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DESCRIPTION OF INELASTIC DIFFRACTION SCATTERING BY THE COMPLEX ANGULAR MOMENTUM METHOD

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A simple analytic expression for the inelastic scattering cross section of a spinless particle is derived on the basis of the complex angular momentum method. The expression obtained yields the well known Blair phase rule. The results are compared with the experimental data on α particle scattering on a number of nuclei. The comparison indicates a very satisfactory qualitative agreement between the theory proposed and the experimental data.

1. INTRODUCTION

THE theory of the inelastic diffraction scattering of nucleons and other nuclear particles on complex nuclei has been in a state of intense development during the last decade. This development has proceeded along two lines. The first approach starts from the Fraunhofer diffraction on a black nucleus, using the adiabatic approximation. This method, first employed in the papers of Drozdov[1] on the excitation of rotational levels of nuclei, was then extended by one of the present authors[2] to the case of vibrational levels, and by Blair[3] to the case of transitions of arbitrary multipolarity. The second approach consists in the application of the nuclear optical model for the calculation of inelastic scattering by the distorted wave method. The results of a number of calculations by this method are presented in the review article of Austern.[4]

Despite the successes in the explanation of the characteristic features of the inelastic scattering process, both these methods suffer from serious deficiencies. The cross sections calculated by the first method, when averaged over the oscillations, increase slowly with scattering angle, whereas the experiments indicate a fast (exponential) fall-off of the cross sections. Moreover, the theoretical cross sections vanish at the diffraction minima, also contrary to experiment. A great difficulty in this method is the account of the Coulomb repulsion between charged particles. The distorted wave method gives results which are in satisfactory agreement with experiment. However, calculations along this line require a great deal of numerical work on electronic computers for each comparison with newly appearing experimental data. Moreover, the agreement with experiment achieved in this way remains to some extent illusory, since the optical model is not a sufficiently clearly defined concept. In particular, it is not clear, to how many parameters one should restrict oneself in calculations using this model. It is also evident that with such an approach it is very difficult to obtain the qualitative characteristics of the inelastic scattering process.

In recent papers of Blair, Sharp, and Wilets[5] and Austern and Blair,[6] a synthesis of the two procedures was achieved. It was shown that the inelastic scattering problem can be reduced to the elastic scattering problem by combining the distorted wave method with the adiabatic approximation. This comes about in that the inelastic scattering amplitude can be expressed through derivatives of the S matrix for elastic scattering with respect to the angular momentum. In particular, if one takes for the S matrix the simplest expression

$$S_l = \begin{cases} 0, & l < l_0 = kR \\ 1, & l > l_0 \end{cases}, \qquad (1)$$

which, in elastic scattering, leads to Fraunhofer diffraction from a black nucleus, one obtains the old results of [1-3] for the inelastic scattering.

In a recent paper,[7] one of us has proposed a new method for the description of elastic diffraction scattering. In this method one is able to take account of the presence of a transition region in the S matrix near $l = l_0$ as well as the Coulomb repulsion and the refraction of the incident wave in the nuclear matter. It was also shown[8] that the comparison of this theory with experiments on the scattering of α particles on many nuclei bears out the correctness of the basic assumptions of the theory.

In the present paper we attempt a unified description of elastic and inelastic scattering on the basis of the theories mentioned above. Such an approach to inelastic scattering is of essential interest not only from the point of view of the inelastic scattering problem itself but also from the point of view of the method of complex angular momenta. The application of this method to inelastic scattering makes the theory developed in [7] much richer in content, since the S matrix parameters introduced in [7] for the description of elastic scattering are used for the inelastic scattering as well. On the other hand, the inelastic scattering is a serious test of the basic premises of the method of complex angular momenta in the theory of diffraction scattering.

