ALGEBRAIC PROPERTIES OF THE ENERGY-MOMENTUM TENSOR AND VACUUM-LIKE STATES OF MATTER

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The physical interpretation of some algebraic structures of the energy-momentum tensor allows us to suppose that there is a possible form of matter, called the μ-vacuum, which macroscopically possesses the properties of vacuum. The assumption that an actually occurring vacuum is a μ-vacuum retains the Lorentz invariance of the Lagrangian (when gravitation is neglected) and preserves the theories based on the requirement of this invariance, and at the same time makes the Mach principle no longer logically convincing. The space time of a μ-vacuum is an Einstein space in the sense of Petrov's definition.[2] A uniform world of μ-vacuum has the de Sitter metric.

In the general theory of relativity the only quantity that describes the properties of matter and its influence on the metric is the energy-momentum tensor. The theory does not establish the possible structures of this tensor which correspond to various forms of matter, but takes them over from other branches of physics. In this sense the general theory of relativity is not a closed theory, and this has been regarded, particularly by Einstein,[1] as a serious shortcoming of the theory. This undesirable feature of the theory could be removed if each algebraic structure of the energy momentum tensor with a complete system of invariants and admitting of a real representation (real observable quantities) could be put in correspondence with a form of matter. Hitherto, however, the physical interpretation of various types of structure of the energy-momentum tensor has been known only for certain special cases.

The present paper looks toward an interpretation of the various structures of the energy-momentum tensor T_{jk} as depending on general properties of the motion of particular forms of matter, as characterized by a set of reference systems comoving with the matter. This restricts the analysis to cases in which the concept of a comoving reference system can be introduced—that is, in which all elementary factors of the matrix $T_{jk} - \theta g_{jk}$ are real and simple, and consequently the eigenvectors of the tensor T_{jk} are nonisotropic and can be chosen so that everywhere where $T_{jk} \neq 0$ they form an orthonormal frame, the eigenframe of the tensor T_{jk}. The timelike vector of the eigenframe represents the four-velocity of the matter.

The classification of the possible algebraic structures of the tensor T_{jk} that satisfy the conditions just stated is given by the following sequence of characteristics:

$$[1111],\quad [(11)11],\quad [(111)1],\quad [11(11)],\quad [(11)(11)]\quad [1(111)],\quad [(1111)]. \tag{1}$$

Here, as usual, each of the symbols 1 correspond to an eigenvalue θ_a of the tensor T_{jk}, and symbols which correspond to equal eigenvalues are enclosed in parentheses. The fourth symbol 1 is assumed to correspond to a timelike (fourth) eigenvalue, so that, for example, the symbol [(11)(11)] means that $\theta_1 = \theta_2$, $\theta_3 = \theta_4$. Because all of the three spacelike eigenvectors are on the same footing, it makes no sense to distinguish symbols [(11)11], [1(11)1], but the characteristics [(11)11] and [11(11)] have different physical meanings.

An eigenframe of the tensor T_{jk} is determined up to a rotation in the plane of eigenvectors that belong to equal eigenvalues. Consequently the four-velocity of the matter, and accordingly also the comoving reference system, are uniquely determined when none of the eigenvalues $\theta_{a'}$ ($a' = 1, 2, 3$) is equal to the number θ_4. Otherwise there is an infinite set of reference systems comoving with the matter. Accordingly, the first three of the characteristics (1) correspond to matter with a unique comoving reference system, and the last four to forms of matter which do not have this property.

Existence and uniqueness of the comoving reference system—that is, the presence at each point of a physical medium of a definite macroscopic three-velocity of mass flux, smaller than the speed of light—are characteristic of physical media which are formed by particles with rest mass different from zero. We shall here call such media ordinary matter. The concept of a reference system is only an idealized representation of a system of bodies of ordinary matter. Accordingly the existence of a comoving reference system characterizes the motion of a given form of matter relative to ordinary matter. That there can be a relation between motion and matter different from that for ordinary matter can be seen from the fact that, for example, the free electromagnetic field ($\mathbf{E}^2 - \mathbf{H}^2 = 0$, $\mathbf{E} \cdot \mathbf{H} = 0$) has no comoving reference system at all. Therefore we cannot regard uniqueness of the comoving system as a priori necessary for a physical medium.

