QUADRUPOLE AND NUCLEAR SPIN INTERACTION VIA THE OPTICAL PHONON VIRTUAL FIELD

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Interaction of nuclear spins and energy migration over orbital levels via the optical phonon virtual field is considered. The mechanism is due to interaction between quadrupole moments (nuclei and electron shells) and the electric field gradient of the optical oscillations. For distances r_{ij} between quadrupoles of the order of the lattice parameters, the cross relaxation transition probability depends on r_{ij} as $1/r_{ij}^{10}$; for distances considerably exceeding the lattice constant it varies as $1/r_{ij}^4$ and $1/r_{ij}^8$. Numerical estimates show that processes of this type are quite effective.

As a result of the fact that neighboring ionic-crystal particles executing optical oscillations are displaced in opposite directions, such oscillations are accompanied by appreciable electromagnetic fields, and the motion of the atoms in such lattices is described by a compatible system made up of the equations of elasticity theory and Maxwell's equations [1,2] The interaction between the multipole moment of the particle and these fields can lead to a number of interesting phenomena. The influence of the interaction between the electric dipole moment operator and the electric field of the optical oscillation on the energy migration over the orbital states of the impurity atoms has been discussed previously [3]; the relaxation of the electronic and nuclear spins, due to interaction between the magnetic moments and the magnetic field, was considered in [4,5].

In this article we discuss the influence of the interaction between the quadrupole moments (of the nuclei and electron shells) and the gradient of the electric field of the optical oscillations on the cross relaxation transitions in a system of nuclear spins and orbital levels of impurity particles. We note that the possibility of interaction between nuclear quadrupoles via the phonon field is pointed out in a paper by Bashkirov and Kopvillem [6].

The action of the discussed mechanism can be intuitively visualized as follows: the particle i, interacting via the quadrupole moment with the gradient of the electric field of the optical oscillations, goes over from the excited state to the ground state and gives rise to a virtual phonon, while the particle j, absorbing this phonon, goes over from the ground state to an excited state. Since the phonons play in this case the role of virtual particles, the entire spectrum of the optical oscillations participates in this process, and the probabilities of the corresponding processes do not depend on the temperature.

We start our analysis of the mechanism in question with a derivation of expressions for the electric field gradient. We direct the z axis along the external magnetic field H. The perturbation Hamiltonian, describing the interaction between the quadrupole moment of the nucleus i and the electric field of the longitudinal ($\mathbf{E}^{\parallel}$) and transverse ($\mathbf{E}^{\perp}$) optical oscillations of the lattice, is written in the form of a scalar product of two tensors—the operator of the quadrupole moment of the nucleus, and the electric field gradient tensor [7]:

$$\mathcal{H}_{\text{nuc}} = \sum_{\mu} (-1)^{\mu} Q_{\mu} (\nabla \mathbf{E}_i^{\parallel, \perp})_{-\mu}, \qquad (1)$$

$$Q_0 = \frac{eQ}{2I(2I-1)} [3\hat{\mathbf{I}}_z^2 - I(I+1)],$$

$$Q_{\pm 1} = \mp \sqrt{\frac{3}{2}} \frac{eQ}{2I(2I-1)} [(\hat{\mathbf{I}}_x \pm i\hat{\mathbf{I}}_y)\hat{\mathbf{I}}_z + \hat{\mathbf{I}}_z(\hat{\mathbf{I}}_x \pm i\hat{\mathbf{I}}_y)], \qquad (2)$$

$$Q_{\pm 2} = \sqrt{\frac{3}{2}} \frac{eQ}{2I(2I-1)} (\hat{\mathbf{I}}_x \pm i\hat{\mathbf{I}}_y)^2,$$

$$(\nabla \mathbf{E}_i^{\parallel, \perp})_{\mu} = (1-\gamma_\infty)(\nabla \mathbf{E}_{1i}^{\parallel, \perp})_{\mu}, \qquad (3)$$

where γ_∞ is a coefficient characterizing the degree of polarization of the electron shell of the ion in question (the anti-screening parameter), and $\nabla \mathbf{E}_{1i}^{\parallel,\perp}$ is the gradient of the electric field

($E_i^{\parallel}$, $E_i^{\perp}$) on the ion in question; eQ is the quadrupole moment of the nucleus, while I, $\hat{I}$, $\hat{I}_s$ (s = x, y, z) are respectively the spin of the i-th nucleus, the spin operator, and the spin projection operator of the i-th nucleus.