2. INELASTIC SCATTERING CROSS SECTIONS

We restrict our discussion to the inelastic scattering by nuclei with zero spin and excitations of the one-phonon type. In the above-mentioned paper of Austern and Blair,[6] it was shown that in this case the inelastic scattering amplitude can be written in the form

$$f_{IM}(\vartheta)=\frac{i}{2}C(I)\sqrt{2I+1}\,[I:M]\sum_{l=M}^{\infty}\sqrt{2l+1}\,\frac{dS_l}{dl}\,Y_l^{-M}(\vartheta,0), \tag{2}$$

where I and M are the angular momentum and its projection for the excited state under consideration. The nuclear matrix element C(I) is given by

$$C(I)=(\varphi_f,\ \xi_{LM}\varphi_i), \tag{3}$$

where φ_i and φ_f are the wave functions of the initial and final states of the nucleus, and the quantities ξ_{LM} are dynamical variables of the nucleus which determine the nuclear surface according to the formula

$$R(\vartheta,\varphi)=R_0+\sum_{L,M}\xi_{LM}Y_{LM}^*(\vartheta,\varphi). \tag{4}$$

We note that C(I) is independent of M for a target nucleus of zero spin. The coefficients [I:M] are equal to

$$[I:M]=\begin{cases}\dfrac{i^I[(I-M)!\,(I+M)!]^{1/2}}{(I-M)!!\,(I+M)!!}, & \text{if } I+M \text{ is even}\\ 0, & \text{if } I+M \text{ is odd.}\end{cases} \tag{5}$$

These coefficients can be written in the following form:[3]

$$[I:M]=i^{-M}\left(\frac{4\pi}{2I+1}\right)^{1/2}Y_I^M\left(\frac{\pi}{2},0\right). \tag{6}$$

From this we easily obtain a relation which we shall need in the following:

$$\sum_M|[I:M]|^2=1. \tag{7}$$

The quantity S_l in (2) is the S matrix element for the l-th partial wave in the elastic scattering of a particle with the same initial energy.

It should also be noted that (2) was obtained by Austern and Blair[6] with neglect of the Coulomb phase shifts. However, the same result is obtained if one assumes one is dealing with a potential which differs from the Coulomb potential only at large distances, where it drops off faster than the Coulomb potential. This leads to different cross sections only at small scattering angles, which we do not consider for other reasons (the discussion of small angles necessitates the inclusion of many poles of the S matrix, which makes our method ineffective). Therefore, with this treatment of the Coulomb repulsion, the scattering amplitude is again expressed by formula (2), where, however, S_l must be taken as the matrix element for elastic scattering with account of the Coulomb repulsion.

The method of performing the summation in (2) is completely analogous to that of [7]. The value of the sum in (2) is entirely determined by the contribution from the residues of the poles of the function dS_l/dl. It is clear that the poles of this function coincide with the poles of S_l with the only difference that they are now double. As in the case of elastic scattering, one can restrict oneself to the contribution from the pair of complex conjugate poles closest to the real axis. Calculations similar to those of [7] lead immediately to the following expression for the cross section:

$$\sigma_{IM}(\vartheta)=2(2I+1)\,|C(I)\,[I:M]\,|^2|a|^2 l_0\vartheta^2\sin^{-1}\vartheta\, e^{-2\beta\vartheta}\times\{b^2+\cos^2[(l_0+{}^1\!/_2)\vartheta+\gamma+{}^1\!/_2\pi(M+1)]\}, \tag{8}$$

$$b=\operatorname{sh}(\beta\theta_0),\quad \theta_0=2\beta^{-1}\operatorname{Im}\delta(l_1),\quad \gamma=\arg a+\pi/4. \tag{9*}$$

Since $\sigma_{IM}=0$ if I + M odd, one can replace M by I in the argument of the cosine in (8). Using also (7), we find for the summed cross section

$$\sigma_I(\vartheta)=\sum_M\sigma_{IM}(\vartheta)=A_I\vartheta^2(\sin\vartheta)^{-1}e^{-2\beta\vartheta}\{b^2+\cos^2[(l_0+{}^1\!/_2)\vartheta+\gamma+{}^1\!/_2\pi(I+1)]\}, \tag{10}$$

$$A_I=2(2I+1)\,|a|^2|C(I)\,|^2 l_0. \tag{11}$$

For comparison, we give the cross section for elastic scattering:[7]

