CONTRIBUTION TO THE THEORY OF THE DIFFRACTION SCATTERING OF PARTICLES BY COMPLEX NUCLEI

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Problems connected with the method of analysis of experimental data on the diffraction scattering of particles by complex nuclei proposed recently by one of the authors[1] are considered. The theory is compared with experiments on the scattering of α particles by a number of nuclei from various regions of the periodic system. The comparison shows that the method considered gives a correct description of diffraction scattering.

1. In a recent paper[1] a method was proposed for the description of the diffraction scattering of particles by complex nuclei, in which, in contrast to the elementary theory of diffraction scattering,[2] the diffuseness of the nuclear boundary and the refraction of the incident wave inside the nucleus are taken into account. In the present paper we compare this theory with the experiments on the scattering of α particles with energies of several tens of MeV, carried out in recent years on a large number of nuclei from various regions of the periodic table.

2. In the elementary diffraction theory the following expression for the S matrix is used:

$$1 - S_l = \begin{cases} 1, & l < l_0 = kR - 1/2 \\ 0, & l > l_0 \end{cases}. \tag{1}$$

In order to take account of the diffuseness of the nuclear surface, one must replace (1) by another expression in which the transition from total absorption to the vanishing of interaction is made not with a jump, but continuously. As an example for a function S_l which satisfies this requirement, one often considers the function[3,4]

$$1 - S_l = \left[1 + \exp\left(\frac{l - l_0}{\lambda}\right)\right]^{-1}. \tag{2}$$

We note that this function also satisfies the physically reasonable requirement that the quantity $1 - S_l$ decrease exponentially for $l \to \infty$. Indeed, for large l, i.e., large impact parameters, the nuclear density decreases exponentially. Hence, the interaction whose strength is determined by the quantity $1 - S_l$ must also decrease exponentially.

Considering expression (2) from the analytic point of view, on which also the method of the previous paper[1] was based, we see that the function has poles at the points

$$l_n = l_0 + i\pi\lambda n \quad (n = \pm 1, \pm 3, \pm 5, \ldots) \tag{3}$$

and the residues of S_l at these poles are equal to

$$a_n = \lambda. \tag{4}$$

Thus, in this model of the S matrix,

$$|a_n| = \beta_n / \pi n, \tag{5}$$

$$\arg a_n = 0, \tag{6}$$

where the β_n denote the imaginary parts of the corresponding poles.

Now the question arises to what extent the relations (5) and (6) are sensitive to the form of the S matrix. May it not be that a model of the S matrix which differs slightly from model (2) but otherwise satisfies the same general requirements (smeared-out step function with exponential decay at infinity) leads to strong deviations from (5) and (6)? Apparently, this is not the case, at least not for the poles closest to the real axis.

Let us consider a model which is rather more general than that corresponding to expression (2):

$$1 - S_l = \left[1 + \chi(l) \exp\left(\frac{l - l_0}{\lambda}\right)\right]^{-1}; \tag{7}$$

here $\chi(l)$ is some function which varies much less rapidly than $\exp[(l - l_0)/\lambda]$. For this function one may choose, for example,

$$\chi(l) = (l / l_0)^m, \tag{8}$$

where m is some small integer.

From (7) follows

$$l_n = l_0 + i\pi\lambda n - \lambda \ln \chi(l_n), \tag{9}$$

$$\operatorname{res} S_{l_n} = \lambda[1 + \lambda\chi'(l_n) / \chi(l_n)]^{-1}. \tag{10}$$

Regarding $\chi(l)$ and its derivative as slowly varying

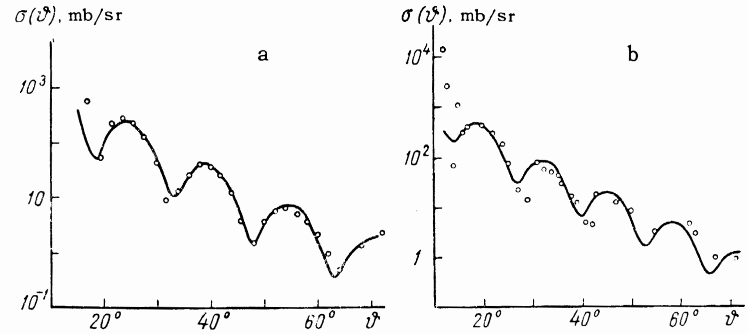

FIG. 1. Elastic scattering cross section for α particles on Mg^{24}: a) for $E_\alpha = 31.5$ MeV, b) for $E_\alpha = 42$ MeV.