In crystals of the NaCl and CsCl type, $E^{\parallel}$ and $E^{\perp}$ can be determined from the formulas

$$\mathbf{E}_{1i}^{\parallel} = R_{\parallel}\mathbf{P}(\mathbf{r}_i),$$

$$\mathbf{P}(\mathbf{r}_i) = \frac{3}{\pi}B_{\parallel}\sum_{\mathbf{q}}\omega_{\mathbf{q}}^{-1/2}\mathbf{q}^0\left(a_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}_i} + a_{\mathbf{q}}^{+}e^{-i\mathbf{q}\mathbf{r}_i}\right),$$

$$R_{\parallel} = \frac{\pi}{3}b\left[1 - \frac{\pi}{3}b\frac{(\alpha_- + \alpha_+)}{v_a}\right]^{-1},$$

$$b = \left[1 - \frac{4\pi}{3}\frac{(\alpha_- + \alpha_+)}{v_a}\right]^{-1},$$

$$B_{\parallel} = \frac{\pi}{3}\frac{Ze}{v_a}\left(\frac{\hbar}{N}\frac{m_- + m_+}{2m_- m_+}\right)^{1/2}, \tag{4}$$

where $\mathbf{r}_i$ is the radius vector of the position of the i-th nucleus, $q = q^0|q|$, α_- and α_+ are the polarizabilities of the negative and positive ions (atoms) of the lattice, m_- and m_+ are the masses of the negative and positive ions of the lattice, a_q^+ and a_q are the operators of creation and annihilation of a normal oscillation with wave vector **q**, N is the number of unit cells in the sample, v_a is the volume of the unit cell, ω_q is the frequency of the phonon with wave vector **q**, e the elementary charge, and Ze the charge of the ion.

For transverse oscillations of the optical branch we have

$$\mathbf{E}_{1i}^{\perp} = \sum_{\mathbf{q},j}R_{\mathbf{q}_j}\mathbf{P}_{\mathbf{q}_j}(\mathbf{r}_i),$$

$$R_{\mathbf{q}_j} = (b_{\mathbf{q}_j} + 4\pi/3)\left[1 - \frac{(b_{\mathbf{q}_j} + 4\pi/3)(\alpha_- + \alpha_+)}{v_a}\right]^{-1},$$

$$b_{\mathbf{q}_j} = 4\pi\omega_{\mathbf{q}_j}^2(c^2\mathbf{q}_j^2 - \omega_{\mathbf{q}_j}^2)^{-1}, \qquad B_{\perp} = 3B_{\parallel}/\pi,$$

$$\mathbf{P}_{\mathbf{q}_j} = B_{\perp}\omega_{\mathbf{q}_j}^{-1/2}\mathbf{i}_j\left(a_{\mathbf{q}_j}e^{i\mathbf{q}\mathbf{r}_i} + a_{\mathbf{q}_j}^{+}e^{-i\mathbf{q}\mathbf{r}_i}\right), \tag{5}$$

where ω_{qIj} is the frequency corresponding to the wave vector **q** of polarization j, c is the speed of light, and $\mathbf{i}_j$ is the unit polarization vector of the normal oscillation, $\mathbf{i}_j \perp \mathbf{q}$.

Let us consider the case of pure ionic crystals and let us neglect polarization of electron shells of the lattice ion (the electron shell of the i-th nucleus). We can then express $(\nabla\mathbf{E}_{1i}^{\parallel,\perp})_{\mu}$ for longitudinal oscillations in the form

$$(\nabla\mathbf{E}_{1i}^{\parallel})_0 = -\frac{i}{2}\sum_{\mathbf{q}}F_{\parallel}(\mathbf{q},\mathbf{r}_i)(\mathbf{q}_z^0)^2,$$

$$(\nabla\mathbf{E}_{1i}^{\parallel})_{\pm1} = \pm\frac{1}{\sqrt{6}}\sum_{\mathbf{q}}F_{\parallel}(\mathbf{q},\mathbf{r}_i)(\mathbf{q}_x^0\mathbf{q}_z^0 \pm i\mathbf{q}_y^0\mathbf{q}_z^0),$$

$$(\nabla\mathbf{E}_{1i}^{\parallel})_{\pm2} = \frac{1}{\sqrt{6}}\sum_{\mathbf{q}}F_{\parallel}(\mathbf{q},\mathbf{r}_i)[(\mathbf{q}_x^0)^2 - (\mathbf{q}_y^0)^2 \pm 2i\mathbf{q}_x^0\mathbf{q}_y^0], \tag{6}$$

where q_l^0 (l = x, y, z) is the l-component of $\mathbf{q}^0$,

$$F_{\parallel}(\mathbf{q},\mathbf{r}_i) = B_{\parallel}\omega_{\mathbf{q}}^{-1/2}q\left(a_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}_i} - a_{\mathbf{q}}^{+}e^{-i\mathbf{q}\mathbf{r}_i}\right).$$

