VIOLATION OF T INVARIANCE IN THE RESONANCE SCATTERING OF PHOTONS BY NUCLEI

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The T-odd angular correlation in the resonance scattering of photons by polarized nuclei is calculated. The effect of an external magnetic field on this correlation is taken into account. Numerical values of the correlation coefficients are given for a large number of nuclei. The experimental data and the predictions of various hypotheses about the degree of non-conservation of T parity in processes involving hadrons are discussed. The question of using the Mössbauer effect to study the T-odd angular correlation experimentally is discussed.

1. INTRODUCTION

THE investigation of effects of nonconservation of T parity (or, what is the same thing in virtue of the CPT theorem, nonconservation of CP parity) in strong and electromagnetic interactions is now especially important owing to the experimental discovery of the decay $K_2^0 \to \pi^+\pi^-$.[1] This decay is forbidden if CP parity is conserved.

One possible explanation of the existence of the decay $K_2^0 \to \pi^+\pi^-$ is given by a hypothesis[2] according to which T parity is not conserved in hadron-hadron interactions. Another explanation is given by a hypothesis[3] which finds the cause of the decay $K_2^0 \to \pi^+\pi^-$ in a nonconservation of C parity in interactions between photons and hadrons. Since P parity is conserved in electromagnetic interactions, according to the CPT theorem a C-parity nonconserving amplitude of the photon-hadron interaction simultaneously violates T invariance. Both of these hypotheses allow the T-noninvariant amplitudes for hadron-hadron or photon-hadron interactions to be of the order of 10^{-2} to 10^{-3} of the corresponding T-invariant amplitudes.

In the present paper we treat theoretically the question of T-noninvariant angular correlations in the resonance scattering of photons by polarized nuclei. In this case the effect of T-parity nonconservation in the hadron-hadron or photon-hadron interaction manifests itself in the fact that the phase difference of the matrix elements found for mixed electric E(L + 1) and magnetic ML transitions between nuclear states is different from 0 or π.[4]

The general formulas derived in this paper can be used if one deals in particular with Mössbauer scattering as the resonance scattering of photons. Expressions are derived in this paper which take into account the effect of an external magnetic field on the angular correlation.

2. THE T-NONINVARIANT ANGULAR CORRELATION

Suppose a photon of mixed multipole character ML + E(L + 1) (L is the total angular momentum of the photon) undergoes resonance scattering by a polarized nucleus with spin j. After resonance absorption of the photon the nucleus passes from the state with spin j to an intermediate excited state with spin j_1. Then, with the emission of a photon of multipole character ML + E(L + 1), the nucleus goes into the final state with spin j. The nucleus is polarized in both the initial and the final states.

To describe the processes of absorption and emission of the mixed photon by the nucleus we introduce a complex mixing parameter δ, which is proportional to the ratio of the reduced matrix elements for the transition between the nuclear states with angular momenta j and j_1:

$$\delta = \langle j_1 | L+1 | j\rangle / \langle j_1 | L | j\rangle = |\delta| e^{i\eta}. \qquad (1)$$

Conservation of T parity is equivalent to the requirement that the mixing parameter δ be real.[4] If, on the other hand, T parity is not conserved in the strong or in the electromagnetic interactions of hadrons, the phase η is different from 0 or π.

Let us denote by $\mathbf{S}_i$ and $\mathbf{S}_j$ the average values of the spin vector of the nucleus in the initial and final states, and by n the normal to the plane of the scattering—$\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$, where $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the directions of the momenta of the incident and scattered photons. Then the expression for the T-odd angular correlation in the resonance scattering of the photons by nuclei with spin j will be

$$W = (1+|\delta|^2)^2 + \sum_g B_g{}^2 P_g(\cos\theta) + R(\mathcal{H}) + 6\left(\frac{j}{j+1}\right)^{1/2} \operatorname{Im}\delta(\mathbf{S}_i - \mathbf{S}_f)\mathbf{n} \times \sum_g \left(\frac{2g+1}{g(g+1)}\right)^{1/2} P_g'(\cos\theta) B_g F_{1g}{}^g(LL+1j_1j), \quad (2)$$

where $P_g(\cos\theta)$ is a Legendre polynomial, $P_g'(z) = dP_g(z)/dz$, and $\cos\theta$ is the cosine of the angle between the momenta of the photons. In the expression (2) we have also used the following notations:

$$F_{1g}{}^g(LL'j_1j) = (-1)^{L-1}\{(2L+1)(2L'+1)(2j+1)(2j_1+1)\}^{1/2} \times C^{g0}_{L1L'-1} X(jj1; LL'g; j_1j_1g), \quad (3)$$

$$B_g = F_g(LLjj_1) - 2\operatorname{Re}\delta F_g(LL+1jj_1) + |\delta|^2 F_g(L+1L+1jj_1), \quad (4)$$

$$F_g(LL'jj_1) = (2g+1)^{1/2} C^{L'1}_{L1g0} U(L'jgj_1; j_1L). \quad (5)$$

In Eqs. (3) and (5) C. U, and X are respectively Clebsch-Gordan, Racah, and Fano coefficients.

