PLASMA CONFINEMENT IN TOROIDAL TRAPS WITH DISRUPTED MAGNETIC SURFACES

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It is shown that the disruption of magnetic surfaces in toroidal traps, due to the presence of external disturbances or to imperfection of the coil design, should result in plasma leakage of the order of the Bohm leakage in a wide range of variation of the plasma parameters.

1. INTRODUCTION

IT is known in toroidal magnetic traps of the "Stellarator" type there is observed an anomalous leakage of plasma, with a lifetime τ close to the so-called Bohm lifetime: $\tau = \pi a^2 eH/cT$, where a is the radius of the plasma column, T the temperature, and H the magnetic field[1,2]. The only exception so far is[3], where much better containment of a cesium plasma was observed, with a lifetime close to the classical value[1)] (we do not consider here axially-symmetrical systems of the Tokamak or Levitron type). To explain the anomalous leakage of the plasma, one customarily introduces instabilities of the drift type, which lead to effects such as turbulent diffusion, and in systems that are strongly elongated in the H direction the corresponding diffusion coefficients can approach the Bohm values. However, the surprising closeness of the experimentally observed lifetime and the Bohm lifetime in a very wide range of temperature variation and plasma-density variation suggests that the plasma leakage can be connected with larger-scale effects. It is shown in this paper that disruption of the magnetic surfaces, due to imperfection of the magnetic system, can lead to a leakage of the Bohm order of magnitude.

2. QUALITATIVE CONSIDERATION

We consider the case when there are no closed magnetic surfaces inside the plasma. In other words, we assume that each force line sooner or later emerges from the plasma, "piercing" as it were the plasma column, entering it on one side and leaving on the other. If we mentally straighten out the plasma column and neglect the influence of the curvature of the force lines on the motion of the electrons and ions, then we can visualize, as the simplest model of the situation under consideration, a cylindrical column with a magnetic field which is inclined somewhat to the cylinder axis: $H = H_0 + h$, where h is the transverse component of the field and H_0 is directed along the cylinder. In the magnetohydrodynamic approximation, the plasma in such a field should scatter along the force lines with a velocity on the order of the thermal velocity of the ions $v_i = \sqrt{T/m_i}$. Outwardly such a scattering would look like the broadening of the column along h with a velocity $v_i \sim hv_i/H_0$.

However, this picture is realistic only at not too small an inclination of the force lines, that is, not too small h/H_0. In fact, since the plasma electrons and the ions drift in an aximuthal direction with a velocity $v_d \sim \rho_i v_i/a$ (where $\rho_i = v_i/\Omega_{Hi} = v_i m_i c/eH$ is the average ion Larmor radius and a is the transverse dimension of the plasma pinch), the magnetohydrodynamic approximation becomes invalid when the scattering velocity becomes comparable with v_d. When $v_t < v_d$, that is,

$$h/H_0 < \rho_i/a, \tag{1}$$

the magnetohydrodynamic approximation cannot be used and it is necessary to employ either the equations of two-fluid magnetohydrodynamics, if the particle collision frequency is insufficient to maintain the Maxwellian distribution, or the kinetic equations, if the plasma is rarefied. We consider here the case of a sufficiently dense plasma, although this is perfectly inessential.

We assume for simplicity that the temperature of the electrons is constant over the section of the column. In addition, we assume that h is not too small, namely

$$\frac{h}{H_0} > \frac{\rho_e}{a} = \left(\frac{m_e}{m_i}\right)^{1/2}\frac{\rho_i}{a}, \tag{2}$$

where ρ_e is the average Larmor radius of the electrons. Condition (2) is equivalent to the assumption

1)A confinement which was somewhat better than that of Bohm was observed also in [4].