in the neighborhood of l_0, we find approximately

$$l_n = l_0 + i\pi\lambda n[1 - \lambda\chi'(l_0) / \chi(l_0)], \qquad (11)$$

$$a_n = \operatorname{res} S_{l_n} = \lambda[1 - \lambda\chi'(l_0) / \chi(l_0)]. \qquad (12)$$

These expressions show that relations (5) and (6) are stable against small deviations of $1 - S_l$ from the form given by (2).

3. The inclusion of the Coulomb interaction at energies well above the Coulomb barrier does not introduce any complications. Singularities due to the Coulomb interaction appear under these conditions only at small scattering angles. Since we do not consider small scattering angles for another reason (the necessity to include many poles), the singularities connected with the Coulomb interaction are thus excluded. Of course, the Coulomb interaction will affect the interpretation of the parameters of the S matrix, for example the parameter l_0 [cf. formula (14)].

4. After these remarks we turn to the comparison of the theory [1] with experiment. The basic formula obtained in [1] contains five parameters: l_0, β, $|a|$, θ_0, and γ. We have

$$\sigma(\vartheta) = \frac{8\pi l_0}{k^2 \sin\vartheta} |a|^2 e^{-2\beta\vartheta} \times\left\{\sinh^2(\beta\theta_0) + \cos^2\left[\left(l_0 + \frac{1}{2}\right)\vartheta + \gamma\right]\right\},$$

$$\gamma = \pi/4 + \arg a. \qquad (13)$$

Here l_0 and β are the real and imaginary parts of the pole of S_l, and $|a|$ is the modulus of the residue of S_l at the pole.

We have compared this formula with the results of the experiments on the elastic scattering of α particles. [5-10] We have considered only those experiments in which $E_\alpha > 40$ MeV (the only exception are the data on Mg^{24} with an energy of the α particles of 31.5 MeV) and $A \geq 24$. This selection of data guarantees that $kR \gg 1$ (in all cases considered $kR > 12$). Moreover, the condition $A \geq 24$ restricts the discussion to those nuclei where one can already speak of a more or less sharply defined nuclear boundary.

Figures 1 to 4 show the differential scattering cross sections in the center-of-mass system for α particles scattered on various nuclei, as calculated according to (13). The corresponding parameter values entering in (13) are given in the table. In the case of Pb^{208} the character of the oscillations in the cross section is extremely weakly defined and thus does not permit a determination of the parameters $|a|$, arg a, and R. For nuclei whose mass numbers are not given in the table, the experiments were done with the natural mixture of isotopes. We see that the theoretical curves describe well the experimentally observed cross sections down to very small scattering angles ($\vartheta \approx 20°$).

Interpreting l_0 as the angular momentum corresponding to the grazing trajectory, we can determine the effective range of interaction from the relation

$$l_0 = kR(1 - B/E)^{1/2}, \qquad (14)$$

where $B = Z_1 Z_2 e^2/R$ is the Coulomb barrier.

	E_{lab}, Mev	β	$\|a\|$	β/π	arg a	θ_0	θ_c	R, F	ΔR, F
$_{12}Mg^{24}$	31,5	2.61	1.14	0.83	0	0,12	0,26	6.46	1.73
$_{12}Mg^{24}$	42	2,65	1.16	0.84	0,6	0.16	0,20	6.21	1.52
$_{22}Ti^{48}$	40	2,56	0,92	0,82	0,7	0,21	0.31	6,97	1,40
$_{29}Cu$	40	2.9	1,15	0,92	1.4	0,19	0,39	7,33	1.55
$_{30}Zn^{64}$	43	4.6	3.48	1.47	0	0,11	0.35	7.64	2.39
$_{41}Nb$	40	3,4	2.83	1.08	1.3	0.15	0.51	8.03	1,79
$_{42}Mo$	40	3,5	2,83	1.12	0.6	0,19	0.52	8,05	1.85
$_{47}Ag$	40	4,28	4,09	1,37	0.2	0,24	0,53	8,73	2.26
$_{82}Pb^{208}$	48	4,2	—	1,34	—	—	—	—	1,97