The summation in (2) is taken over all admissible even values of the index g, except the value g = 0. In each concrete case the number of terms in the sum (2) is determined by the properties of the functions F_g and $F_{1g}{}^g$. For some special cases these functions are tabulated in [5] and [6].

We note that in the expression (2) the symbol $R(\mathcal{H})$ stands for additional terms which arise in the angular correlation when the influence of an external magnetic field is taken into account.

3. THE EFFECT OF AN EXTERNAL MAGNETIC FIELD

The effect of an external magnetic field on angular correlations arises because under the influence of the field there are changes in the populations of the levels corresponding to different values of the spin projection μ_1 along the direction of the magnetic field for the intermediate state of the nucleus. In this case there is a change of the angular correlation of the photons if the lifetime τ of the intermediate state is comparable with or greater than the time $\tau(\mathcal{H})$ which is necessary for a change of the spin projection of the nucleus under the action of the external field $\mathcal{H}$. In general the magnetic field changes the populations of the levels not only in the intermediate state but also in the initial and final states. But nonstationary perturbations by neighboring particles lead to the establishing of an equally probable distribution over levels with different values of the spin projection in the initial and final states, if these states have large lifetimes.[5] In the case of Mössbauer scattering the initial and final nuclear states are in general stable. In this case, if the external magnetic field is independent of the time and has axial symmetry, its effect on the angular correlation reduces to the affixing of an additional factor [5]

$$\exp(i\omega_{\mu_1\mu_1'} - 1/\tau)t, \quad (6)$$

where τ is the lifetime of the intermediate state, $\omega_{\mu_1\mu_1'} = (\mu_1 - \mu_1')g\mu_N\mathcal{H}$, g is the gyromagnetic ratio, and μ_N is the nuclear magneton. The factor (6) allows for the fact that the external magnetic field removes the degeneracy of the levels with different values of the spin projection μ_1 in the intermediate state, and also the fact that the nucleus in the intermediate state has a finite lifetime τ. When there is no magnetic field this factor does not depend on the spin projection and therefore does not affect the angular correlation.

In the expression (6) t means the time interval that elapses between the acts of absorption and emission of the photon by the nucleus. It is obvious that the expression (6) must be integrated over times in the range $[0, \infty]$, and the result is

$$H(m) = (1 - ima)^{-1}, \quad (7)$$

where $a = g\mu_N\mathcal{H}\tau$, $m = \mu_1 - \mu_1'$. Confining ourselves to the first three terms in the series expansion of (7) for $ma < 1$, we have

$$H(m) = 1 + ima - (ma)^2. \quad (8)$$

The condition $ma < 1$ means that the splitting of the intermediate state in the external magnetic field is smaller than the width $\Gamma = 1/\tau$ of this level.

It follows from the experimental data that for practically attainable magnetic fields and the majority of nuclei $ma \ll 1$ ($ma \sim 10^{-2}$—10^{-5}, if there is no very large internal field acting on the nucleus), and therefore the expansion (8) is legitimate. Under this condition the additional term $R(\mathcal{H})$ in the angular correlation (2), which takes into account the effect of the external field, will be of the form

$$R(\mathcal{H}) = \sum_g B_g^2 \{a(\mathbf{n}\mathbf{n}_{\mathcal{H}}) P_g'(\cos\theta) + a^2(\mathbf{n}\,\mathbf{n}_{\mathcal{H}})^2 P_g''(\cos\theta) - a^2(\mathbf{n}_1\mathbf{n}_2) P_g'(\cos\theta)\} + 6\sqrt{\frac{j}{j+1}}\,\mathrm{Im}\,\delta\,(\mathrm{S}_i - \mathrm{S}_f)\,\mathbf{n} \times a^2 \sum_g \left(\frac{2g+1}{g(g+1)}\right)^{1/2} B_g F_{1g}^g(LL+1j_1j)\{P_g'''(\cos\theta)(\mathbf{n}\mathbf{n}_{\mathcal{H}})^2 - P_g'(\cos\theta) - 3P_g''(\cos\theta)(\mathbf{n}_1\mathbf{n}_2)\}, \quad (9)$$

where $\mathbf{n}_{\mathcal{H}}$ is the unit vector in the direction of the external magnetic field. The summation in (9) is taken over all admissible even values of g, except the value g = 0. We note that the term in (9) which depends on the polarization of the nucleus is proportional to the factor a^2, and therefore in view of what has been said earlier the effect of a magnetic field on the spin part of the angular correlation is small.

4. CONCLUSION

It follows from the expressions (3) and (5) for the functions F_g and F_{1g}^g that the angular correlation (2) for resonance scattering of photons vanishes if the spin j_1 of the intermediate state of the nucleus is equal to 0 or $1/2$.