that $hv_e/H_0 > v_d$, where $v_e = T/m_e$ is the average thermal velocity of the electrons. This means that the electrons have time to enter into equilibrium along the force lines of the magnetic field, that is, they satisfy the equilibrium equation

$$\nabla(nT) = -en\mathbf{E} - enc^{-1}[\mathbf{v}_e\mathbf{H}], \quad (3)^*$$

where $\mathbf{v}_e$ is the average (hydrodynamic) velocity of the electrons. Inasmuch as T = const, Eq. (3) can be represented in the form

$$\mathbf{E} = -\frac{1}{c}[\mathbf{v}_e\mathbf{H}] - \nabla\left(\frac{T\ln n}{e}\right). \quad (4)$$

Substituting this expression in Maxwell's equation, we get

$$\partial\mathbf{H}/\partial t = -c\,\mathrm{rot}\,\mathbf{E} = \mathrm{rot}\,[\mathbf{v}_e\mathbf{H}]. \quad (5)$$

It follows therefore that the magnetic field is frozen into the electrons. This means that even if a plasma that is standing still is produced at the initial instant, the frozen-in magnetic field inside the plasma will be dragged by the electrons, as shown by the dashed lines in Fig. 1, and the stretching of the force lines will set the plasma rotating at a speed on the order of $v_d \sim \rho_i v_i/a$. The angular momentum is transferred here, obviously, from the external winding. Plasma rotation of this type was considered by Thonemann and Kolb[5,6] as applied to θ-pinches. If the kinetic energy of the rotation $m_i n v_d^2/2 \sim \rho_i^2 nT/a^2$ is much smaller than the energy $h^2/8\pi$ of the transverse field, that is,

$$\beta = 8\pi nT/H^2 \ll h^2a^2/H_0^2\rho_i^2, \quad (6)$$

then the perturbation of the transverse field due to the rotation can be neglected. Inasmuch as according to (2) we have $h^2a^2/H_0^2\rho_i^2 > m_e/m_i$, the condition (6) is certainly satisfied when $\beta < m_e/m_i$.

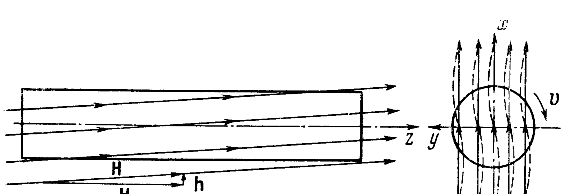

FIG. 1.

Thus, if $h/H_0 > \rho_e/a$, then the low-pressure plasma should start rotating with a velocity of the order of the drift velocity, and when $h/H_0 < \rho_i/a$ the ions will not have time to move appreciably along H during the time of one azimuthal revolution.

However, the rotation itself leads to the appearance of a centrifugal force $m_i g \sim m_i v_d^2/a \sim m_i v_i^2 \rho_i^2/a^3$. Since nothing prevents the plasma from expanding along **h** in the absence of magnetic surfaces, this centrifugal force should lead to an outward scattering of the plasma with a speed on the order of $\sqrt{ga} \sim v_i\rho_i/a$. In other words, the plasma should decay with a lifetime $\tau \sim a^2/v_i\rho_i \sim a^2 eH/cT$. Since the electrons escape in this case principally along the force lines and the ions transversely to the force lines, the entire process is similar to the diffusion of a weakly ionized plasma in a tilted metallic chamber[7], where Simon's short-circuit effect takes place.[8]

We shall present below a more detailed analysis of the stationary state of a plasma and the development of flow as a result of the centrifugal instability.

3. FLOW OF PLASMA WITH COLD IONS

An examination of the oscillations which develop as a result of the centrifugal instability is best started with consideration of the simpler case of a plasma with cold ions ($T_i = 0$). In addition, we shall consider first flows that are homogeneous along the column and then proceed to the more general case of two-dimensional flow.

