpha-\frac{e_\alpha}{m_\alpha}\nabla\varphi\frac{\partial f^\alpha}{\partial \mathbf{v}}-\frac{\partial f^\alpha}{\partial \mathbf{v}}\nabla\sum_k\frac{n_k{}^s}{nm_\alpha}\frac{\omega_{p\alpha}{}^2}{(\partial\varepsilon/\partial\omega_k)(\omega-\mathbf{kv})^2}=0, \tag{1}$$

$$\frac{\partial n_k{}^s}{\partial t}+\frac{\partial\omega}{\partial\mathbf{k}}\nabla n_k{}^s-\nabla\omega_k\frac{\partial n_k{}^s}{\partial\mathbf{k}}=0, \tag{2}$$

$$\varepsilon=1+\sum_{\alpha=i,e}\frac{\omega_{p\alpha}{}^2}{nk^2}\int\mathbf{k}\frac{\partial f^\alpha}{\partial\mathbf{v}}\frac{d\mathbf{v}}{\omega-\mathbf{kv}}=1-\frac{\omega_{pi}{}^2}{\omega^2}+\frac{\omega_{pe}{}^2m}{k^2T_e}=0, \tag{3}$$

$$\Delta\varphi=-4\pi\sum_\alpha e_\alpha\int f^\alpha\,d\mathbf{v}. \tag{4}$$

The last term of Eq. (1) describes the change effected in the averaged particle distribution function by the "high-frequency force"

$$-\nabla\sum_k\frac{n_k{}^s\omega_k}{n}\frac{\omega_{p\alpha}{}^2}{\omega_k\,\partial\varepsilon/\partial\omega_k}\frac{1}{(\omega-\mathbf{kv})^2}$$

(2) is the Liouville equation for the ion-sound plasmons, (3) is the dispersion equation for ion sound, and (4) is the Poisson equation; $\omega_{p\alpha}$ are the plasma frequencies.

We first consider short-wave sound:

$$\omega=\omega_{pi}[1-(kr_{De})^{-2}]$$

(r_D is the Debye radius). This condition can be realized when $T_e \gg T_i$, which is simultaneously the condition for the existence of weakly-damped ion sound. In this region of the spectrum we can neglect the perturbation of the thermal motion of the electrons under the influence of the wave (ionic plasma oscillations).

We shall solve (2) by successive approximations. We note that since $\partial\omega/\partial\mathbf{r}=0$ when $n_k^s=0$, we get in the zeroth approximation

$$\frac{\partial n_k{}^s}{\partial t}+\frac{\partial\omega}{\partial\mathbf{k}}\nabla n_k{}^s=0.$$

The solution of this equation is

$$n_k{}^s=N_k{}^s+\sum_{q\neq0}N_{kq}{}^s\exp(-i\Omega t+i\mathbf{qr}),\quad \Omega=\mathbf{q}\frac{\partial\omega}{\partial\mathbf{k}}, \tag{5}$$

N_{kq} is none other than the sum of all possible "beats" of the ion-sound oscillation with close values of the wave vector:

$$N_{kq}=\frac{k^2}{8\pi}\frac{\partial\varepsilon}{\partial\omega_k}\varphi_k\overset{+}{\varphi}_{k-q}.$$

Writing down similar expansions for φ and δf, we substitute them together with (5) into the system (1)—(4). We recall that all the initial equations are valid only for $q \ll k$.

Obtaining from (1)—(4) equations for the Fourier components and eliminating φ_{kq}, we get

$$\delta f_{kq}{}^i = -\frac{\omega^3 N_{kq}{}^s}{2Mn}\frac{\mathbf{q}\,\partial f^i/\partial \mathbf{v}}{\Omega - \mathbf{qv}}\left\{\frac{1}{(\omega-\mathbf{kv})^2} - \int \mathbf{q}\frac{\partial f^i}{\partial \mathbf{v}}\frac{d\mathbf{v}}{(\omega-\mathbf{kv})^2(\Omega-\mathbf{qv})}\Big/\int \mathbf{q}\frac{\partial f^i}{\partial \mathbf{v}}\frac{d\mathbf{v}}{\Omega - \mathbf{qv}}\right\}. \quad (6)$$

