INTERACTION OF AN ELECTRON BEAM WITH A NONISOTHERMAL PLASMA AND NONLINEAR STABILIZATION OF THE TWO-STREAM INSTABILITY

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A method of stabilizing the plasma two-stream instability is proposed; this method makes use of intense ion-acoustic oscillations in the plasma. It is shown that a "pulsating" spectrum can exist in a system consisting of a beam and a nonisothermal plasma.

It is well known that in the nonlinear theory an electron beam transmitted through a plasma is unstable against excitation of longitudinal waves;[1] in turn, this instability leads to a very rapid retardation and smearing of the beam.[2] However, in a number of cases, (for example, in the gas betatron) the two-stream instability and the associated retardation and smearing of the beam are undesirable effects.1) For this reason it is of interest to investigate ways of avoiding this instability and possible means of stabilization. One such possibility based on the nonlinear interaction of plasma waves (nonlinear damping) has been noted earlier.[4] In the present note we wish to direct attention to another promising method in which the two-stream instability is stabilized by using the nonlinear interaction between the plasma waves and ion acoustic waves in a nonisothermal plasma. In addition we indicate certain new "oscillatory" regimes which can exist in a system consisting of a nonisothermal plasma and an electron beam; the conditions under which these modes appear are investigated qualitatively. In analyzing the problems indicated above the basic nonlinear processes we consider are the induced scattering of Langmuir waves (l) on plasma particles, leading to the production of ion acoustic waves (s), and vice versa.2) Using the expressions developed earlier for ls scattering[4] we can write a system of equations for the spectral energy density of the Langmuir waves $W_l(\mathbf{k})$ and the ion acoustic waves $W_s(\mathbf{k})$.3)

$$\frac{dW_l}{dt} = [\gamma_l(\mathbf{k}) - \gamma_l^{(n)}(\mathbf{k})]\, W_l,$$

$$\gamma_l^{(n)}(\mathbf{k}) = \sqrt{\frac{\pi}{2}}\,\frac{\omega_{0e}}{n_e T_e}\int d\mathbf{k}_1\, W_s(\mathbf{k}_1)\,\frac{\cos^2\widehat{\mathbf{k}\mathbf{k}_1}}{k_1 r_{De}}\exp\left[-\frac{1}{2(k_1 r_{De})^2}\right]. \quad (1)$$

$$\frac{dW_s}{dt} = -[\gamma_s(\mathbf{k}) - \gamma_s^{(n)}(\mathbf{k})]\, W_s,$$

$$\gamma_s^{(n)}(\mathbf{k}) = \left(\frac{\pi Z m}{2M}\right)^{1/2}\frac{\omega_{0e}}{n_e T_e}\,\frac{\exp[-1/2(k r_{De})^2]}{k r_{De}} \times \int d\mathbf{k}_1\, W_l(\mathbf{k}_1)\cos^2\widehat{\mathbf{k}\mathbf{k}_1}. \quad (2)$$

Here, $\gamma_{l,s}(\mathbf{k})$ is the growth rate (damping) derived from the linear theory; Z and M are the charge and mass of the ion; ω_{0e} is the plasma frequency; $r_{De} = [\omega_{0e}/v_{Te}]^{-1}$ is the Debye radius; $v_{Te} = (T_e/m)^{1/2}$ is the thermal velocity; m, T_e, and n_e are the mass, temperature and density of the electrons in the plasma and the quantities W_l and W_s are normalized so that

$$\int d\mathbf{k}\, W_l(\mathbf{k}) = U_l, \qquad \int d\mathbf{k}\, W_s(\mathbf{k}) = U_s, \quad (3)$$

where U_l and U_s are the mean energy densities of the plasma waves and the ion acoustic waves.

The last terms on the right sides of (1) and (2) are nonlinear in the energy and take account of the induced scattering of the plasma waves and the ion acoustic waves on plasma particles. Since the quantities $\gamma_l^{(n)}$ and $\gamma_s^{(n)}$ are inherently positive, it follows from (1) and (2) that taking account of the

1)The conditions for which the two-stream instability is not dangerous are treated in[3] (cf. also[1]).

2)Estimates show that other nonlinear processes, in particular the decay (addition) of plasma waves and acoustic waves, are not important under the present conditions.

3)For simplicity we limit ourselves here to a nonmagnetic weakly anisotropic plasma.

nonlinearity must always lead to stabilization of the high-frequency plasma oscillations and destabilization of the low-frequency ion acoustic oscillations.[4)]

It follows that by producing sufficiently strong ion acoustic fluctuations in the short-wave region, where $kr_{De} \gtrsim 1$, it should be possible to stabilize the most dangerous high-frequency instabilities. A sufficient condition is the inequality

$$\gamma_l(\mathbf{k}) < \sqrt{\frac{\pi}{2}}\frac{\omega_{0e}}{n_e T_e}\int d\mathbf{k}_1\, W_s(\mathbf{k}_1)\frac{\cos^2\widehat{\mathbf{k}\mathbf{k}_1}}{k_1 r_{De}}\exp\left[-\frac{1}{2(k_1 r_{De})^2}\right]. \quad (4)$$

Here, the quantity n_1 denotes the number density of particles in the beam, v_0 is the mean velocity of these particles, Δv is the velocity spread in the beam, and it is assumed that the maximum value of the growth rate γ_l^{max} is of order

$$\frac{n_1}{n_e}\left(\frac{v_0}{\Delta v}\right)^2\omega_{0e},$$

in which case the instability condition (4) can be replaced by the simpler relation:

$$\frac{\gamma_l^{max}}{\omega_{0e}} \approx \frac{n_1}{n_e}\left(\frac{v_0}{\Delta v}\right)^2 < \frac{1}{2}\frac{U_s^{(1)}}{n_e(T_eT_i)^{1/2}}, \quad (4')$$

where

$$U_s^{(1)} = \int_{r_{De}^{-1}}^{r_{Di}^{-1}} k^2\, dk \int d\Omega\, W_s(\mathbf{k})$$

is the energy density of the ion acoustic fluctuations in the wave number range $r_{De}^{-1} \lesssim k \lesssim r_{Di}^{-1}$; T_i and $r_{Di} = (T_i/ZT_e)^{1/2} r_{De}$ are the temperature and Debye radius of the plasma ions. It is then evident that if the noise is not too dense the stability criterion is a relatively weak one.