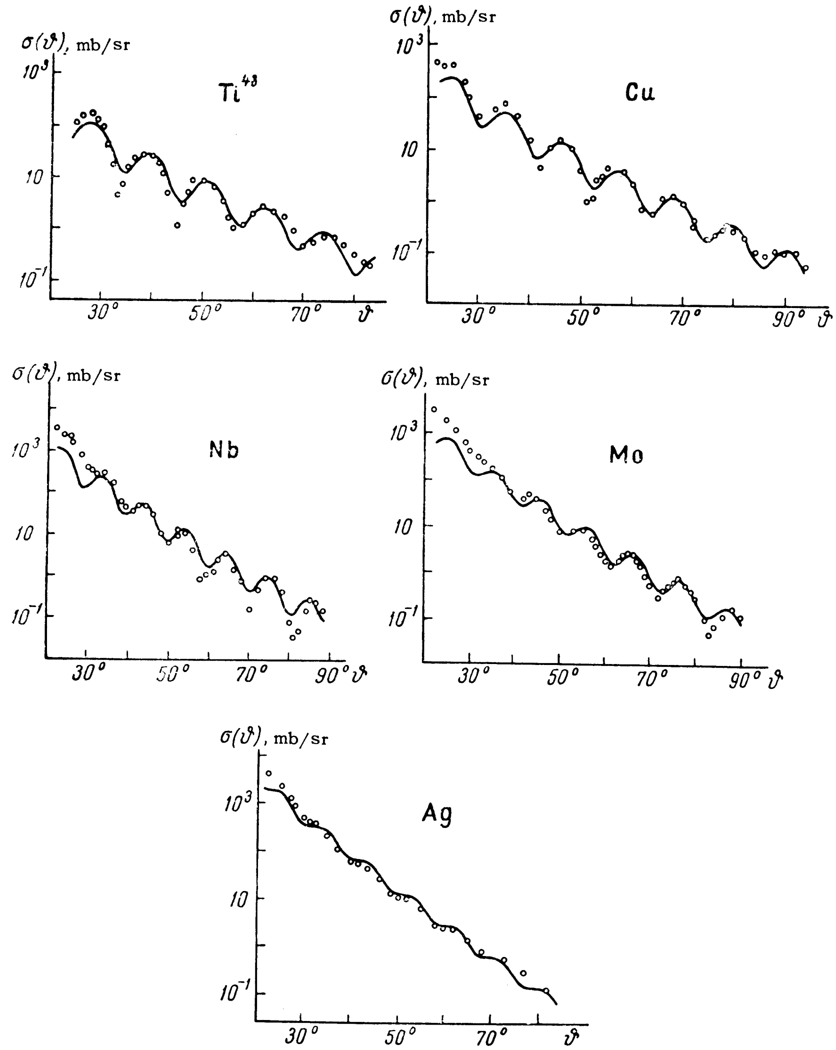

FIG. 2. Elastic scattering cross section for α particles on Ti^{48}, Cu, Nb, Mo, and Ag for $E_\alpha = 40$ MeV.

Figure 5 shows the dependence of R on $A^{1/3}$. This dependence is well described by the relation

$$R = 1.24A^{1/3} + 2.60 \ [\mathrm{F}]. \qquad (15)$$

The value found, $r_0 = 1.24$ F, is in complete agreement with the value of r_0 obtained by other methods.[11] The value 2.60 F obtained for the constant term in (15) can be well explained through the radius of the α particle $R_\alpha \sim 2$ F plus the range of the nuclear forces, $r_{nuc} \sim 1$ F.