Using (3), we can express the frequency shift $\delta\omega_k$ in terms of δf_{kq}:

$$\delta\omega_{kk'q} = -\frac{\omega_k^3}{2nk^2}\int \frac{\mathbf{k}\partial\delta f^i_{k'q}/\partial\mathbf{v}}{\omega - \mathbf{kv}}d\mathbf{v}. \quad (7)$$

Of importance to further calculations are the two parameters kc_i/ω and Ω/qc_i, where $c_i = \sqrt{2T_i/M}$. For weakly-damped ion sound ($kr_{Di} \ll 1$ and $T_i/T_e \ll 1$) we always have $kc_i/\omega \ll 1$. Further, when $kr_{De} \gg 1$, we have

$$\Omega = \mathbf{q}\frac{\partial\omega}{\partial\mathbf{k}} = \frac{(\mathbf{kq})}{k}\sqrt{\frac{T_e}{M}}\,(kr_{De})^{-3}.$$

We see therefore that when $kr_{De} > (T_e/T_i)^{1/6}$ the parameter Ω/qc_i is small. When $kr_{De} < (T_e/T_i)^{1/6}$ the parameter Ω/qc_i is large everywhere except the region defined by the following conditions:

$$|\cos\widehat{\mathbf{kq}}| < \sqrt{T_i/T_e}\,k^3r^3_{De} \quad \text{for } 1 < kr_{De} < (T_e/T_i)^{1/6},$$
$$|\cos\widehat{\mathbf{kq}}| < \sqrt{T_i/T_e} \quad \text{for } 1 \gtrsim kr_{De}. \quad (8)$$

In this region the indicated parameter always remains small.

The unperturbed function f^i will be assumed to be Maxwellian. In this case, all the integrals contained in $\delta\omega$ reduce to the Kramp function and can be calculated in final form in the presence of small parameters.

Let us consider first the region $kr_{De} > (T_e/T_i)^{1/6}$. In this case the calculation of $\delta\omega_{kk'q}$ yields

$$\delta\omega_{kk'q} = -\frac{N^s_{k'q}}{Mn}\left[(\mathbf{kk'}) + i\sqrt{\pi}\frac{[\mathbf{kq}][\mathbf{k'q}]}{q^2}\frac{\Omega}{qc_i}\right]. \quad (9)^*$$

From (2) we can get the rate of change of the plasmon number N_k by assuming that the phase distribution of the beats N_{kq} is random and carrying out statistical averaging over the phases[1]:

$$(\dot N_k)^{(1)} + (\dot N_k)^{(2)} = \left\langle \frac{\partial}{\partial\mathbf{k}}(n_k + \delta n_k)\frac{\partial\delta\omega_k}{\partial\mathbf{r}}\right\rangle$$
$$= \frac{\partial}{\partial\mathbf{k}}\sum_{q(\mathbf{k}=-\mathbf{k'})}\mathbf{q}N_{kq}{}^s\,\mathrm{Im}\,\delta\omega_{k,k',-q}$$
$$+\pi\frac{\partial}{\partial\mathbf{k}}\sum_{k',q}\mathbf{q}\delta\omega_{kk'q}\,\delta\omega_{k,-k',-q}\,\delta\left(\mathbf{q}\frac{\partial\omega}{\partial\mathbf{k}} - \mathbf{q}\frac{\partial\omega}{\partial\mathbf{k'}}\right)\mathbf{q}\frac{\partial N_k{}^s}{\partial\mathbf{k}}. \quad (10)$$

*$[\mathbf{kq}] \equiv \mathbf{k}\times\mathbf{q}$.