For transverse oscillations

$$(\nabla\mathbf{E}_{1i}^{\perp})_0 = -\sum_{\mathbf{q},j}F_{\perp}(\mathbf{q}_j,\mathbf{r}_i)[(\mathbf{i}_j)_z\mathbf{q}_z^0],$$

$$(\nabla\mathbf{E}_{1i}^{\perp})_{\pm1} = \pm\sqrt{2/3}\sum_{\mathbf{q},j}F_{\perp}(\mathbf{q}_j,\mathbf{r}_i)[(\mathbf{i}_j)_x \pm i(\mathbf{i}_j)_y]\mathbf{q}_z^0,$$

$$(\nabla\mathbf{E}_{1i}^{\perp})_{\pm2} = \sqrt{2/3}\sum_{\mathbf{q},j}F_{\perp}(\mathbf{q}_j,\mathbf{r}_i) \times [(\mathbf{i}_j)_x\mathbf{q}_x^0 - (\mathbf{i}_j)_y\mathbf{q}_y^0 \pm 2i(\mathbf{i}_j)_x\mathbf{q}_y^0],$$

$$F_{\perp} = 2\pi B_{\perp}\frac{(2\omega_{\mathbf{q}_j}^2 + c^2q_j^2)\,q_j\omega_{\mathbf{q}_j}^{-1/2}}{c^2q_j^2 - \omega_{\mathbf{q}_j}^2}\left(a_{\mathbf{q}_j}e^{i\mathbf{q}_j\mathbf{r}_i} - a_{\mathbf{q}_j}^{+}e^{-i\mathbf{q}_j\mathbf{r}_i}\right). \tag{7}$$

Further, using the usual formalism of the evolution operator [8], we can show that the probability per unit time of the process in which the nucleus i goes over from the state with magnetic quantum number m + 1 to the state with quantum number m with creation of a virtual phonon, while nucleus j goes over from m to m − 1 with absorption of the same phonon, is calculated by means of the following formulas:

1. Longitudinal oscillations:

$$W_{ij}^{\parallel}(m+1) = (1-\gamma_\infty)^4F_1^{\parallel}\frac{1}{2\pi a^2\omega_{\parallel}^4r_{ij}^4}\left[\sum_{n=0}^{\infty}(-1)^n\left(\frac{\pi}{a}\right)^{2n} \times \frac{r_{ij}^{2n}}{(2n+1)!}\left(\frac{\pi}{a^2}(n-3) - \frac{2[13n(2n+1)-6]}{r_{ij}^2(2n+1)}\right)\right]^2 \times \int_0^\infty g_{m+1}^2(\omega)\,d\omega,$$

$$F_1^{\parallel} = B_{\parallel}^4V^2(eQ)^4(2m+1)^4 \times [(I+m+1)(I-m)]^2/[\hbar 2I(2I-1)]^4; \tag{8}$$

$$W_{ij}^{\parallel}(m+2) = (1-\gamma_\infty)^4F_2^{\parallel}\frac{32}{2\pi a^2\omega_{\parallel}^4r_{ij}^8} \times \left[\sum_{n=0}^{\infty}(-1)^n\left(\frac{\pi r_{ij}}{a}\right)^{2n}\frac{(5n+1)}{(2n+1)!}\right]^2\int_0^\infty g_{m+2}^2(\omega)\,d\omega,$$

$$F_2^{\parallel} = B_{\parallel}^4V^2(eQ)^4 \times \frac{[(I+m+2)(I+m+1)(I-m)(I-m-1)]}{\hbar^4[2I(2I-1)]^4}, \tag{9}$$

where a is the lattice constant, $\mathbf{r}_{ij}$ the distance between particles i and j, m the spin projection on the z axis (which is directed along $\mathbf{r}_{ij}$), $\omega_{\parallel}$ the end-point frequency of the longitudinal optical oscillations, $g_{m+\mu}(\omega)$ the normalized distribution function of the state density of the level $m + \mu$, and V the volume of the sample.