We introduce a rectangular coordinate system with a z axis coinciding with the plasma-column axis and an x axis directed along h. Assuming homogeneity, that is, $\partial/\partial z = 0$, we get from the projection of the balance equation for electrons (3) on the direction of the magnetic field H

$$\frac{\partial}{\partial x} nT_e = en\frac{\partial\varphi}{\partial x}, \quad (7)$$

that is,

$$\varphi = \frac{T_e}{e}\ln n + \varphi_0(y), \quad (8)$$

where φ is the potential of the electric field: $\mathbf{E} = -\nabla\varphi$. (The electron temperature T_e is assumed constant over the cross section of the column.) For ions, we can use at $T_i = 0$ the equation

$$m_i n\frac{d\mathbf{v}}{dt} = -en\nabla\varphi + \frac{en}{c}[\mathbf{vH}], \quad (9)$$

where **v** is the ion velocity and $d\mathbf{v}/dt = \partial\mathbf{v}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{v}$.

When $h/H_0 < c_s/a\Omega_{Hi}$ where $c_s = \sqrt{T_e/m_i}$ is the speed of sound we can neglect the longitudinal ion velocity (along H), and in the same approximation we can take H in (9) to stand for H_0. In other words, in our approximation Eq. (9) describes two-dimensional flow with $v_z = 0$.

*$[\mathbf{v}_e\mathbf{H}] \equiv \mathbf{v} \times \mathbf{H}$.

If we neglect the small inertial term in the left side of (9) and take (8) into account, we obtain

$$\mathbf{v} = \frac{cT_e}{eH^2 n}[\mathbf{H}_0 \nabla n] + \mathbf{u}, \tag{10}$$

where **u** stands for the quantity $(c/eH^2)[H_0 \times \nabla\varphi_0]$. In our case, when the oscillations are homogeneous along z, the only nonvanishing component of **u** is along x, with $u_x = u_x(t, y)$. Substituting (10) into the continuity equation and recognizing that the first term in (10) does not lead to a change in density, since the corresponding current lines are on the surface n = const, we get

$$\partial n/\partial t + u_x \partial n/\partial x = 0. \tag{11}$$

We see therefore that in the stationary state $u_x = 0$, that is, the potential φ_0 should be constant.

We now turn to Eq. (9) with an inertial term, which is small enough to be able to replace **v** by (10). In order to obtain a closed equation for u_x, we must take into account the following circumstance. It follows from Eq. (3) with $\partial/\partial z = 0$ that the equilibrium along the magnetic field leads automatically to compensation of the electron pressure gradient along x by the electric field E_x, that is, to relation (7), from which it follows that $v_{ey} = 0$. In other words, the electrons can move along the axis and along the force lines, but not transversely to the magnetic surfaces, which in our case coincide with the surfaces y = const. From this, taking the quasineutrality into account and assuming that the circuit for the currents is not closed by external conductors (such as a conducting chamber), we obtain the following relation for the ions:

$$\int n v_y dx = 0, \tag{12}$$

that is, the flux of ions through any magnetic surface should also vanish.

Integrating the x-component of (9) and taking (8), (11), and (12) into account, we get

$$\int n \frac{dv_x}{dt} dx = N \frac{\partial u_x}{\partial t} + \frac{cT_e}{eH}\frac{\partial u_x}{\partial y} \int \frac{\partial n}{\partial x} dx + \frac{c^2 T_e^2}{e^2 H^2} \int \left(\frac{\partial n}{\partial y}\frac{\partial^2 n}{\partial x \partial y} - \frac{\partial n}{\partial x}\frac{\partial^2 n}{\partial y^2}\right)\frac{dx}{n} = 0, \tag{13}$$

where $N = \int n dx$ is the running density.

Let us assume that the plasma column does not touch the walls. Then

$$\int \frac{\partial n}{\partial x} dx = 0,$$

and (13) takes the form

$$N \frac{\partial u_x}{\partial t} = -\frac{c^2 T_e^2}{e^2 H^2} \int \left(\frac{\partial n}{\partial y}\frac{\partial^2 n}{\partial x \partial y} - \frac{\partial n}{\partial x}\frac{\partial^2 n}{\partial y^2}\right)\frac{dx}{n}. \tag{14}$$