Substitution of (9) in (10) yields

$$(\dot N_k{}^s)_{pi} = \frac{\partial}{\partial\mathbf{k}}\left(\frac{\sqrt{\pi}}{Mn}\sum_q\frac{\mathbf{q}}{q}\frac{\Omega}{c_i}\frac{[\mathbf{kq}]^2}{q^2}N_k{}^sN^s_{k-q}\right), \quad (11)$$

$$(\dot N_k{}^s)_{pp} = \frac{\partial}{\partial\mathbf{k}}\frac{\pi}{(Mn)^2}\times\sum_{k',q}\mathbf{q}(\mathbf{kk'})^2\,\delta\left(\mathbf{q}\frac{\partial\omega}{\partial\mathbf{k}} - \mathbf{q}\frac{\partial\omega}{\partial\mathbf{k'}}\right)N_{k'}{}^sN^s_{k'-q}\,\mathbf{q}\frac{\partial N_k{}^s}{\partial\mathbf{k}}. \quad (12)$$

Formulas (11) and (12) correspond to plasmon-ion and plasmon-plasmon interaction, respectively.

Let now $1 \ll kr_{De} < (T_e/T_i)^{1/6}$. The foregoing condition can be realized only in a strongly nonisothermal plasma. In this case the contribution of the electrons to $\delta\omega$ is still a small correction. Calculating the integrals in (7) for $kc_i/\omega \ll 1$ and $qc_i/\Omega \ll 1$, we get

$$\delta\omega_{kk'q} = -\frac{N^s_{k'q}}{Mn}\frac{(\mathbf{kq})(\mathbf{k'q})}{q^2}. \quad (13)$$

The imaginary addition to $\delta\omega_{kk'q}$ is in this limiting case exponentially small everywhere except in the region U_{kq} defined by the first condition of (8), where the solution (9) remains in force. Taking this circumstance into account, we get

$$(\dot N_k{}^s)_{pi} = \frac{\partial}{\partial\mathbf{k}}\left(\frac{\sqrt{\pi}}{Mn}\sum_{q\in U_{kq}}\frac{\mathbf{q}}{q}\frac{\Omega}{c_i}\frac{[\mathbf{kq}]^2}{q^2}N_k{}^sN^s_{k-q}\right), \quad (14)$$

$$(\dot N^s_k)_{pp} = \frac{\partial}{\partial\mathbf{k}}\frac{\pi}{(Mn)^2}\sum_{k'}\left(\sum_{q\in U_{k'q}}\mathbf{q}(\mathbf{kk'})^2\delta\left(\mathbf{q}\frac{\partial\omega}{\partial\mathbf{k}} - \mathbf{q}\frac{\partial\omega}{\partial\mathbf{k'}}\right)\times N^s_{k'}N^s_{k'-q}\mathbf{q}\frac{\partial N^s_k}{\partial\mathbf{k}} + \sum_{q\bar\in U_{k'q}}\mathbf{q}\,(k_q^2k_q'^2)\,\delta\left(\mathbf{q}\frac{\partial\omega}{\partial\mathbf{k}} - \mathbf{q}\frac{\partial\omega}{\partial\mathbf{k'}}\right)\times N^s_{k'}N^s_{k'-q}\mathbf{q}\frac{\partial N^s_k}{\partial\mathbf{k}}\right). \quad (15)$$

We note that, strictly speaking, it is necessary to use in (14) and (15) the exact expressions for the imaginary part of the Kramp function when $\Omega/qc_i \sim 1$.

To take into account the contribution of the electrons when $kr_{De} \lesssim 1$, it is convenient to use in lieu of the system (1)—(4) another self-consistent system of equations:

$$\frac{\partial\delta f^i}{\partial t} + \mathbf{v}\nabla\delta f^i - \frac{e}{M}\nabla\varphi\frac{\partial f^i}{\partial\mathbf{v}} - \frac{\partial f^i}{\partial\mathbf{v}}\nabla\sum_k\frac{n_k{}^s}{2Mn}\frac{\omega^3}{(\omega-\mathbf{kv})^2} = 0, \quad (16)$$

$$n^e = n\exp\left[\frac{1}{T_e}\left(e\varphi - \sum_k\frac{n_k{}^s}{2n}\frac{M}{m}\frac{\omega^3}{k^2[v_e^2]}\right)\right] \quad (17)$$

and the equations (3) and (4). Here $[v_e^2] = \kappa T_e/m$, where $\varkappa$ is a numerical factor on the order of unity, the exact calculation of which is quite difficult. The physical meaning of (17) is simple: this is the Boltzmann distribution in a field which is almost static for the electrons.

