CONDITIONS FOR SELF EXCITATION OF A LASER OPERATING IN AN INHOMOGENEOUSLY BROADENED BAND

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An analysis of threshold characteristics of a laser with a tunable selective resonator is performed. The results of measurements and computation of generation frequencies and threshold pumping are given for a neodymium laser with a continuously tunable frequency within the 9434 cm^{-1} band. It is shown that the basic characteristics of stimulated emission at the threshold of generation are determined by the shape of the luminescence band of the active medium and the Q-factor curve of the selective resonator.

1. INTRODUCTION

CONTINUOUSLY tunable lasers furnish new opportunities to study the spectral properties and the energy structure of materials used as the active media. The simplest device to vary the generation frequency within the emission band is the dispersion resonator [1,2] consisting of two plane reflectors on both sides of a dispersive optical system. The Q-factor of such a resonator depends substantially upon the frequency. It reaches a maximum at a frequency at which the emission is normal to both reflectors and drops off away from that frequency. Tuning is accomplished by rotating one of the mirrors. Given a sufficiently steep response of the Q-factor, the tuning of the dispersion resonator may in principle yield generation at all lasing transitions.

Dispersion resonators with active media having wide luminescence bands (hundreds and thousands of cm^{-1}) are extremely promising with respect to continuously tunable lasers. Glass activated with neodymium ions [3,4] is suitable from this point of view. Therefore, having accomplished generation at the R_2 ruby line [5], we began to study generation of neodymium glass in a dispersion resonator. In [6] we presented the first results of the continuous tuning of the generation frequency within 500 cm^{-1} in the 9434 cm^{-1} (1.06 μ) band. This paper presents the results of theoretical and experimental investigation of threshold conditions and generation frequencies involved in tuning a dispersion resonator.

2. THEORY OF THE PROBLEM

We consider the problem of threshold pump power and emission frequency of a laser with a dispersion resonator tuned to any frequency, for active media having homogeneously and inhomogeneously broadened luminscence bands. For the sake of simplicity we limit ourselves to the case of nondegenerate working transitions. Since stimulated emission occurs when the gain of the active medium equals the losses of the resonator, it is necessary to consider the spectral response of these coefficients to find the threshold parameters of generation at various frequencies.

In the case of homogeneously broadened band the spectral dependence of the gain α has the form [7]

$$\alpha(\nu)=\frac{A_{21}}{8\pi\nu^2\mu^2(\nu)c}(n_2-n_1)g_1(\nu). \tag{1}$$

Here A_{21} is the probability of spontaneous emission of an active center in the $2\rightarrow 1$ transition, n_2 and n_1 are population densities of the upper and lower working levels respectively, ν is the light frequency in cm^{-1}, $\mu(\nu)$ is the index of refraction of the active medium, c is the velocity of light, and $g_1(\nu)$ is the form factor normalized so as to give $\int_{-\infty}^{+\infty} g_1(\nu)\,d\nu = 1$.

The luminescence intensity at a frequency ν is determined by

$$I(\nu)=A_{21}n_2g_1(\nu)=I(\nu_0)g(\nu).$$

Here $I(\nu_0)$ is the luminescence intensity at the band maximum ν_0, and $g(\nu)$ is the form factor of the luminescence band normalized to give $g(\nu_0) = 1$. Then

$$\alpha(\nu) = \frac{1}{8\pi\nu^2\mu^2(\nu)c}\frac{n_2 - n_1}{n_2} I(\nu_0) g(\nu). \quad (2)$$

It follows that for a homogeneously broadened band the spectral dependence of the gain coincides with the contour of the luminescence band if the factor $1/\nu^2\mu^2(\nu)$ depends on frequency weakly, as is often the case.