In the stationary state, the integral on the right side of (14) should vanish. It is easy to see that it vanishes, in particular, for any state that is symmetrical with respect to the x plane, i.e., n(x, y) = n(−x, y). Let us assume now that a small perturbation n′ is superimposed on the symmetrical equilibrium state n, and let us consider the evolution of this perturbation in the linear approximation. Linearizing (14), and then differentiating it with respect to t and taking (11) into account, we obtain after integrating by parts

$$N \frac{\partial^2 u_x}{\partial t^2} = -\frac{c^2 T_e^2}{e^2 H^2}\frac{\partial}{\partial y}\left\{ \frac{\partial u_x}{\partial y} \int \frac{1}{n}\left(\frac{\partial n}{\partial x}\right)^2 dx \right\}. \tag{15}$$

Since this equation is of the elliptic type, it corresponds to instability, that is, to solutions that increase exponentially with time. For example, for one of the simplest distributions, $n = n_0 \exp(-r^2/a^2)$, Eq. (15) takes the form

$$\frac{\partial^2 u_x}{\partial t^2} = -\frac{2c^2 T_e^2}{e^2 H^2 a^2}\frac{\partial^2 u_x}{\partial y^2}, \tag{16}$$

from which we get, for perturbations of the type $\exp(\gamma t + ik_y y)$, a small-perturbation growth increment

$$\gamma = \sqrt{2}\frac{cT_e}{eHa} k_y. \tag{17}$$

In order of magnitude, for large-scale perturbations with $k_y \sim 1/a$, we have $\gamma \sim cT_e/eHa^2 \sim D_B/a^2$, where $D_B = (1/16)cT_e/eH$ is the Bohm diffusion coefficient.

In order to visualize more clearly the mechanism of the instability, let us consider a column deformation in which the profile n = const becomes elliptical (Fig. 2), and let us isolate an individual layer AB. We see from Fig. 2 that the curvature of the curve n = const, which coincides in first approximation with the current line, is larger at the point B than at the point A. Therefore the centrifugal force F_B at the point B exceeds the centrifugal force F_A at the point A, and consequently the layer AB becomes accelerated along the x axis, that is, the initial perturbation increases. Total cancellation of the forces applied to the given layer

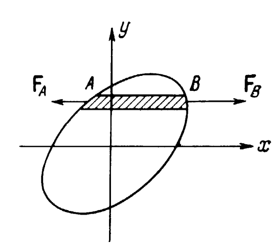

FIG. 2.

is possible only for particular types of distributions, for example for symmetrical ones $n(x, y) = n(-x, y)$. In the general case, a rotating plasma cannot be in equilibrium, and its separate layers should ''glide'' along the magnetic surfaces, that is, in our case along x. Since the characteristic velocity of such slippage is of the order of cT_e/eHa, the plasma leakage time should be of the order of $\sim eHa^2/cT_e$.

We have considered here one-dimensional ''platelike'' flows, corresponding to the slipping of plasma layers which are homogeneous along z. Flows that are periodic in z are also possible. Let us choose a certain section z = 0 and let us examine the picture of two-dimensional flow in this plane. From the equation for the equilibrium of the electrons along the magnetic field

$$T_e\left(H_0\frac{\partial n}{\partial z}+h\frac{\partial n}{\partial x}\right)=en\left(H_0\frac{\partial\varphi}{\partial z}+h\frac{\partial\varphi}{\partial x}\right) \quad (18)$$

we get

$$\varphi=\frac{T_e}{e}\ln n+\varphi_0\left(x-\frac{h}{H_0}z,y\right), \quad (19)$$

so that in the z = 0 plane the potential φ_0 can be regarded as an arbitrary function of x and y with a period $d = hL_0/H_0$ along x (L_0—length of column). Taking (19) into account we obtain from the equilibrium condition (3), accurate to terms of order h/H_0

$$\mathbf{v}_e=\frac{c}{H^2}[\mathbf{H}\nabla\varphi_0]=\frac{c}{H}\left\{-\frac{\partial\varphi_0}{\partial y},\frac{\partial\varphi_0}{\partial x},\frac{h}{H_0}\frac{\partial\varphi_0}{\partial y}\right\}. \quad (20)$$