2. Transverse oscillations:

$$W_{ij}^{\perp}(m+1) = (1-\gamma_\infty)^4 F_1^{\perp} \frac{2^6 2\pi}{\omega_\perp^4 r_{ij}^2} [f_1(a, \omega_\perp, r_{ij})]^2 \tag{10}$$

$$\times \int_0^\infty g_{m+1}^2(\omega)\, d\omega;$$

$$W_{ij}^{\perp}(m+2) = (1-\gamma_\infty)^4 F_2^{\perp} \frac{2^6 2\pi}{\omega_\perp^4 r_{ij}^4} [f_2(a, \omega_\perp, r_{ij})]^2$$

$$\times \int_0^\infty g_{m+2}^2(\omega)\, d\omega; \tag{11}$$

$$F_{1,2}^{\perp} = \left(\frac{B_\perp}{B_\parallel}\right)^4 F_{1,2}^{\parallel};$$

$$f_1(a, \omega_\perp, r_{ij}) = \Big[\frac{1}{r_{ij}^4} \cos \zeta_{ij} + \frac{1}{r_{ij}^3}\left(9\pi \frac{\omega_\perp}{c} \sin \zeta_{ij} - \frac{1}{a} \cos \varphi_{ij}\right) + \frac{3}{r_{ij}^2}\left(\pi \frac{\omega_\perp^2}{c^2} \cos \zeta_{ij} + \frac{\pi^2}{a^2} \sin \varphi_{ij} + 8 \frac{\omega_\perp^2}{c^2} \sin \varphi_{ij}\right) + \frac{\pi}{r_{ij} a}\left(\frac{6\omega_\perp^2}{c^2} - \frac{\pi^2}{a^2}\right) \cos \varphi_{ij} - \frac{57}{4} \pi \frac{\omega_\perp^4}{c^4} \cos \zeta_{ij} + \frac{9\pi}{4} \frac{\omega_\perp^5}{c^5} r_{ij} \sin \varphi_{ij}\Big];$$

$$f_2(a, \omega_\perp, r_{ij}) = \Big[\frac{1}{r_{ij}^3}(\sin \varphi_{ij} + \cos \zeta_{ij}) + \frac{1}{r_{ij}^2}\left(9\pi \frac{\omega_\perp}{c} \sin \zeta_{ij} - \frac{7}{a} \cos \varphi_{ij}\right) + \frac{3\omega_\perp^2}{r_{ij} c^2}(\pi \cos \zeta_{ij} - 6 \sin \varphi_{ij}) - \frac{27}{4} \pi \frac{\omega_\perp^4}{c^4} r_{ij} \cos \zeta_{ij}\Big];$$

$$\zeta_{ij} = \frac{\omega_\perp r_{ij}}{c}, \qquad \varphi_{ij} = \frac{\pi}{a} r_{ij}.$$

It follows from expressions (8)—(11) that for two nuclear spins located at distances on the order of the lattice constant, the greatest contribution to the probability $W_{ij}^{\parallel,\perp}$ is made by terms that are inversely proportional to the tenth power of a. For more remote neighbors ($r_{ij} \gg a$) the dependence of $W_{ij}^{\parallel,\perp}$ on r_{ij} is different. Retaining in the square brackets of (8)—(11) only the most essential terms, we find that for such r_{ij}

$$W_{ij}^{\parallel,\perp}(m+1) \sim \frac{1}{a^6 r_{ij}^4}, \quad W_{ij}^{\parallel,\perp}(m+2) \sim \frac{1}{a^2 r_{ij}^8}. \tag{12}$$

So far we have considered the interaction of only two particles. To obtain the total probability of the transition of a particle i from $m + \mu$ into m with simultaneous transition of any of the particle j from m into $m + \mu$, expressions (8)—(11) must be summed over all the particles j, which are in the ground state and are removed from i by a distance r_{ij} which does not exceed the phonon mean free path. It may turn out here that a noticeable contribution to

$$W_i^{\parallel,\perp} = \sum_j W_{ij}^{\parallel,\perp}$$

is made by the terms whose dependence on r_{ij} differs from (12).