For an inhomogeneously broadened band formed by the superposition of i lines of various kinds of centers, the gain is equal to

$$\alpha(\nu) = \frac{1}{8\pi\nu^2\mu^2(\nu)c}\sum_i A_{21}{}^i (n_2{}^i - n_1{}^i) g_1{}^i(\nu), \quad (3)$$

and the luminescence intensity is written in the form

$$I(\nu) = \sum_i A_{21}{}^i n_2{}^i g_1{}^i(\nu) = I(\nu_0) g(\nu).$$

A comparison of the last two formulas shows that the frequency dependence of the gain differs in general from the shape of the luminescence band in active media operating according to the three level system with inhomogeneously broadened luminescence bands. This difference, however, can be small, in particular in the case of the four-level generation system satisfying the condition $n_2 \gg n_1$. In this case we naturally obtain the condition

$$\alpha(\nu) = \frac{1}{8\pi\nu^2\mu^2(\nu)c} I(\nu_0) g(\nu), \quad (4)$$

which implies that here, too, the spectral dependence of the gain practically coincides with the shape of inhomogeneously broadened luminescence band.

For the majority of solid-state lasers based on the four level system we can neglect the variation in the number of active centers in ground state during the pumping operation. Thus the population density of the upper working level n_2, and consequently the gain and luminescence intensity, increases linearly with pumping up to the generation threshold. Then

$$\alpha(\nu) = \frac{1}{8\pi\nu^2\mu^2(\nu)c} I'(\nu_0) g(\nu) W. \quad (5)$$

Here W is the pump intensity and $I'(\nu_0)$ luminescence intensity at band maximum for a single pump pulse. We note that (5) is valid for any type of interaction between centers (migration of energy, quenching or sensitization of luminescence, etc.), since all are automatically accounted for by the shape of band $g(\nu)$.

Let us now consider the spectral response of the loss coefficient of the dispersion resonator. If the latter is tuned to a frequency ν_t at which the emission is normal to the mirrors, then the loss coefficient $\gamma_{\nu_t}(\nu_g)$ for a generation frequency ν_g can be written in the form

$$\gamma_{\nu_t}(\nu_r) = \gamma_{\nu_t}(\nu_t)\Gamma(\nu_g, \nu_t), \quad (6)$$

where $\gamma_{\nu_t}(\nu_t)$ is the loss coefficient at the tuning frequency and $\Gamma(\nu_g, \nu_t)$ determines the increasing losses of the dispersion resonator as the generation frequency departs from the tuning frequency; $\Gamma(\nu_g \nu_t)$ is normalized so that $\Gamma = 1$ when $\nu_g = \nu_t$. The quantity $\gamma_{\nu_t}(\nu_t)$ is determined by the mirror transparency, diffraction losses, scattering, and other factors whose dispersion can be neglected within the limits of the emission band. Therefore $\gamma_{\nu_t}(\nu_t)$ is the same at all usable frequencies.

To find the function $\Gamma(\nu_g, \nu_t)$ we can make use of the fact that the effect of the dispersive prism in the case of monochromatic radiation is reduced to rotating the direction of a light wave by a so-called "deviation angle."[8] At a frequency $\nu_g \neq \nu_t$ the dispersion resonator is mismatched by an angle $\Delta\varphi$ given by

$$\Delta\varphi = (\nu_g - \nu_t) d\varphi/d\nu, \quad (7)$$

where $d\varphi/d\nu$ is the angular dispersion of the prism and is practically constant within the limits of the spectral region under investigation.

Near the minimum of the deviation angle, the beam rotation occurs without distortion of the angular scale. At the frequency ν_g therefore the losses in a resonator whose dispersive prism is set at minimum deviation coincide with the losses of a resonator with the same parameters (nonselective resonator) that is mismatched by an angle $\Delta\varphi$. The increase in losses of a nonselective mismatched resonator is practically independent of the generation frequency within the luminescence band. Therefore we can determine $\Gamma(\nu_g, \nu_t)$ by studying generation in a nonselective resonator at the frequency of the luminescence line maximum for various mismatch angles.

In the four level generation system the threshold gain, and therefore the losses, are proportional to the pump power. Then

$$\Gamma(\nu_g, \nu_t) = w_{thl}(\Delta\varphi), \quad (8)$$

where $w_{thl}(\Delta\varphi)$ is the relative generation threshold in a nonselective resonator that is mismatched by the angle $\Delta\varphi$ determined by (7).