We solve this system in the same manner as the system (1)—(4). Taking into account the smallness of the argument of the exponential in (17), we get

$$\delta f_{kq}{}^{i} = \frac{\mathbf{q}\,\partial f^{i}/\partial \mathbf{v}}{\Omega - \mathbf{q}\mathbf{v}} \frac{\omega^{3} N_{kq}{}^{s}}{2Mn}\Big[\frac{1}{(\omega - kv)^{2}} - \Big\{\int \mathbf{q}\frac{\partial f^{i}}{\partial \mathbf{v}} \times \frac{d\mathbf{v}}{(\omega - \mathbf{k}\mathbf{v})^{2}(\Omega - \mathbf{q}\mathbf{v})} - \frac{M^{2}n}{\varkappa k^{2}T_{e}{}^{2}}\Big\}\Big/\Big\{\int \mathbf{q}\frac{\partial f^{i}}{\partial \mathbf{v}}\frac{d\mathbf{v}}{\Omega - \mathbf{q}\mathbf{v}} + \frac{Mn}{T_{e}}\Big\}\Big], \tag{18}$$

$$\delta n_{kq}{}^{e} = -\frac{N_{kq}{}^{s}\,\omega^{3}}{2T_{e}}\Big[\Big\{\int \mathbf{q}\frac{df^{i}}{\partial \mathbf{v}}\frac{d\mathbf{v}}{(\omega - \mathbf{k}\mathbf{v})^{2}(\Omega - \mathbf{q}\mathbf{v})} - \frac{M^{2}n}{\varkappa k^{2}T_{e}{}^{2}}\Big\} \times \Big\{\int \mathbf{q}\frac{\partial f^{i}}{\partial \mathbf{v}}\frac{d\mathbf{v}}{(\Omega - \mathbf{q}\mathbf{v})} + \frac{Mn}{T_{e}}\Big\}^{-1} + \frac{M}{\varkappa k^{2}T_{e}}\Big],$$

$$\delta\omega_{kk'q} = \frac{\omega^{3}}{2n}\Big[\int \frac{\delta f_{k'q}\,d\mathbf{v}}{(\omega - \mathbf{k}\mathbf{v})^{2}} + \frac{M}{3T_{e}}\,\delta n^{e}_{k'q}\Big]. \tag{19}$$

The final expression for $\delta\omega$ is quite unwieldy. We present it only for the limiting case $kr_{De} \ll 1$:

$$\delta\omega_{kk'q} = \frac{N^{s}_{k'q}\,kk'}{Mn}\,\nu_{kk'q}, \tag{20}$$

where

$$\nu_{kk'q} = \frac{1}{4}\Big[\frac{1 + (2 + 1/\varkappa)\cos^{2}\widehat{\mathbf{k}'\mathbf{q}}}{1 - \cos^{2}\widehat{\mathbf{k}'\mathbf{q}} + 3k'^{2}r^{2}_{De}}\Big(\frac{1}{3} - 2\frac{\cos\widehat{\mathbf{k}\mathbf{q}}}{\cos\widehat{\mathbf{k}'\mathbf{q}}} - \frac{1}{\cos^{2}\widehat{\mathbf{k}'\mathbf{q}}}\Big) + \frac{1}{3\varkappa} + 2 + 2\frac{\cos\widehat{\mathbf{k}\mathbf{q}}}{\cos\widehat{\mathbf{k}'\mathbf{q}}} + \frac{1}{\cos^{2}\widehat{\mathbf{k}'\mathbf{q}}}\Big].$$

We note that in the region of V_{kq} defined by the condition $|\cos\widehat{\mathbf{kq}}| \lesssim \sqrt{T_i/T_e}$ the solution (9) remains in force.