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*sh ≡ sinh.

$$\sigma_{el}(\vartheta) = (8\pi l_0 |a|^2 / k^2 \sin\vartheta) e^{-2\beta\vartheta} \times \{b^2 + \cos^2[(l_0 + 1/2)\vartheta + \gamma]\}. \quad (12)$$

Thus the inelastic scattering is described by the same parameters as the elastic scattering, except for the factor A_I which contains the nuclear matrix element. It follows hence that, with the parameters l_0, β, θ_0, $|a|$, and arg a determined from the data on elastic scattering, we can uniquely determine the angular distribution of the inelastically scattered particles, and from the absolute values of the experimental inelastic scattering cross section we find the value of the nuclear matrix element. This means that formulas (10) and (12) allow one, in particular, to extract important information on nuclear structure, since the nuclear matrix elements obtained in the manner described can be compared with the values found on the basis of different versions of nuclear models.

An important result expressed by (10) and (12) is the fact that they contain the so-called Blair phase rule.[3] This rule, which is very well confirmed by experiment, states that the oscillations of the inelastic scattering cross sections are in opposite phase with the oscillations of the elastic scattering cross section if the spin of the excited state I is even, and they are in phase in the opposite case. The phase of the cosine in (10) does indeed contain an additional term as compared to the elastic scattering case, $(1/2)(I + 1)\pi$, which shifts the phase by $\pi/2$ when I is even and leaves it unchanged when I is odd.

The phase rule was obtained by Blair on the very unreliable basis of Fraunhofer diffraction theory. In the paper by Shekhata and one of the authors[9] a more fundamental derivation of this rule was attempted. However, also these authors remained within the framework of Fraunhofer diffraction theory. The discussion of the present paper and the result obtained about the correctness of the Blair phase rule can evidently be regarded as a completely satisfactory justification of this rule.

3. COMPARISON WITH EXPERIMENT AND DISCUSSION OF RESULTS

Figures 1 to 5 show the results of the calculation of the cross sections for the elastic and inelas-

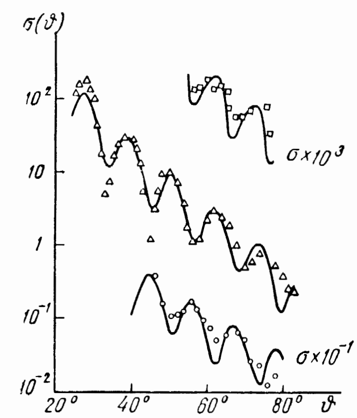

FIG. 2. Elastic and inelastic (with excitation of the 2^+ level at 1 MeV and the 3^- level at 3.5 MeV) scattering cross section for α particles of 41 MeV[12,13] on Ti^{48}.

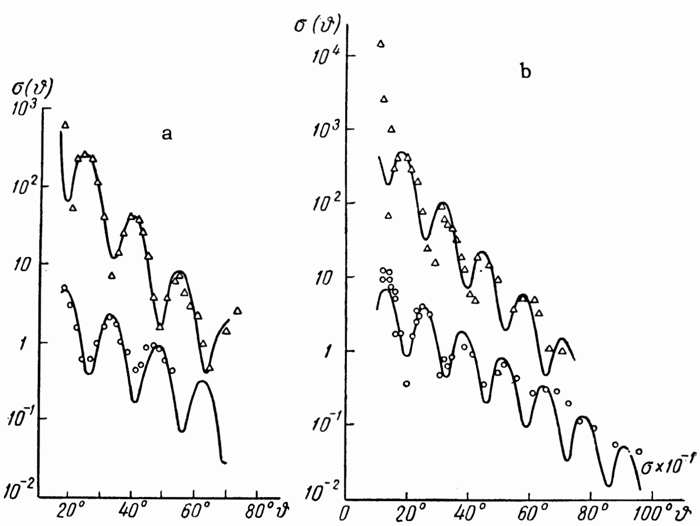

FIG. 1. Elastic and inelastic (with excitation of the 2^+ level at 1.37 MeV) scattering cross section for α particles with energies a) 31.5 MeV,[10] and b) 42 MeV[11] on Mg^{24}.