Another method of determining $\Gamma(\nu_g, \nu_t)$ is to investigate the generation threshold in a dispersion resonator at a fixed frequency ν_g for various tuning frequencies ν_t. A selector tuned to a given frequency can be inserted into the resonator to fix the generation frequency. By retuning the selector we can determine $\Gamma(\nu_g, \nu_t)$ for all combinations of generation and tuning frequencies.

The condition for laser action at a frequency ν_g in a dispersion resonator tuned to the frequency ν_t is

$$\alpha(\nu_g) = \gamma_{\nu_t}(\nu_g). \qquad (9)$$

Substituting the obtained values of gain (5) and losses (6) and (8) into (9) we arrive at the pump power $W_{\nu_t}(\nu_g)$ required to achieve generation at the frequency ν_g:

$$W_{\nu_t}(\nu_g) = 8\pi\nu_g^2\mu^2(\nu_g)\,c\,\frac{\gamma_{\nu_t}(\nu_g)\,w_{th1}(\Delta\varphi)}{I'(\nu_0)\,g(\nu_g)}. \qquad (10)$$

It is convenient to consider quantities in relative units, normalized with respect to the case when the tuning frequency equals the generation frequency ν_0. We then obtain a general expression for the threshold pump power:

$$w_{\nu_t}(\nu_g) = w_{th1}(\Delta\varphi)/g^*(\nu_g), \qquad (11)$$

where

$$w_{\nu_t}(\nu_g) = \frac{W_{\nu_t}(\nu_g)}{W_{\nu_0}(\nu_0)}, \quad g^*(\nu_g) = \frac{\nu_0^2\mu^2(\nu_0)}{\nu_g^2\mu^2(\nu_g)}\,g(\nu_g).$$

It is clear that when the dispersion resonator is tuned to the frequency ν_t, laser action starts at the threshold pump power w_{th} and frequency ν_{th} determined by the ordinate and abscissa respectively of the minimum point on the curve $w_{\nu_t}(\nu_g)$.[1)] The set of values of w_{th} and ν_{th} for various ν_t yields the basic characteristics of a laser with a dispersion resonator, i.e., the threshold curve $w_{th}(\nu_t)$ and the resonator tuning curve $\nu_{th}(\nu_t)$.

When the dispersion resonator is tuned, the threshold generation frequency smoothly follows the tuning frequency if the frequency-dependent loss coefficient is steeper than the contour of the luminescence band. Otherwise, when the resonator is tuned to the wings of the band, the gain in the tuning frequency region falls faster than the losses increase near ν_0. Therefore the generation starts at or near the band maximum and the working frequency of the laser is not tunable.

The general conclusion of the above analysis is as follows: the threshold pump powers and generation frequencies in a laser with tunable selective resonator are determined by the shape of the luminescence band of the working transition and the Q-factor curve of the resonator. This conclusion does not extend to active media with an inhomogeneously broadened luminescence band operating according to the three level system.

3. EXPERIMENTAL RESULTS

The investigation was performed with the equipment shown schematically in Fig. 1. Samples of KGSS-7 glass were used with Nd^{3+} ion concentration of the order of 6%. Sample No. 1 was 16 mm in diameter and 240 mm long, and No. 2 was 8 mm in diameter and 80 mm long. The end-face parallelism was ~4" and the lateral surface was rough-polished. The flat dielectric mirrors of the resonator had reflection coefficients of 99.5 and 95%, constant within 0.1% between the limits of the luminescence band. The resonator length was 70 cm.

