CONCERNING THE PROBLEM OF THE EFFECT OF PRESSURE ON THE TEMPERATURE OF THE SUPERCONDUCTING TRANSITION

V. G. BAR'YAKHTAR and V. I. MAKAROV

Physico-technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 20, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 1934–1937 (December, 1965)

Expressions for the derivatives of the superconducting transition temperature T_k and the critical magnetic field strength H_k with respect to the pressure are obtained in terms of the experimentally obtained quantities, i.e., the electron and lattice compressibility and the Grüneisen constants. The derived formulas describe satisfactorily the magnitude and sign of the derivatives $\partial T_k/\partial P$, $\partial H_k/\partial P$ for most of the pure superconductors.

1. A large amount of data is at present available on the effect of the pressure on the superconducting transition temperature.[1] In a number of superconductors the superconducting transition temperature T_k increases under pressure, in others it decreases linearly. There is however no quantitative explanation of these experimental data on the basis of contemporary superconductivity theory.

In this paper, starting from the well-known expression for the temperature of the superconducting transition, an expression is found for the derivative of T_k with respect to the pressure in terms of experimentally observable quantities: the Grüneisen constants of the electrons and of the lattice, and the compressibility of the metal. Knowing these quantities, it is possible to calculate the derivative $\partial T_k/\partial P \equiv T_{k,P}$ and to compare the value of $T_{k,P}$ thus obtained with its experimentally determined value. Satisfactory agreement is obtained between the calculated and experimentally determined values of $T_{k,P}$ (see the Table).

2. The temperature of the superconducting transition is determined by the formula[2]:

$$T_k = 1.14\omega_D \exp\left(-\frac{1}{F}\right), \quad F = \frac{8}{9}\frac{C^2}{Ms^2}\frac{Vp_F^2}{2\pi^2 N_0 v_F}, \tag{1}$$

where ω_D is the Debye temperature, C the constant of electron-phonon interaction,[3] N_0 the number of atoms in the solid, V the volume of the solid, M the mass of the ion, s the speed of sound, and p_F and v_F the momentum and velocity of the electron. The electron-phonon interaction constant can be expressed in terms of the number of atoms per unit volume (N_0/V) and the screening radius r_0[4]:

$$C = \frac{3N_0}{2V} e^2 r_0^2. \tag{2}$$

For the case of a quadratic dispersion law of the conduction electrons the screening radius is given by the formula[5]

$$r_0^{-2} = \frac{4e^2 p_F^2}{\pi v_F}. \tag{3}$$

Superconductor	$\varkappa \cdot 10^6$ atm^{-1} [6]	γ_p [1]	γ_e	F [7]	T_k, °K [1]	$(\partial T_k/\partial P)\cdot 10^5$ deg/atm, experiment	$(\partial T_k/\partial P)\cdot 10^5$ deg/atm, theory	H_k [1]	$(\partial H_k/\partial P)_{T=0}\cdot 10^3$ Oe/atm, experiment [1]	$(\partial H_k/\partial P)_{T=0}\cdot 10^3$ Oe/atm, theory
Al	1.34	2.8 [13]	1.7 [13]	0.193	1.19	−2.0 [8]	−2.1	99	−3.0	−1.82
In	2.55	2.4		0.395	3.407	−4.4 [1]	−4.5	283	−3.6	−3.94
Pb	2.37	2.8	1.7 [12]	0.493	7.19	−4.5 [1]	−6.0	803	−7.9	−8.0
Hg_α	4.0	2.2		0.446	4.153	−3.6 [1]	−4.75	411	−7.2	−5.1
Sn	1.87	2.4	1.1 [11]	0.296	3.733	−4.6 [1]	−5.4	306	−5.0	−4.8
Cd	2.2	2.2		0.196	0.54	−1.8	−1.1	28	−1.0	−0.56
Zn	1.7	2.0		0.200	0.91		−1.05	53	−1.4	−0.67
La_β	3.55	0.8		0.370	5.95	+14.0 [9]	+4.75	1600	+5.5	+11.2
Nb	0.575 [10]	1.35	1.5 [10]	0.357	9.22		−0.1	1944	−1.2	−0.44
Ta	0.5 [10]	1.5	1.3 [10]	0.296	4.46	−0.26	−0.45	830	−1.05	−0.94
V	0.636 [10]	0.9	1.65 [10]	0.274	5.30		0.97	1310	+1.3	2.0
Zr	1.04	1.05		0.178	0.56	+0.9– 1.4 [1]	0.1	47		+0.1
Mo	0.36	1.0		0.18	0.91	+0.1± ±0.1 [1]	0.04	86		+0.1

Introducing the density of states per unit energy interval,

$$\nu(\varepsilon_F) = V p_F^2 / 2\pi^2 v_F$$

and using formulas (2) and (3), we express F in the form

$$F = \frac{1}{2(4\pi)^2} \frac{N_0}{Ms^2 \nu(\varepsilon_F)}$$

In this formula it is convenient to go over from the speed of sound s to the Debye temperature $\omega_D = \pi s/a$, where a is the lattice constant equal to $(V/N_0)^{1/3}$; then the expression for T_k can be rewritten in the form

$$T_k = 1.14\omega_D \exp\left\{-\left(\frac{N_0}{V}\right)^{2/3} \frac{N_0}{32\nu(\varepsilon_F) M\omega_D^2}\right\}^{-1}, \quad (4)$$

whence

$$\frac{\partial T_k}{\partial P} = T_k \varkappa \left\{\gamma_p \left(1 - \frac{2}{F}\right) + \frac{\gamma_e}{F} + \frac{2}{3F}\right\}, \quad (5)$$

where we have introduced the notation

$$\varkappa\gamma_p = \frac{1}{\omega_D} \frac{\partial \omega_D}{\partial P},$$

$$\varkappa\gamma_e = -\frac{1}{\nu(\varepsilon_F)} \frac{\partial}{\partial P} \nu(\varepsilon_F), \quad \varkappa = -\frac{1}{V} \frac{\partial}{\partial P} V. \quad (5')$$

The quantities γ_p and γ_e are the Grüneisen constants for the lattice and for the electrons and κ is the compressibility.