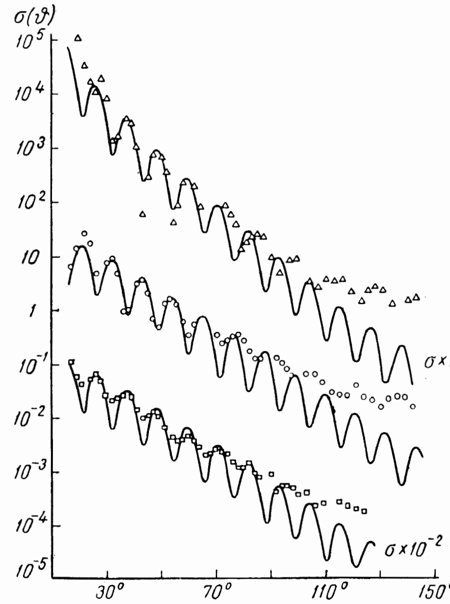

FIG. 3. Elastic and inelastic (with excitation of the 2^+ level at 1.45 MeV and the 3^- level at 43 MeV) scattering cross sections for α particles of 43 MeV[14] on Ni^{58}.

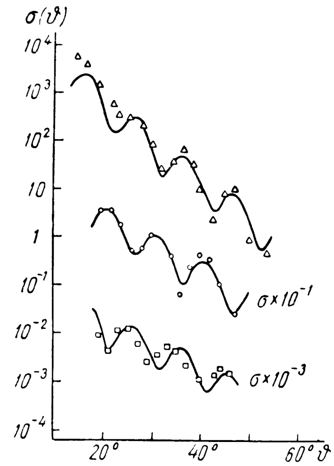

FIG. 4. Elastic scattering cross sections for α particles of 43 MeV[15] on Zn^{64} and inelastic scattering cross sections for the excitation of the 2^+ level at 1.04 MeV and the 3^- level at 2.8 MeV for α particles of 43 MeV[16] on Zn^{66}.

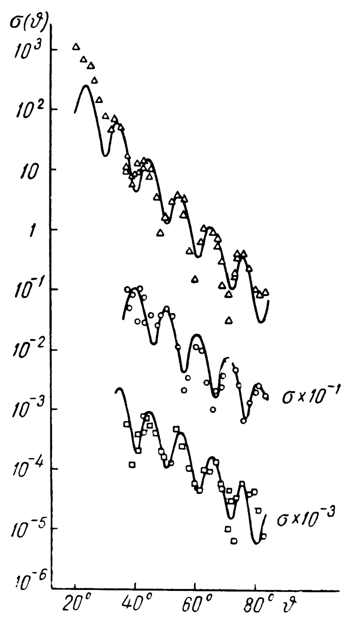

FIG. 5. Elastic and inelastic (with excitation of the 2^+ level at 1.8 MeV and the 3^- level at 2.8 MeV) scattering cross sections for α particles of 41 MeV on Sr^{88}.[12] The experimental data have been taken with a natural mixture of isotopes.

tic scattering of α particles on five different nuclei according to formulas (10) and (12), and the corresponding experimental data.