The generation frequency was tuned from R_1 to R_2 (the ruby line), a distance of merely 29 cm^{-1}, by means of a three-prism system with an angular dispersion of the order of 10 angular sec/cm^{-1}. In view of the large width of the 9434 cm^{-1} band in the investigated glasses and of the high mismatch sensitivity of the glass laser [9], a dispersion of the order of 0.4 angular sec/cm^{-1}, i.e. 25 times lower than in the case of the ruby, is quite sufficient. We used single prisms of glass with an index of refraction μ equal to 1.7 and 2.0. The refracting angle was selected so as to make the beam fall on the prism face at the Brewster angle with a minimum beam deflection. This reduced to the minimum the losses of one of the polarization components. The angular dispersion of the prism was 0.75 and 2 angular sec/cm^{-1} for μ equal to 1.7 and 2.0 respectively. Since variation in the least deflection angle did not exceed ± 15' in our case, we consider that the minimum deflection was obtained over the entire tuning range. The spectral analysis of the emission was carried out with the STÉ-1 spectrograph and an electron-optical con-

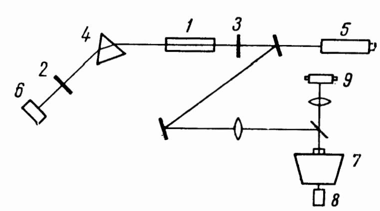

FIG. 1. Diagram of the experimental setup. 1—sample of neodymium glass; 2 and 3—flat dielectric resonator mirrors; 4—dispersion prism; 5—autocollimator; 6—photoreceptive; 7—STE-1 spectrograph; 8—camera with electron-optical converter; 9—mercury lamp.

1) The use of the functions $w_{\nu_t}(\nu_g)$ for frequencies unobtainable at the threshold is of interest in the analysis of the spectral composition of laser emission at above-threshold pump powers.

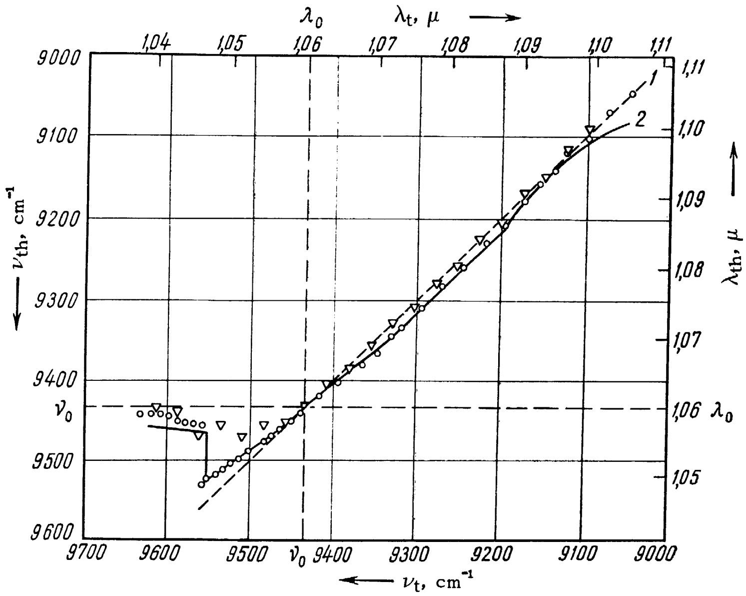

FIG. 2. Threshold generation frequency ν_{th} as a function of tuning frequency of the dispersion resonator. ▽—experimental points for sample 1, prism dispersion 2 angular sec/cm^{-1}, ○—experimental points for sample 2, prism dispersion 0.75 angular sec/cm^{-1}; curve 1—ideal tuning curve ($\nu_t = \nu_{th}$), 2—computed tuning curve for sample 2 and prism dispersion of 0.75 angular sec/cm^{-1}.

verter. The dispersion of the system (including the recording equipment) was 25 cm^{-1}/mm and the resolving power was about 0.5 cm^{-1}.

Below we give the experimental results of investigating the threshold conditions of generation for various tuning frequencies ν_t of the dispersion resonator. The value of ν_t was determined with an accuracy to 5 cm^{-1} from the angle of mirror rotation from the position at which ν_{th} = 9434 cm^{-1}. We assumed that the resonator was tuned to the band maximum when generation occurred was 9434 cm^{-1} at minimum threshold. The tuning curves were plotted for pump powers not above 1.05 times threshold for a given ν_t. In view of the resolving power used, the emission usually consisted of a single line; in many cases, however, 2—3 lines were emitted 3—10 cm^{-1} apart. In that case the average frequency of the packet was taken as the threshold generation frequency ν_{th}.