If we use formulas (10), we get for $kr_{De} \ll 1$:

$$(\dot{N}_{k}{}^{s})_{pi} = \frac{\partial}{\partial \mathbf{k}}\Big(\frac{\sqrt{\pi}}{Mn}\sum_{q\in V_{kq}} \frac{\mathbf{q}}{q}\frac{\Omega}{c_{i}}\frac{[\mathbf{kq}]^{2}}{q^{2}} N_{k}{}^{s}N^{s}_{k-q}\Big), \tag{21}$$

$$(\dot{N}_{k}{}^{s})_{pp} = \frac{\pi}{(Mn)^{2}}\frac{\partial}{\partial \mathbf{k}}\sum_{k'}\Big(\sum_{q\in V_{k'q}} \mathbf{q}(\mathbf{l}\,\mathbf{k}')^{2}\,\delta\Big(\mathbf{q}\frac{\partial\omega}{\partial \mathbf{k}} - \mathbf{q}\frac{\partial\omega}{\partial \mathbf{k}'}\Big) \times N^{s}_{k'}N^{s}_{k'-q}\mathbf{q}\frac{\partial N^{s}_{k}}{\partial \mathbf{k}} + \sum_{q\in V_{k'q}} \mathbf{q}k^{2}k'^{2}\nu^{2}_{kk'q}\delta\Big(\mathbf{q}\frac{\partial\omega}{\partial \mathbf{k}} - \mathbf{q}\frac{\partial\omega}{\partial \mathbf{k}'}\Big) \times N^{s}_{k'}N^{s}_{k'-q}\mathbf{q}\frac{\partial N^{s}_{k}}{\partial \mathbf{k}}\Big). \tag{22}$$

We note that inasmuch as all the foregoing results were obtained for $q \ll k$, they take into account only the contribution made to the general expressions for $\dot{N}_k$ by the small-angle scattering.

2. INTERACTION OF LANGMUIR OSCILLATIONS WITH ION SOUND

Vedenov and Rudakov[1] derived expressions similar to (11) and (12) for $(\dot{N}^{L}_{k})_{pe}$ and $(\dot{N}^{L}_{k})_{pp}$, describing the change in the number of Langmuir plasmons as a result of plasmon-electron and plasmon-plasmon scattering. The influence of the ions on these processes was assumed to be small. The region of applicability of such an approximation was shown to be $kr_{De} \gg \sqrt{m/M}$.

Let us consider the broader region $kr_{De} \gtrsim \sqrt{m/M}$. Solving the system (1)—(4) for the Langmuir oscillations and investigating the result, we can show that in this region the ion influence can be neglected in the cone

$$|\cos\widehat{\mathbf{kq}}| > \frac{1}{kr_{De}}\sqrt{\frac{m}{M}}.$$

Outside this cone, the scattering by ions is comparable in order of magnitude with the scattering by electrons.

Let the plasma be strongly nonisothermal, $T_e \gg T_i$. Then in the region

$$\frac{1}{kr_{De}}\sqrt{\frac{T_i}{T_e}}\sqrt{\frac{m}{M}} < |\cos\widehat{\mathbf{kq}}| < \frac{1}{kr_{De}}\sqrt{\frac{m}{M}}$$

allowance for the ion motion introduces into $\delta\omega$ a pole term corresponding to Cerenkov radiation of ion sound. The resonance condition is

$$\Big(\frac{\Omega}{qc_s}\Big)^{2} = 1, \qquad \Omega = \mathbf{q}\frac{\partial\omega}{\partial \mathbf{k}}. \tag{23}$$

Condition (23) can be satisfied only if $|\partial\omega/\partial\mathbf{k}| \geq \sqrt{T_e/M}$ and determines the angle at which the ion sound is emitted.