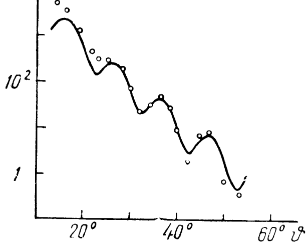

FIG. 3. Elastic scattering cross section for α particles on Zn^{64} for $E_\alpha = 43$ MeV.

Assuming that the function $1 - S_l$ has the form (2), we see that $1 - S_l$ changes from 0.9 to 0.1 as l varies from $l_0 - 2\lambda \ln 3$ to $l_0 + 2\lambda \ln 3$. One can therefore say that changing the impact parameter by $\Delta R = 4\lambda k^{-1} \ln 3$ leads to a change of the absorption coefficient from 0.9 to 0.1. As is seen from the table, ΔR varies rather slowly with increasing atomic weight. It follows from this that the properties of the nuclear surface layer are almost the same for all nuclei considered. This result is also in good agreement with the known data on the scattering of electrons on nuclei.

The modulus of the residue, $|a|$, is very close to the value obtained for it by using the expression (2) for light nuclei, and exceeds this value by a factor of two or three for the heavier nuclei. The parameter θ_0, interpreted as the scattering angle of the α particle on the grazing trajectory, is for all nuclei smaller than θ_C, the Coulomb scattering angle on the grazing trajectory. This should be expected, since the action of the nuclear forces on

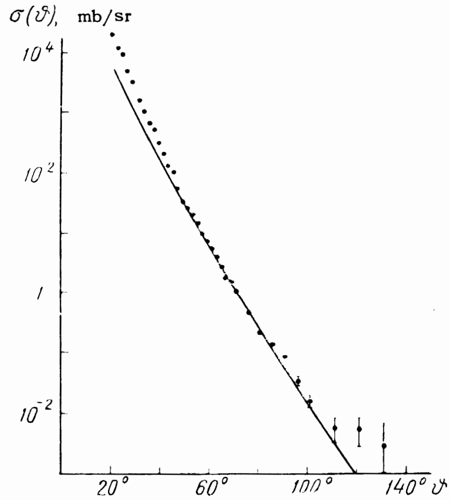

FIG. 4. Elastic scattering cross section for α particles on Pb^{208} for E_α = 48 MeV.

the α particle must reduce the scattering angle on the grazing trajectory as compared to the Coulomb angle.

The present results of the comparison of the theory with experiment allow one to conclude that the method proposed in [1] gives a completely satisfactory description of a wide range of data on the elastic diffraction scattering. For the parameters entering in the formula for the differential cross section physically reasonable parameters are obtained. The behavior of the parameters R and ΔR describing the radii of the nuclei and the properties of the surface layer, are in good accord with the data obtained from the evaluation of the results of experiments on the scattering of high-energy electrons. In electron scattering experiments one determines, of course, only the proton component of the density. Therefore, our results indicate that the behavior of the neutron component does not differ appreciably from that of the proton component.

We note another aspect of our method of analysis of experimental data on diffraction scattering. If for some nucleus an appreciable deviation of the parameters from a gradual change with increasing mass number A is observed, one should naturally expect that this nucleus has some peculiarities whose nature one should try to explain by nuclear structure theory. Thus, for example, it is seen from the table that for the nucleus Zn^{64} the parameters β and $|a|$ experience a very sharp deviation from a gradual change with increasing A. We are thus justified in expecting peculiarities in the structure of this nucleus. One may expect the appearance of peculiarities of this type for nuclei with closed shells, for nuclei at the limits of the regions of nonsphericity, etc. However, the limited data on diffraction scattering available at present do not permit such a detailed analysis.

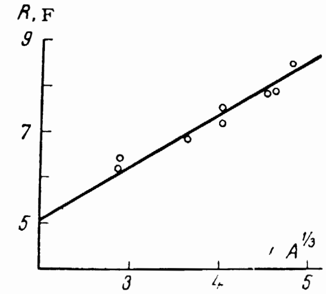

FIG. 5. Dependence of the effective range of interaction, R, on $A^{1/3}$.

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