The general characteristic of the generation frequency tuning curve was the same for all specimens under investigation. Typical plots are shown in Fig. 2. According to the figure, there is a 300—400 cm^{-1} region, located mainly near the long-wave wing of the luminescence band, in which the generation frequency follows smoothly the resonator tuning. When the angular dispersion is 2 angular sec/cm^{-1} the tuning curve is close to a straight line and the generation frequency practically coincides with the tuning frequency (Fig. 2, experimental points for sample 1). When the dispersion is lower, the generation frequency is displaced towards the band maximum relative to the tuning frequency for all $\nu_t \neq \nu_0$, bending the function $\nu_{th}(\nu_t)$ (Fig. 2, experimental points for sample 2).

A different picture appears when the dispersion resonator is tuned to the short-wave wing of the 9434 cm^{-1} band. Commencing with a tuning frequency whose value depends upon the specimen and prism dispersion, the generation frequency ν_{th} no longer follows the tuning frequency ν_t but returns to the band maximum instead. Two typical plots of $\nu_{th}(\nu_t)$ can be distinguished here. In the first case (Fig. 2, experimental points for sample 1), ν_{th} lags behind ν_t beginning with ν_t = 9460 cm^{-1} and then smoothly returns to the band maximum. In the second case (Fig. 2, experimental points for sample 2), there is a smooth shift to 9560-9520 cm^{-1} followed by a backward jump of 100 cm^{-1} and stabilization of the generation frequency near the band maximum. In both cases there is a larger number of lines emitted at the threshold and a larger scatter of experimental points on the tuning curve than in the course of generation near the long-wave wing.

Some measurements of the generation threshold as a function of tuning frequency of the dispersion resonator are shown in Fig. 3.[2)] These were obtained under the same conditions as the tuning curves given in Fig. 2. As in the latter case, the threshold curves of various specimens practically

[2)] The absolute generation threshold in a dispersion resonator tuned to 9434 cm^{-1} amounted to 50-200 J for various pumping parameters and configurations and exceeded by 20-30% the generation threshold of the usual resonator of the same length. The value of the threshold pump energy was determined with an error not exceeding 3%.

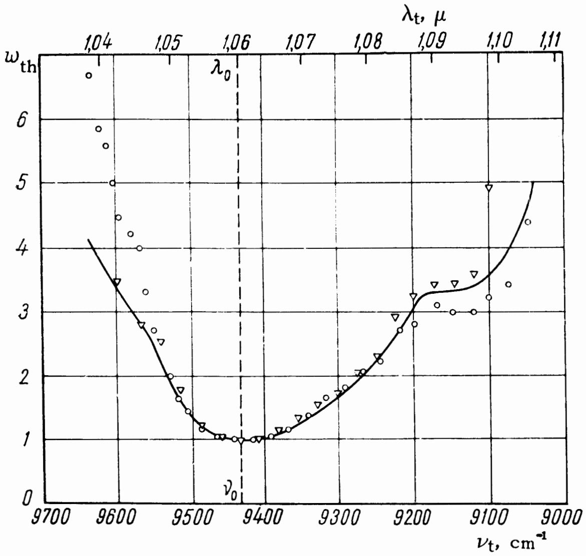

FIG. 3. Threshold pumping w_{th} as a function of tuning frequency ν_t of the dispersion resonator; ▽—experimental data for sample 1, ○—experimental data for sample 2; solid line—computed curve for sample 2.