Solving the system (1)—(4) just as in Sec. 1, we can obtain an expression for $\delta\omega_{kk'q}$ (see also[1]). In the vicinity of the poles defined by the condition (23) we can obtain

$$\mathrm{Im}\,\delta\omega_{kk'q} = \frac{3N^{L}_{k'q}}{4mnr^{2}_{De}}\,\frac{\sqrt{\pi}\,|\Omega/qc_{e}|}{\pi(\Omega/qc_{e})^{2} + (1 - 3Mm^{-1}k'^{2}r^{2}_{De}\cos^{2}\widehat{\mathbf{k}'\mathbf{q}})^{2}}, \tag{24}$$

$$|\delta\omega_{kk'q}|^{2} = \frac{9N^{L}_{k'}N^{L}_{k'-q}}{16(mn)^{2}r^{4}_{De}}\,\frac{1}{\pi(\Omega/qc_{e})^{2} + (1 - 3Mm^{-1}k'^{2}r^{2}_{De}\cos^{2}\widehat{\mathbf{k}'\mathbf{q}})^{2}}. \tag{25}$$

Substituting (24) and (25) in (9) and (10) we get $(\dot{N}^{L}_{k})^{(1)}$ and $(\dot{N}^{L}_{k})^{(2)}$, which describe in this case the change in the number of Langmuir plasmons as a result of three-plasmon processes ($\dot{N}^{(1)}$) and higher-order processes with emission of ion sound ($\dot{N}^{(2)}$). In the resonance region they are larger than

$(\dot{\bar{N}}_k^L)_{pe}$ and $(\dot{\bar{N}}_k^L)_{pp}$ by $(kr_{De})^{-4}$ and $(kr_{De})^{-6}$, respectively. It must be remembered, however, that the resonance region itself is a narrow space between two cones, so that the contribution of various processes to the total derivative $(\dot{N}_k)$ can be compared only when the specific function N_k and the degree of non-isothermicity T_i/T_e are specified.

Using the second-quantization formalism, we can readily obtain the general kinetic equation for the quasiparticles[2]:

$$\dot{N}_q{}^s = \sum_k W_{k,\,k-q}[(N_q{}^s+1)N_k{}^L(N_{k-q}^L+1) - (N_k{}^L+1)N_{k-q}^L N_q{}^s],$$

$$\dot{N}_k{}^L = \sum_q \{W_{k,\,k-q}[N_{k-q}^L N_q{}^s(N_k{}^L+1) - N_k{}^L(N_{k-q}^L+1)(N_q{}^s+1)] + W_{k,\,k+q}[N_{k+q}^L(N_k{}^L+1) \times (N_q{}^s+1) - N_k{}^L N_q{}^s(N_{k+q}^L+1)]\}.$$

Going to the limit $q \ll k$, we obtain for $N^L \gg 1$

$$\dot{N}_q{}^s = \sum_k W_{k,\,k-q}\left[N_k{}^L N_{k-q}^L + N_q{}^s\,\mathbf{q}\frac{\partial N_k{}^L}{\partial \mathbf{k}}\right],$$

$$\dot{N}_k{}^L = \sum_q \mathbf{q}\frac{\partial}{\partial \mathbf{k}}\left[W_{k,\,k-q}\left(N_k{}^L N_{k-q}^L + N_q{}^s\mathbf{q}\frac{\partial N_k{}^L}{\partial \mathbf{k}}\right)\right]. \quad (26)$$

Here $W_{kk'}$ is the corresponding transition probability with allowance for the conservation laws.

Going over in (26) to the limit $N^S \ll N^L$ and comparing with (10) and (24), we get

$$W_{k,\,k-q} = \frac{3\sqrt{\pi}}{4mnr_{De}^2}\,\delta\left[\left(1-\frac{3M}{m}k^2r_{De}^2\cos^2\mathbf{kq}\right)^2\right]; \quad (27)$$

the δ-function in (27) automatically ensures satisfaction of the conservation laws. We note that it is impossible to go in (26) to the limit $N^L \lesssim 1$ (although it is possible to go to the limit $N^S \lesssim 1$), since the condition $N^L \gg 1$ is essential for the applicability of this method.

Most of the results of this article were obtained by regular perturbation-theory methods (see, for example, the reviews[3,4]). They have been obtained here by a very simple method. The new results are expressions (8), (13), and (20) for the correction to the real part of the frequency of long-wave sound in a plasma containing a narrow packet of short-wave ion-sound oscillations, and also (formulas (12), (15), and (22), which describe nonlinear scattering of sound by sound.

In conclusion, I am sincerely grateful to L. I. Rudakov for suggesting the topic and for guidance.

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