The experimental data[10-16] on the elastic scattering and the excitation of the 2^+ and 3^- levels are in all figures indicated by triangles, circles, and squares, respectively. The parameter values used are given in Table I. The comparison shows that a single set of parameters fits satisfactorily the elastic as well as the inelastic scattering of α particles on nuclei. An appreciable

Table I

Nucleus	E_α	R	β	$\|a\|$	arg a	θ_0
Mg^{24}	31.5	6.46	2.61	1.14	0	0.12
Mg^{24}	42	6.21	2.65	1.16	0.6	0.16
Ti^{48}	41	6.97	2.56	0.92	0.7	0.21
Ni^{58}	43	7.03	2.80	1.13	1.0	0.14
Zn^{64}	43	7.64	4.60	3.48	0	0.11
Zn^{66}	43	7.64	4.60	3.48	0.3	0.11
Sr^{88}	41	7.93	3.20	1.48	1.0	0.14

Table II

Nucleus	E_α	Level E	Level I^π	$\|C(I)\|$	β_I	β_I^*
Mg^{24}	31.5	1.37	2+	0.66	0.33	0.36
Mg^{24}	42	1.37	2+	0.63	0.34	0.36
Ti^{48}	41	1	2+	0.35	0.16	0.22
		3.5	3−	0.15	0.08	0.09
Ni^{58}	43	1.45	2+	0.36	0.16	0.21
		4.45	3−	0.23	0.12	0.08
Zn^{66}	43	1.04	2+	0.31	0.12	0.26
		2.8	3−	0.20	0.09	0.14
Sr^{88}	41	1.8	2+	0.18	0.07	0.11
		2.8	3−	0.16	0.07	0.10

discrepancy between theory and experiment occurs only in the case of Ni^{58} for angles $\vartheta \gtrsim 100°$. It is very probable that it is no longer permissible to restrict oneself to only one pair of poles in the analysis of the scattering into such large angles.

In Table II we give the values of the nuclear matrix element C(I) in units of 10^{-13} cm. There we also include the values of the parameters β_I obtained from C(I) by

$$\beta_I = \sqrt{2I+1}\,C(I) / R_N, \tag{13}$$

where R_N is the actual nuclear radius defined by

$$R_N = R - R_\alpha, \tag{14}$$

for R_α, the radius of the α particle, the value of 2.0 F was chosen.[12] In the last column the values of β_I^* are listed. These are obtained from the β_I given by McDaniels et al.[12] by multiplication with $R_0/(R_0 - R_\alpha)$, where R_0 is taken from the same reference. It is clear that in this way account is taken of the difference between R_0 and R_N, the actual nuclear radius.

We see that our values of β_I, the nuclear deformation parameters, are in general agreement with those obtained in [12] on the basis of the old theory of inelastic diffraction scattering, although there are in some cases noticeable discrepancies (by a factor of 1.5 to 2). Such an agreement should have been expected from the comparison of formulas (10) and (12) with the corresponding formulas of the old diffraction theory. Indeed, let us consider the case of odd spin I. Then we have from (10) and (12)

$$\sigma_I(\vartheta) = (4\pi)^{-1}(kR_N\vartheta\beta_I)^2\sigma_{el}(\vartheta). \quad (15)$$

This relation between the elastic and inelastic scattering cross sections also holds in the usual diffraction theory[3] for odd I, though only in the asymptotic region $kR\vartheta \gg 1$.

The situation is analogous for even I. Here

$$\sigma_I(\vartheta) = (4\pi)^{-1}(kR_N\vartheta\beta_I)^2 \exp(\pi\beta/l_0)\sigma_{el}(\vartheta + \pi/2l_0). \quad (16)$$

Almost the same relation has also been derived for the asymptotic region in the usual diffraction theory. The only difference is that in the usual theory there is no factor $\exp(\pi\beta/l_0)$. This must lead to a reduction of β_I by the factor $\exp(-\pi\beta/2l_0)$ as compared with the usual theory. As a rule the quantity $\pi\beta/2l_0$ is much less than unity, so that the effect of this factor is small, although it may somewhat alter the magnitude of β_I.

In conclusion the authors express their deep gratitude to N. Austern and J. Blair for sending a preprint of their work[6] before publication and also to A. A. Kresnin for valuable discussions.

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