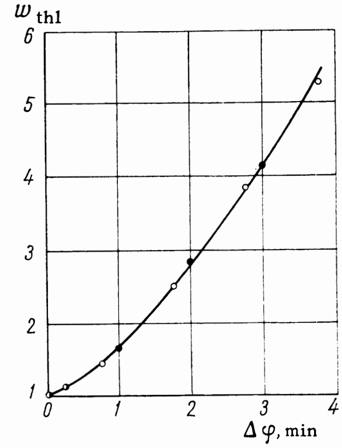

FIG. 4. Threshold pumping w_{th1} as a function of mirror angle $\Delta\varphi$ of the nonselective resonator. Resonator length, 70 cm, sample 2; ○—experimental points for mirrors with reflection coefficients $r_1 = 99.5\%$, $r_2 = 95\%$, ●—experimental points for mirrors with reflection coefficients $r_1 = 99.5\%$, $r_2 = 50\%$; ◑—coincident points for the above cases.

coincide near the maximum, except for some deviations observed in tuning to the wings of the luminescence band.

The general character of the threshold curve $w_{th}(\nu_t)$ matches the shape of a luminescence band that has a steep short-wave wing and a small maximum or plateau in the long-wave wing near 9150 cm^{-1}, due to a transition from a lower sublevel of the $^4F_{3/2}$ state to one of the sublevels of the $^4J_{11/2}$ state.

Far-field investigation of neodymium-glass generation in a dispersion resonator at various ν_t showed that in the continuous-tuning range the emission indicatrix is symmetrical and has practically the same width as in a nonselective resonator. In the tuning range where generation returns to the band maximum, the indicatrix is stretched out in the direction of resonator dispersion and has the same shape as in the case of generation in a misaligned resonator.[9,10]

The spectral dependence of the loss coefficients in a dispersion resonator was determined by measuring the threshold pumping as a function of the mirror misalignment angle in a nonselective resonator. Mode selection with respect to polarization was performed by means of a glass Brewster plate to bring the parameters of the nonselective resonator into close conformity with the characteristics of the dispersion resonator. The thickness of the Brewster plate was 10 mm which, in conjunction with an angle of incidence of ~60° and a laser beam diameter of up to 10 mm, precluded any additional mode selection in terms of frequency. Indeed, when $\mu = 1.5$, the transverse shift of the beam due to a double reflection from the plate surface is about 10 mm; this means that beams of various orders are not superimposed. Consequently, they do not interfere and there is thus no mode selection.

Laser operation in a dispersion resonator at frequencies other than ν_0 was simulated by varying the mirror transparency in the nonselective resonator and thus changing the threshold gain according to (9). Threshold curves of the nonselective resonator measured at various values of mirror transparency are given in Fig. 4. It can be seen that with different initial losses the threshold curves $w_{the}(\Delta\varphi)$ practically coincide. This confirms the feasibility of using the misalignment curves of the nonselective resonator in the analysis of the spectral dependence of the loss coefficient for all tuning frequencies.

4. DISCUSSION OF RESULTS

A comparison of theory with experiment was carried out by computing the threshold pump curves[3)] $w_\nu(\nu_g)$ for a known shape of the luminescence curve of neodymium glass and the obtained function of the angular losses. Figure 5 shows the computation results for sample 2. Points on each pump curve denote the absolute

3)The computation was based on pump energy rather than power. However, the difference in the computation results was small, since the pulse length was only weakly dependent upon the flash energy in our experiments.