The angular correlation (2) has been calculated for a number of particular values of the spin of the nucleus in the initial and intermediate states. Here the case considered is that which is evidently the most interesting—that in which the photons are a mixture of magnetic dipole and electric quadrupole radiation. In this calculation we have used the numerical values of the function F_g from a paper by Dolginov.[5] The numerical values of the function $F_{1g}^g(LL+1j_1j)$ were obtained by using properties of the Fano coefficients (cf. [7]).

The results of the calculation are presented in the form of a table. The first column shows the values of the spins of the initial (j), intermediate (j_1), and final (j) states of the nucleus. The other columns of the table give the numerical values of the coefficients that determine the angular correlation (2) when it is written in the form

$$W = (1+|\delta|^2)^2 + (A + B\,\mathrm{Re}\,\delta + C|\delta|^2)^2 P_2(\cos\theta) + D|\delta|^4 P_4(\cos\theta) + \mathscr{E}\,\mathrm{Im}\,\delta\{(\mathrm{S}_i - \mathrm{S}_f)\mathbf{n}\}(\mathbf{n}_1\mathbf{n}_2)(A + B\,\mathrm{Re}\,\delta + C|\delta|^2). \quad (10)$$

It follows from the expressions (2) and (10) that for the interpretation of experimental data it is necessary to know the mixing parameter. It can be determined from the data found in an angular-correlation experiment on the resonance scattering of photons by unpolarized nuclei.

We emphasize that the T-odd angular correlation (2) derived in this paper is proportional to the polarization of the nucleus in the initial state. The calculations show that there is no T-odd correlation if the initial nuclei are aligned. This assertion is also true in the general case in which the polarization state of the nucleus is described by even tensors in the expansion of its polarization density matrix.

(jj_1j)	A	B	C	D	$\mathscr{E}$
$1/2,\ 3/2,\ 1/2$	0.5	1.732	−0.5	0	$2\sqrt{3}$
$3/2,\ 5/2,\ 3/2$	0.374	1.898	−0.191	0.497	$36/5\sqrt{10}$
$5/2,\ 7/2,\ 5/2$	0.327	1.890	−0.078	0.406	$30/7\sqrt{7}$
$7/2,\ 9/2,\ 7/2$	0.303	1.870	−0.020	0.333	$\sqrt{7/2}$

In connection with Eq. (2) we point out one very important fact. An angular correlation of the type of that derived in this paper can in principle also occur when there is conservation of T parity. The cause of this is the exchange of virtual photons between the initial, intermediate, and final states of the nucleus. If the correlation that arises as the result of this exchange is of the same order of magnitude as the T-odd correlation (2), its experimental observation would still not mean that we really have to do with a nonconservation of T parity. Estimates made by Henley and Jacobsohn[8,9] have shown, however, that the terms arising from the exchange of virtual photons and leading to a correlation of the form (2) are smaller than the terms conserving (sic) T parity by a factor of about 10^{-6}.

For the experimental detection of an effect of nonconservation of temporal parity by this method it is necessary to observe the resonance scattering of γ rays by a polarized target. Optimaal conditions for the observation arise if the mixing parameter δ is close to unity. The right-left asymmetry of the scattering should be observed at an angle of deflection of the γ rays of 45° or 135°. The direction of polarization of the nucleus at which the scattering occurs must be perpendicular to the plane of the scattering, to make the asymmetry as large as possible. It is convenient to use the Mössbauer effect for the observations, measuring the difference of the counting rates for a stationary source and for a source moving with a speed v chosen so that the conditions for resonance scattering without recoil are definitely not satisfied ($v/c \gg \Gamma/E_\gamma$).

Among the nuclei suitable for such experiments, we should call attention to the nucleus Ir^{191} ($3/2$, $5/2$, $3/2$) (129 keV transition). The Mössbauer effect was first discovered with this nucleus, and the mixing coefficient (defined as the ratio of the reduced matrix elements of the multipoles E2 and M1) has been determined recently by Davydov, Tarasov, and Khrudev[10] and found to be $\delta = 0.38$. Davydov first pointed out the possibility of using this nucleus to solve the problem in question. This nucleus can be polarized, at least by the method proposed by Samoĭlov.[11] The experiment does not require observation of coincidences, and this makes it easier to interpret the results. The main difficulty in doing the experiment is that the polarized target has to be kept at a very low temperature while a rather intense beam of γ rays is striking it.

At the present time the most important experiments that verify time-reversal invariance in weak and electromagnetic interactions in the low-energy region are experiments on the decay of polarized neutrons,[12] on the decay of RaE,[13] and on the triple correlation in Rh^{106}[14] (cf. also[15]). The estimate for η that follows from all the experiments is

$$\eta \leqslant (3 \div 4) \cdot 10^{-2}.$$

If we pose the question of increasing the experimental accuracy so that one could observe $\eta \sim 10^{-3}$, preliminary calculations show that for the experiment to be done in a reasonable time the γ-ray energy that would be absorbed in the polarized target would be of the order of 800 erg/sec.

This proposed type of experiment to investigate time-parity nonconservation by means of the Mössbauer effect is obviously not the only possible one.

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