Let us consider from this point of view an energy-momentum tensor with the characteristic [(1111)], i.e., with all the eigenvalues equal. In this case any orthonormal set of four nonisotropic vectors is an eigenframe, and consequently any reference system is a comoving one. To see the meaning of this assertion, let us imagine a moving test particle interacting in some way with matter which has an energy-momentum tensor with the characteristic [(1111)]. According to what has been said, the rest system of the particle can always be regarded as also comoving for the matter, so that all interactions between the matter and the particle do not depend on its velocity. Therefore velocity can not be determined by the study of such interactions. In other words, for the interactions of a particle with the matter under consideration we have precisely the same principle of relativity as for the interaction of a particle with vacuum. Accordingly, from the point of view of a macroscopic description, this kind of matter has the fundamental property of vacuum, and consequently a tensor with the characteristic [(1111)] can be interpreted as the energy-momentum tensor of a vacuum-like medium. In the framework of the formalism of the general theory of relativity there is no reason not to regard such a medium as a form of matter, and to put it in contrast with other forms of matter. And conversely: in assuming the reality of a vacuum-like state of matter, we are not going outside the framework of the general theory of relativity.

This hypothetical form of matter differs from vacuum in the sense of the usual definition, which assumed that the energy-momentum tensor of vacuum is equal to zero. In particular, we can assign to it a proper mass density $\mu = -\theta$, where θ is the value of the equal eigenvalues of the energy-momentum tensor. For brevity we shall call this form of matter a μ-vacuum. We shall return later to the choice of the sign of the scalar μ.

It is interesting to compare the properties of μ-vacuum and the free electromagnetic field. Both of these states of matter are limiting cases in relation to sets of states with a unique comoving system; the corresponding passages to the limit are: for the former, a transition to eigenvalues of the energy-momentum tensor which are all equal, and for the latter, the transition from a time-like eigenvector to an isotropic one. Both the μ-vacuum and the electromagnetic field are characterized by paradoxical behavior in relation to ordinary bodies, their properties being mutually complementary: for the former any reference system is a comoving one, and for the latter no reference system is a comoving one. Owing to this neither medium can be characterized by specifying a comoving reference system. In particular, from the dilation (or contraction) of any of the reference systems comoving with a μ-vacuum one cannot draw direct conclusions about its intrinsic characteristics, say about a change of its density.

Because of the multiplicity of the comoving reference systems we cannot introduce the concept of localization of an element of the μ-vacuum matter, and consequently cannot introduce the concepts of particle and of the number of particles of the μ-vacuum in a given volume, if we understand by a particle an object singled out in a classical sense relative to the remaining "part" of the matter. Similarly, one cannot introduce the classical concept of a photon. This comparison shows that the concept of μ-vacuum fills up at least a logical gap in the classification of the states (forms) of matter.

In the spirit of this approach we can also interpret the states with energy-momentum tensors which have the characteristics [11(11)], [(11)(11)], and [1(111)]. We call the eigenvalues of the tensor T_{jk} which are equal to the timelike eigenvalue vacuum eigenvalues. The first two of these characteristics obviously correspond to a state of matter in which its interactions with ordinary matter do not depend on the component of the velocity of the latter along the eigenvector which belongs to the vacuum eigenvalue. It is natural to call the direction of this vector a vacuum direction. With the characteristic [1(111)] there are two vacuum eigenvalues and a whole "vacuum plane" spanned by the vectors which belong to the vacuum eigenvalues. The projection of the velocity of ordinary matter onto this plane cannot be detected by its interaction with matter which has an energy-momentum tensor with the characteristic in question.

The only one of these states with a vacuum direction actually known is that with the characteristic [(11)(11)]. This is the electromagnetic field with $\mathbf{E}^2 - \mathbf{H}^2 \neq 0$. In a coordinate system K in which the vectors **E** and **H** are parallel, the common direction of these fields is a vacuum direction. In fact, the components of the vectors **E** and **H** are the same in all coordinate systems which move relative to K with velocities directed along the common direction of the fields. Therefore these systems cannot be distinguished from one another by any sort of measurements of the fields.