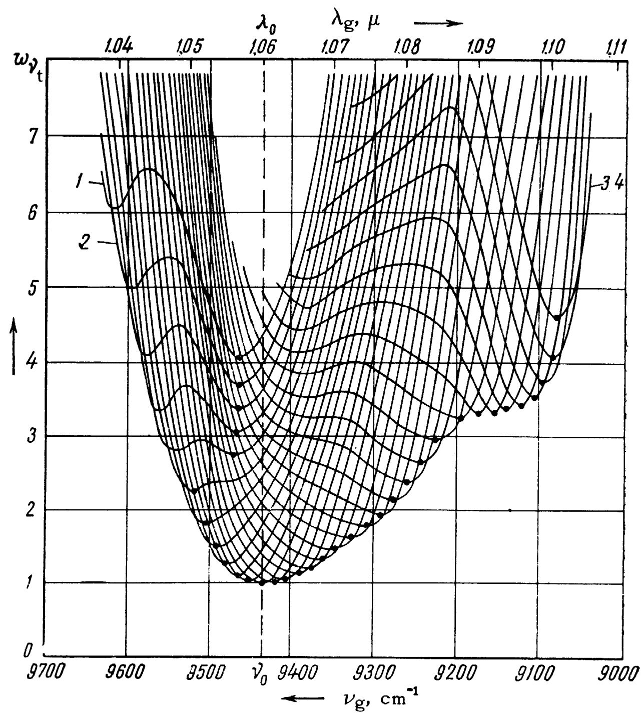

FIG. 5. Computed curves of threshold pumping w_{ν_t} as a function of generation frequency ν_g for various tuning frequencies ν_t of the dispersion resonator; $1-\nu_t = 9634\ cm^{-1}$, $2-\nu_t = 9615\ cm^{-1}$ ... $34-\nu_t = 9058\ cm^{-1}$. Sample 2, prism dispersion, 0.75 angular sec/cm^{-1}.

minimum that, as it was noted above, determines the threshold pump power w_{th} and threshold generation frequency ν_{th}.

When the dispersion resonator is tuned to the region near ν_0, the frequencies corresponding to threshold pump minima $w_{\nu t}(\nu_g)$ little differ from the corresponding tuning frequencies. When the tuning frequency is shifted towards lower frequencies an additional minimum appears in the curve $w_{\nu t}(\nu_g)$ near ν_0. Since the latter corresponds to larger threshold pump powers, generation begins at a deeper minimum near the tuning frequency. Therefore, a continuous tuning frequency shift occurs over the entire long-wave wing.[4)]

A different picture appears when the resonator is tuned to frequencies corresponding to the short-wave wing of the luminescence band. The luminescence band is steeper here than the loss coefficient at the values of angular dispersion used in this work. Thus a wide plateau appears in the computed pump curve as early as $\nu_t = 9540\ cm^{-1}$, indicating generation instability with respect to the emission frequency. A further increase in the tuning frequency deepens the minimum near ν_0 and returns the generation frequency to the band maximum.

Figures 3 and 4 show the computed tuning curves and generation thresholds together with the experimental points. The theory provides a very good explanation of the experimental results and, in particular, of the bend in the tuning curve of the resonator at low angular dispersion and the sharp jump of threshold generation frequency with a steep luminescence curve.

The computed threshold curve of the dispersion resonator also agrees fairly well with experimental data. The slight difference observed between theory and experiment in tuning in the far regions of the band wings is probably due to the approximate method of determining the losses of a dispersion resonator and the use of pump energy to define the generation threshold.

The computation of threshold generation frequencies based on (7) yields the angular misalignment of the optical resonator that, in turn, can be used to find the variation in emission divergence accompanying the tuning frequency shift.

An analysis of (11) for the threshold pump power taking account of (8) shows that when the generation frequency is close to the tuning frequency, the quantity $\Gamma(\nu_g, \nu_t)$ and the frequency dependence of threshold pumping practically coincides with the inverse curve of the luminescence band. This allows for an independent determination of the shape of the emission band of the working transition from the generation conditions.

5. CONCLUSIONS

Our research confirmed the feasibility of using dispersion resonators to study the spectral properties of active media. At the same time it is apparent that the study of threshold conditions of generation alone is insufficient to determine the origin of the shape of the luminescence band of active media. An answer to this problem and also the study of the processes of excitation energy transfer by active centers, according to preliminary experiments, requires an analysis of the evolution of the spectral composition of the emission that accompanies increased above-threshold pump powers. Furthermore, the study of generation in selective tunable resonators with several high-Q ranges should also yield valuable data.

[4)]In some cases it is possible to achieve generation also at the frequency of the second minimum near ν_0, with increased pump power. However, the interesting and important problem of the spectral composition of generation at above-threshold pump power is beyond the scope of this paper and will be discussed in a separate communication.

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