Assuming that formula (5) is correct not only for a quadratic dispersion law, but also for an arbitrary law $\epsilon(p)$, we compare the theory and experiment.

The experimental values of the quantities γ_p and γ_e and κ known at present are presented in the Table for a number of metals. Using these values it is easy to calculate $T_{k,P}$ from formula (5). It can be seen from the Table that the experimentally observed values of $T_{k,P}$ and those calculated according to formula (5) are generally in good agreement.

In those instances in which the Grüneisen constant γ_e for electrons is not known, we assumed that $\gamma_e = 1.5$ (this corresponds approximately to the average of the known values). The average value of γ_e yields the correct sign of the derivative of the temperature of the superconducting transition with respect to the pressure.

The calculated values of $T_{k,P}$ for all metals differ nevertheless from the experimentally found values of $T_{k,P}$. In principle this could be related to the fact that the constant γ_e has not been determined sufficiently well, for an error in γ_e (or γ_p) of 10—20 per cent leads to an error of 50—100 per cent in the calculated value of $T_{k,P}$ because of the presence of the large parameter F^{-1}.

Analyzing the data presented in the Table (the seventh and eighth column) one can assume that the Grüneisen constant γ_e for the metals In, Hg, Zn, Cd, Zr, Mo, and La_β can differ from $\gamma_e = 1.5$ by no more than ±(0.1—0.2).

3. The value of the critical magnetic field in the superconductor at temperatures T much smaller than T_k is connected with the superconducting transition temperature by the relation

$$H_k(0) = 1.75[4\pi\nu(\varepsilon_F)/V]^{1/2} T_k. \quad (6)$$

Differentiating this expression with respect to the pressure, one can express the derivative $H_{k,P} \equiv \partial H_k(0)/\partial P$, as was done for $T_{k,P}$, in terms of the Grüneisen constants γ_e and γ_p and the compressibility κ:

$$\frac{\partial H_k}{\partial P} = \varkappa H_k \left\{\left(1 - \frac{2}{F}\right)\left(\gamma_p - \frac{1}{2}\gamma_e\right) + \frac{2}{3}\frac{1}{F} + \frac{1}{2}\right\}. \quad (7)$$

On the basis of this formula one can calculate the magnitude and the sign of $H_{k,P}$ for various metals and compare it with the experimentally found values of $H_{k,P}$ (see the Table). As can be seen from the Table, the agreement for $H_{k,P}$, just as for $T_{k,P}$, between the calculated and experimentally determined values is not bad.

The values of $T_{k,P}$ and $H_{k,P}$ are determined for 19 pure metals. For Ga, Re, Ru, Th, and Ti γ_e and γ_p have not been determined and therefore a comparison with the theory is difficult.

For Tl, if we assume $\gamma_e \approx 1.5$, then according to the theory $T_{k,P} = -5 \times 10^{-5}$ deg/atm; the experimentally determined value $T_{k,P}$ is -1.4×10^{-5} deg/atm at 20,000—28,000 atm. The thallium anomalies in the pressure range up to 6000 atm are apparently connected with features of the energy spectrum of the conduction electrons.[14,15]

The authors thank B. G. Lazarev for a discussion of the work.

[1] N. B. Brandt and N. I. Ginzburg, UFN 85, 485 (1965), Soviet Phys. Uspekhi 8, 224 (1965).

[2] J. Bardeen, L. Cooper, and J. Schrieffer, Phys. Rev. 108, 1175 (1957).

[3] H. Bethe and A. Sommerfeld, The Electron Theory of Metals (Russ. Transl.) ONTI, 1938.

[4] V. I. Bonch-Bruevich, DAN SSSR **124**, 1233 (1959), Soviet Phys. Doklady **4**, 172 (1959).

[5] V. L. Bonch-Bruevich and S. V. Tyablikov, Metod funktsiĭ Grina v statisticheskoĭ mekhanike (The Green's Function Method in Statistical Mechanics), Fizmatgiz, 1961.

[6] P. W. Bridgman, The Physics of High Pressure, Bell, London, 1949.

[7] D. Pines, Phys. Rev. **109**, 280 (1958).
[8] N. Muench, Phys. Rev. **99**, 1814 (1955).
[9] W. E. Gardnen and T. F. Smith, Phys. Rev. **138**, A484 (1965).
[10] G. K. White, Cryogenics **3**, 292 (1962).
[11] G. K. White, Phys. Letters **8**, 294 (1964).
[12] G. K. White, Phil. Mag. **7**, 271 (1962).
[13] G. K. White, Phil. Mag. **6**, 815 (1961).
[14] V. I. Makarov and V. G. Bar'yakhtar, JETP **48**, 1717 (1965), Soviet Phys. JETP **21**, 1151 (1965).
[15] B. G. Lazarev, L. S. Lazareva, V. I. Makarov, and T. A. Ignat'eva, JETP **48**, 1065 (1965), Soviet Phys. JETP **21**, 711 (1965).

Translated by Z. Barnea
248