For order of magnitude estimates of the probabilities $W_{ij}^{\parallel,\perp}$, we use the following values of the parameters: $Q = 2 \times 10^{-25}$ cm^2, $z = 1$, $m_+ = 6.5 \times 10^{-23}$ g, $m_- - 5.9 \times 10^{-23}$ g, $a = 3.14 \times 10^{-8}$ cm (the parameters of the KCl lattice), $N = 10^{23}(3.14)^{-1}$, $\omega_\perp = 6 \times 10^{13}$ sec^{-1}, $\omega_\parallel = 10^{13}$ sec^{-1}, $\int g_p^2(\omega)d\omega = 10^{-4}$ sec, I = 2, m = – 1, V = 1 cm^3, $\gamma_\infty \sim 10^2$. Substituting them in (8)—(11) we obtain $W_{ij}^{\parallel}(m+1) \sim 10^2$ sec^{-1}, $W_{ij}^{\parallel}(m+2) \sim 10^3$ sec^{-1}, $W_{ij}^{\perp}(m+1) = 10^3$ sec^{-1}, $W_{ij}^{\perp}(m+2) = 2 \times 10^3$ sec^{-1}. It is of interest to compare the obtained values for the analogous probabilities for the case of magnetic nuclear dipole-dipole interaction. Assuming that the order of magnitude of the nuclear magnetic moment is 10^{-23} erg/G, we find that the probability of reorientation of two neighboring spins at a distance is of the order $W_{ij}^{nuc} \sim 10^3$ sec^{-1}.

Thus, at distances of the order of the lattice constant, the probabilities of reorientation of the nuclear spins, due to the interaction of the quadrupole moments of the nuclei via the virtual field of the optical phonons can be of the same order as the probabilities of analogous processes due to magnetic dipole-dipole interactions. And even at a distance $r_{ij} = 3a$ the value of W_{ij}^{nuc} decreases by only two orders, i.e., the interaction in question has a much longer range than the dipole-dipole interaction.

It is quite obvious that analogous processes can occur in the optical and in the microwave bands as a result of the interaction between the quadrupole moments of the electron shells with the gradient of the electric field produced by the optical oscillations. Unlike the nuclear quadrupole tensor component, we shall denote the components of the electron shell tensor of the impurity particle by $D_{\pm\mu}$,

$$D_{\alpha\beta} = e \sum_k (3r_\alpha^{(k)} r_\beta^{(k)} - r^{(k)2} \delta_{\alpha\beta}); \quad \alpha, \beta = x, y, z,$$

$$D_0 = \frac{3}{2} D_{zz}, \quad D_{\pm 1} = \mp \frac{\sqrt{6}}{2} (D_{xz} \mp D_{yz}),$$

$$D_{\pm 2} = \frac{\sqrt{6}}{2} (D_{xx} - D_{yy} - 2iD_{xy}), \tag{13}$$

where the summation is over all the electrons of the impurity particle.

Calculations similar to those given above lead to the following expressions for the probabilities of the cross relaxation transitions between the quadrupole sublevels of the particle:

$$W_{ij}^{\parallel}(D_{\pm 1}) = \frac{F_{1e}^{\parallel}}{2\pi (\Delta^2 - \hbar^2 \omega_\parallel^2)^2 a^2 r_{ij}^4} \Big[\sum_{n=0}^{\infty} \Big]^2_{(8)} \int_0^\infty g_{\alpha_2}^2(\omega)\, d\omega, \tag{14}$$

$$W_{ij}^{\parallel}(D_{\pm 2}) = \frac{32 F_{2e}^{\parallel}}{2\pi (\Delta^2 - \hbar^2 \omega_\parallel^2)^2 a^2 r_{ij}^8} \Big[\sum_{n=0}^{\infty} \Big]^2_{(9)} \int_0^\infty g_{\alpha_2'}^2(\omega)\, d\omega, \tag{15}$$

$$W_{ij}^{\perp}(D_{\pm 1}) = \frac{2^6 2\pi F_{1e}^{\perp}}{(\Delta^2 - \hbar^2 \omega_\perp^2)^2 r_{ij}^2} [f_1(a, \omega_\perp, r_{ij})]^2 \int_0^\infty g_{\alpha_2}^2(\omega)\, d\omega, \tag{16}$$

$$W_{ij}^{\perp}(D_{\pm 2}) = \frac{2^6 2\pi F_{2e}^{\perp}}{(\Delta^2 - \hbar^2 \omega_\perp^2)^2 r_{ij}^4} [f_2(a, \omega_\perp, r_{ij})]^2 \times \int_0^\infty g_{\alpha_2'}^2(\omega)\, d\omega,$$