We note, in conclusion, that in certain cases the stabilization mechanism indicated above can evidently arise automatically in a plasma in an external electric field because this situation provides a mechanism for continuous acceleration of a small group of "runaway" electrons.

We now consider the excitation of oscillations in a system consisting of a beam plus a plasma in which the parameters are such that the system is stable against the excitation of ion acoustic waves in the linear approximation, but unstable against the excitation of plasma waves. In other words, we consider the solution of the system consisting of (1) and (2) when the relation $\gamma_l > 0$ holds in some region of wave numbers while the quantity γ_s is always positive. We shall not try to find a rigorous solution of (1) and (2), but shall limit ourselves to a qualitative analysis.

We denote by γ_l^{max} and γ_s^{min} the maximum value of the growth rate γ_l and the damping γ_s at the point at which the product $kr_{De}\gamma_s(k) \times \exp[-1/2(kr_{De})^2]$ is a minimum. Analysis of (1) and (2) yields the following qualitative pattern of excitation by an electron beam in a nonisothermal plasma. In passing through the plasma the beam first excites plasma waves, whose amplitudes increase rapidly. As a result of nonlinear effects there follows a reduction of the effective damping of the ion-acoustic waves; then, when the intensity of the plasma waves becomes large enough (so that $\gamma_s^{(n)} > \gamma_s^{min}$) the ion-acoustic waves are excited. On the other hand, the increasing amplitude of the acoustic waves leads to a retardation of the growth of the plasma waves so that (when $\gamma_l^{(n)} > \gamma_l^{max}$) the plasma waves are damped. In turn, however, the reduction of the amplitude of the plasma waves retards the growth of the ion acoustic waves, which reach some maximum intensity, and then are also damped, approaching their initial value. The process then repeats itself.

Thus, the energy densities of the plasma waves U_l and the acoustic waves U_s (which depend on the time in the case of an initial value problem, or on the spatial coordinate in the case of a stationary boundary-value problem) become bounded functions which oscillate about certain mean values $\overline{U}_l$ and $\overline{U}_s$ with a characteristic time T_0 (or spatial period $L_0 \approx (v_l v_s)^{1/2} T_0$ where v_l and v_s are the group velocities of the plasma waves and the acoustic waves). The values of the quantities $\overline{U}_l$ and $\overline{U}_s$ can be estimated and are of order

$$\overline{U}_l \approx \frac{\gamma_s^{min}}{\omega_{0e}}\left(\frac{M}{m}\right)^{1/2} n_e T_e, \qquad \overline{U}_s \approx \frac{\gamma_l^{max}}{\omega_{0e}} n_e T_e. \quad (5)$$

The order of the characteristic pulsation time T_0 (or the period L_0) is

$$T_0 \approx [\gamma_s^{min}\gamma_l^{max}]^{1/2}. \quad (6)$$

Thus, we see that taking account of the nonlinear interactions leads to a bound on the growth of

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[4)] This is a very general consequence of the original nonlinear equations for a weakly anisotropic plasma in which the scattering processes always lead to the transfer of energy from high-frequency plasmons to lower frequency plasmons.[4] It should be noted, however, that this statement does not hold[5] in the general case of a plasma that is not weakly anisotropic (in particular, a system of two interacting plasmas).

the plasma waves and to the establishment of stationary pulsating spectra.[5)]

One further remark should be made at this point. In analyzing (1) and (2) we have neglected completely the interaction of the beam with the plasma waves excited by the beam and the reduction in the growth rate γ_l caused by this interaction. Obviously we can neglect this interaction only when the characteristic time for the smearing of the beam T_D (or the characteristic length $L_D \approx v_0 T_D$) is much larger than the pulsation time T_0 (or the pulsation period L_0). If this is not the case, so that $T_D \ll T_0$ (or $L_D \ll L_0$) because of the rapid smearing of the beam the nonlinear effects treated here are not important and the development of the instability (in any case the initial stage) can be described within the framework of the quasilinear theory.[2]

Assuming that the time $T_D \approx n_e m (\Delta v)^3/(\omega_{0e} v_0 U_l)$ and taking account of (5) we can write the condition for the existence of the pulsating solutions in the form

$$\frac{(\Delta v)^3}{v_0 v_{Te}^2} \gg \left[\frac{M\gamma_s^{min}}{m\gamma_l^{max}}\right]^{1/2} \quad (T_D \gg T_0) \qquad (7)$$

for the initial value problem and

$$\frac{(\Delta v)^3}{v_{Te}^3} \gg \left[\sqrt{\frac{M}{m}}\,\frac{v_{Te}}{v_0}\,\frac{\gamma_s^{min}}{\gamma_l^{max}}\right]^{1/2} \quad (L_D \gg L_0) \qquad (8)$$

for the stationary boundary-value problem.

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5)We note that static solutions are excluded. The condition that must be satisfied for static solutions to exist is $\gamma_{l,s}(\mathbf{k}) - \gamma_{l,s}^{(n)}\mathbf{k} \equiv 0$. It is evident that this situation cannot hold [cf. the definition of $\gamma_{sl}^{(n)}(\mathbf{k})$].

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