We see that the electron velocity is directed along the equipotential surface φ_0 = const, that is, the flux of electrons through such a surface should be equal to zero. And by virtue of the quasineutrality, when the currents are not short circuited by the chamber, the flux of ions through the surface φ_0 = const must vanish. Thus, if we multiply (9) by $\mathbf{v}_e = cH^{-2}[\mathbf{H}\times\nabla\varphi_0]$ and integrate over the surface φ_0 = const, which is a cylinder of arbitrary cross section with generators along H, then the last integral will vanish, since it reduces to an integral of $n\mathbf{v}\cdot\nabla\varphi_0$. The integral of the first term in the right side of (9) also vanishes, since, when (8) is taken into account, it is the integral of a surface divergence taken over a closed surface. We thus have

$$\int_{\varphi_0=\mathrm{const}} n\left[\frac{d\mathbf{v}}{dt}\mathbf{H}\right]\nabla\varphi_0\,dS=0. \quad (21)$$

H can be taken here to mean H_0, since we neglect the longitudinal motion of the ions. Together with the continuity equation

$$\frac{\partial n}{\partial t}+\frac{c[\mathbf{H}_0\nabla\varphi_0]}{H^2}\nabla n=0$$

Eq. (21) provides a complete description of arbitrary two-dimensional flow. In the particular case when $\partial\varphi_0/\partial x = 0$, it reduces to the equation of motion (14) given above for one-dimensional flow along the x axis. If we choose φ_0 in the form $\varphi_0 = \varphi_0(t, y-\alpha(x-hz/H_0))$, then the equipotential lines in the z = 0 plane should again be straight lines, y = ax + const, but now inclined to the x axis. The flow is here again one-dimensional and represents slipping of plasma layers, while the equations of motion must have the form (11) and (14), but in a rotated system of coordinates. It follows, in particular, that stationary state takes place only in the case of axial symmetry, when the surfaces n = const represent coaxial cylinders of circular cross section.

In the general case, the flow represents convection in which the electrons move in individual tubes arranged along the force lines, as in the case when flute instability of a plasma develops in magnetic traps[9]. It can be thought that such a flow is turbulent, and in this case the small factor 1/16 in the expression for the Bohm diffusion coefficient $D_B = (1/16)cT_e/eH$ corresponds to smallness of the mixing length compared with characteristic plasma dimension a, as is the case in hydrodynamic turbulence[10].

4. CONVECTION OF PLASMA WITH HOT IONS

Inasmuch as convection due to centrifugal instability is similar, with respect to electrons, to two-dimensional convection in flute instability, it is necessary to take into account when $T_i \neq 0$ the effect of the finite Larmor radius, which leads in the case of flute instability to stabilization of small oscillations[11].

To describe this effect we can use the equations of two-fluid hydrodynamics with allowance for ''oblique'' collisionless viscosity[12]. In our case of incompressible two-dimensional flow, the equations of motion for the ions can be written in the form

$$m_i n\frac{d\mathbf{v}}{dt}-\frac{T_i}{\Omega_{Hi}H}([\mathbf{H}_0\nabla n]\nabla)\mathbf{v}-\nabla\left\{\frac{T_i n}{2\Omega_{Hi}}\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\right\}$$
$$=-\nabla nT_i-en\nabla\varphi+\frac{en}{c}[\mathbf{v}\mathbf{H}_0]. \quad (22)$$

Let us consider again one-dimensional and homogeneous flow along z. Substituting for φ in (22) the expression (8) and neglecting small terms in the left side of (22), we get

$$\mathbf{v} = \frac{c(T_e + T_i)}{eH^2 n}[\mathbf{H}_0 \nabla n] + \mathbf{u}, \qquad (23)$$

where u again has only an x component u_x that is independent of x and z.