$$F_{\mu e}^{\parallel, \perp} = (B_{\parallel, \perp})^4 V^2 |\langle \alpha_1 | D_{-\mu} | \alpha_2 \rangle|^2 |\langle \alpha_2 | D_\mu | \alpha_1 \rangle|^2, \quad \mu = 1, 2, \tag{17}$$

where $[\sum_{n=0}^{\infty}]_{(8),(9)}$ are the expressions in the square brackets in formulas (8) and (9), and $g_p(\omega)$ is the normalized density distribution function of the states of the level p. It must be emphasized that the expressions obtained are valid when $\Delta = \hbar\omega_0 > \hbar\omega_\perp$ and $\Delta < \hbar\omega_\perp$. The inclusion of $\Delta = \hbar\omega_\perp$ entails no difficulty if one knows the analytic relation $\omega_\perp = f(q)$.

For the spectroscopy of the impurity centers, the most interesting case is $r_{ij} \gg a$. It is easy to see that for this case we can rewrite (14)—(17) in the form

$$W_{ij}^{\parallel\prime}(D_{\pm 1}) = \frac{\pi^2 F_{1e}^{\parallel}}{2^3 (\Delta^2 - \hbar^2 \omega_\parallel^2)^2 a^6 r_{ij}^4} \cos^2 \zeta_{ij} \times \int_0^\infty g_{\alpha_2}^2(\omega)\, d\omega, \tag{14a}$$

$$W_{ij}^{\parallel\prime}(D_{\pm 2}) = \frac{32 F_{2e}^{\parallel}}{2\pi (\Delta^2 - \hbar^2 \omega_\parallel^2)^2 a^2 r_{ij}^8} \cos^2 \zeta_{ij} \int_0^\infty g_{\alpha_2'}^2(\omega)\, d\omega, \tag{15a}$$

$$W_{ij}^{\perp\prime}(D_{\pm 1}) = \frac{(2\pi)^7 F_{1e}^{\perp}}{(\Delta^2 - \hbar^2 \omega_\perp^2)^2 a^6 r_{ij}^4} \cos^2 \varphi_{ij} \int_0^\infty g_{\alpha_2}^2(\omega)\, d\omega, \tag{16a}$$

$$W_{ij}^{\perp\prime}(D_{\pm 2}) = \frac{2^6 10^2 \pi^3 F_{2e}^{\perp}}{(\Delta^2 - \hbar^2 \omega_\perp^2)^2 a^2 r_{ij}^8} \cos^2 \varphi_{ij} \int_0^\infty g_{\alpha_2'}^2(\omega)\, d\omega. \tag{17a}$$

To estimate the order of magnitude of the quantities, we use the following values of the parameters entering into the formulas:

$$|\langle \alpha_1 | D_\mu | \alpha_2 \rangle| \sim e \cdot 10^{-16} \text{cm}^2, \quad \Delta = \hbar\omega_0 \gg \hbar\omega_\perp,$$

$$\omega_0 \sim 10^{15} \text{ sec}^{-1}, \quad r_{ij} \sim 10^{-7} \text{cm}, \quad \int_0^\infty g_p^2(\omega)\, d\omega \approx 10^{-10} \text{sec}.$$

The remaining parameters are the same as used above. The calculations yield

$$W_{ij}^{\parallel}(D_{\pm 1}) \sim 10^9 \text{ sec}^{-1}, \quad W_{ij}^{\parallel}(D_{\pm 2}) \sim 10^3 \text{ sec}^{-1},$$

$$W_{ij}^{\perp}(D_{\pm 1}) \sim 10^{13} \text{ sec}^{-1}.$$

Consequently, in the optical branch of the spectrum, the mechanism under discussion is also a very effective means of transferring the excitation energy between the orbital states of the impurity centers. Such a mechanism effects migration of the energy over the levels for which the dipole-dipole interactions are equal to zero. In addition, $W_{ij}^{\parallel,\perp}(D_{\pm 1})$ decreases with increasing r_{ij} much more slowly than the analogous probabilities due to dipole-dipole interactions.

In conclusion, we note that upon simultaneous action of the operator

$$\mathscr{H}_e = \sum_\mu (-1)^\mu D_\mu (\nabla \mathbf{E}^{\parallel, \perp})_{-\mu}$$

and the spin-orbit operator V_{S0}, cross correlation transitions are possible in the electron-spin system, and also relaxation of the spin systems with conversion of the magnetic energy into the energy of the optical branches of the lattice oscillations. It is obvious that the latter mechanism can be effective only at sufficiently high temperatures.

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