Integrating (22) with respect to x, we obtain, taking quasineutrality into account, again an equation similar to (13):

$$\int \left(n \frac{dv_x}{dt} - \frac{cT_i}{eH^2}[\mathbf{H}_0 \nabla n] \nabla v_x \right) dx = 0. \qquad (24)$$

Substituting here the approximate value of v from (23) under the assumption that the plasma does not touch the walls, we obtain

$$N \frac{\partial u_x}{\partial t} = - \frac{c^2 T_e (T_e + T_i)}{e^2 H^2} \int \left(\frac{\partial n}{\partial y} \frac{\partial^2 n}{\partial x \partial y} - \frac{\partial n}{\partial x} \frac{\partial^2 n}{\partial y^2} \right) \frac{dx}{n}. \qquad (25)$$

This equation differs from (14) only in the fact that T_0^2 is replaced by $T_e(T_e + T_i)$.

Thus, when $T_i \neq 0$ the convection has exactly the same character as in a cold plasma, except that the flow velocity, and consequently also the coefficient of turbulent diffusion, increases by a factor $(1 + T_i/T_e)^{1/2}$, that is, $D \sim (cT_e/eH)(1 + T_i/T_e)^{1/2}$. We see that in the case of a "layered" flow along the magnetic surfaces, the effect of the finite Larmor radius not only fails to stabilize the small oscillations, but also does not change the character of the flow at all.

5. DISCUSSION OF THE GENERAL CASE

We have considered above a very simple system without magnetic surfaces that are closed inside the plasma, namely, we have assumed that the magnetic field is homogeneous. However, the assumption of the homogeneous field is utterly unrealizable, and the conclusion that convection develops in the absence of magnetic surfaces is more general. In fact, to deduce the presence of instability of the stationary state it would be important only to assume that the force lines do not remain within the plasma, that is, that there exists a certain transverse magnetic-field component penetrating through the plasma column in a transverse direction and causing it to rotate, and that the electrons have a Boltzmann distribution. The first assumption holds in any case of disrupted surfaces or surfaces that emerge from the plasma pinch to the outside, since the "disruption" is taken here to mean precisely the ability of the force lines to emerge from the plasma to the outside. As to the second assumption, it is justified in a rarefied plasma, if within the time $t \sim a^2/\rho_i v_i \sim a^2/\rho_e v_e$ of the drift oscillations the electrons have time to travel along the force lines, that is, if the length L_H of the force line prior to the emergence from the plasma does not exceed $v_0 t \sim a^2/\rho_e$:

$$L_H < a^2/\rho_e. \qquad (26)$$

This condition is equivalent to (2).

In a dense plasma there may appear, owing to the finite conductivity, an additional electric field when the electrons flow along the force lines, $E_{\parallel} \sim j_{\parallel}/\sigma$. Taking account of the fact that $j_{\parallel} \sim L_H j_{\perp}/a$, the potential of this field, $\varphi \sim E_{\parallel} L_H$, amounts to $\varphi \sim L_H^2 env_i\rho_i/a^2$ when $j_{\perp} \sim enD/a \sim en\rho_i v_i/a$. From the condition that this potential be small compared with T_e/e, we obtain the condition for establishment of a Maxwellian distribution:

$$L_H < a\sqrt{\Omega_{He}\tau_e}, \qquad (27)$$

where $\Omega_{He} = eH/m_e c$ and τ_e is the average time of electron-ion collisions.

When applied to the already performed experiments, the conditions (26) and (27) are very lenient, that is, the conditions (26) and (27) are satisfied even in the case when the force line goes through very many revolutions along the torus before emerging from the plasma.

As to the concrete details of decay of the magnetic fields, they are not significant, it merely suffices that it be possible to construct on the force lines emerging from the plasma certain magnetic surfaces along which the electron tubes could glide. (Since the ends of these tubes are situated outside the plasma, the crossing of the force lines does not change qualitatively the convection picture.)

To complete the picture, let us consider also the opposite case, when closed surfaces exist in the plasma. Since the electrons easily enter into equilibrium and assume a Boltzmann distribution, their temperature at any specified magnetic surface can be regarded as constant, and from the longitudinal component of the equilibrium condition (3) we get $\varphi = (T_e/e) \cdot \ln n + \varphi_0$, where T_e and φ_0 are functions of the magnetic surfaces only. It follows therefore that the vector $\nabla(nT_e) - en\nabla\varphi$ is perpendicular to the magnetic surface, and consequently, as seen from (3), the electron velocity component normal to the surface, v_{en}, vanishes. In other words, in the presence of a Boltzmann distribution the electrons cannot move transversely to the magnetic surfaces. It is obvious that this statement is equivalent to the freezing-in of the force lines in the electrons.

Thus, anomalous diffusion can result only from a deviation of the electrons from a Boltzmann distribution, that is, either from friction of the electrons against the ions, or from interaction with the oscillations of resonant electrons. Since all these effects are small in a high-temperature plasma, that the confinement in the presence of magnetic surfaces is expected to be much better than in accordance with Bohm's formula.

The foregoing explains one more question. As is well known, Bohm diffusion is observed only in toroidal systems, and plasma confinement is much better in open traps provided the flute instability is suppressed (see, for example,[13]). If the Bohm diffusion is connected with the disruption of the magnetic surfaces, this situation can be readily explained. In fact, if the column pinch of Fig. 1 were not a torus but a cylinder of limited length, then it could exist also without rotation. Here, as usual, a radial electric field $\mathbf{E} = \nabla p_i/en$ should be produced inside the plasma to balance the ion pressure. As to the slight disparity between the surfaces n = const and the force lines, in a very strongly elongated cylinder it would lead simply to small oscillations of the drift type, which would have the macroscopic appearance of rotation of the cylinder around a central force lines in the direction of the electron drift. There would be no real rotation of the ions in this case.

When the column is closed on itself (or made very strongly elongated) the picture changes greatly—the radial electric field $\mathbf{E} = \nabla p_i/en$ can no longer be established, for this would give rise to electron currents. The electrons freeze to the force lines, the column begins to rotate, and a centrifugal instability develops. Thus, in the absence of closed magnetic surfaces, a toroidal column differs essentially from a plasma column that is bounded on the ends.

Let us consider one more effect which can appear on going over to a geometry more complicated than in Fig. 1, namely, the effect of the minimum average magnetic field. If we add to the centrifugal force dealt with above also the force due to the diamagnetic effect, which leads to additional stabilization of the plasma, then we obtain, in order of magnitude,

$$\gamma^2 \sim \frac{v_i^2}{a^2} k^2 \rho_i^2 - \frac{v_i^2}{aR}, \qquad (28)$$

where γ is the growth increment of the small perturbations, and R is the average radius of curvature of the force lines. The first term in (28) corresponds to the centrifugal instability (see (16) and (17)), and the second takes into account the inhomogeneity of the magnetic field. It is seen from (28) that when the average magnetic field has a sufficiently deep minimum, stabilization of the long-wave perturbations takes place, but the instability remains when $k^2\rho_i^2 > a/R$. It is natural to assume that this effect decrease the effective diffusion coefficient by a factor $1/ka \sim (\rho_i/a)\sqrt{R/a}$.

6. CONCLUSION

We have shown in this paper that even a small disruption of the magnetic surfaces, such that the length of the force line L_H prior to the emergence from the plasma does not exceed (26) or (27), centrifugal instability and convection develop in the plasma; the convection leads to a leakage of the same order as the Bohm leakage, if $L_H > a^2/\rho_i$. Since the corresponding conditions can be very easily satisfied, the Bohm diffusion should be observed in a very wide range of variation of the plasma parameters. Inasmuch as it is very difficult to close the magnetic surfaces in toroidal systems of complicated geometry, preference should be given to axially-symmetrical systems such as Tokamak, Levitron, etc, where it is easier to produce closed magnetic surfaces under identical requirements with regards to the precision of winding construction.

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