Let us try to see some of the consequences of the hypothesis that a μ-vacuum is actually possible. We consider a world consisting of ordinary matter immersed in a μ-vacuum, and we assume that there is so little matter that its effect on the metric, on the scale of the world as a whole, can be neglected. According to what has been said, the principle of relativity holds for the interactions of the matter with the μ-vacuum. Since our treatment has been local, we are concerned only with the local validity of this principle. This is enough, however, to secure the validity of physical theories (for example, quantum field theory) based on the assumption of Lorentz invariance of the Lagrangian and dealing with processes which involve a region of space time in which there is essentially no variation of the gravitational field. Accordingly the hypothesis of the μ-vacuum makes no changes in the formal apparatus of these theories, although in principle there can be interactions of the fields with the μ-vacuum describable in the framework of these theories but different from the interactions with ordinary matter. We shall return to this later, after first examining the cosmogonic consequences of the replacement of the vacuum by a μ-vacuum.

We take here the signature (+++-) for the metric and use a system of units in which mass and energy units are the same (c = 1). In an arbitrary coordinate system the energy-momentum tensor with equal eigenvalues is of the form

$$T_{jk} = -\mu g_{jk}, \tag{2}$$

where g_{jk} is the metric tensor. Since by hypothesis the amount of ordinary matter is negligibly small, the Einstein equations are of the form

$$G_{jk} = 8\pi\mu g_{jk}, \tag{3}$$

where G_{jk} is the Einstein tensor.

Because $G^k_{j|k} = 0$ and $g_{jk|l} = 0$ everywhere, it immediately follows that if, as has been assumed, the amount of ordinary matter is negligibly small, then the space-time of the μ-vacuum world has the properties of an Einstein space in the sense of Petrov's definition.[2] Consequently (cf. [2], Sec. 19):

1) there are three possible types of μ-vacuum, corresponding to three algebraic structures of the Riemann tensor allowed by the condition (3);

2) the various local states of the μ-vacuum can be characterized in the following ways: for type I, by five independent real invariants, for type II by three, and for type III by one; and the invariants can be chosen so that the scalar μ is a homogeneous linear function of them;

3) there exists a uniquely defined orthogonal frame relative to which the Riemann tensor takes the canonical form.

Thus the scalar μ is not an exhaustive characteristic of a μ-vacuum. In particular, all three types of μ-vacuum are possible for $\mu = 0$, and in their local properties they are obviously not distinguished in any way relative to $\mu \neq 0$. The types of perturbations possible in a μ-vacuum within the framework allowed by the condition μ = const are of the nature of gravitational waves analogous to those considered by Pirani.[3] From this point of view the μ-vacuum can also be regarded as a form of gravitational field. The essential point in our argument is that such a form of gravitational field cannot be rejected on the basis of a priori considerations, nor can it be regarded as something contrasted with the forms of matter, if we describe these last by means of the single formalism of the energy-momentum tensor.

The space-time of a homogeneous μ-vacuum world is a Riemannian space with constant curvature $K = 8\pi\mu/3$. As is well known, a Riemannian space of constant negative curvature has properties whose physical interpretation is extremely difficult (cf., e.g., [4], Chapter VII, Sec. 1). Therefore it is natural to set $\mu > 0$, in agreement with usual ideas about the sign of a mass density. Then the μ-vacuum world is a homogeneous spherical de Sitter world. From the present point of view the de Sitter world is not "empty," but contains matter in a vacuum-like state. It is easy to show that the mass of the de Sitter world is equal to the mass which would be ascribed to it by an observer outside it who measures the gravitational field.

As long as the metric is determined only by the μ-vacuum, the density of the μ-vacuum, in view of the condition μ = const, plays the role of the cosmological constant, which accordingly can be interpreted in the framework of the ordinary formalism of the general theory of relativity. If, on the other hand, we cannot neglect the matter other than the μ-vacuum, the analogy of the μ-vacuum density with the cosmological constant can be maintained only in so far as the interaction of this matter with the μ-vacuum is unimportant. Otherwise the condition μ = const does not hold, and the analogy with the cosmological constant is destroyed.

The differences between the structure of the energy-momentum tensor of μ-vacuum and that for ordinary matter, and the consequent differences between its equations of motion and its properties and the equations of motion and properties for ordinary matter show that if the μ-vacuum is real, then it is a specific form of matter. Since the equations of the general theory of relativity do not contain adequate information about the conditions of transition between different forms of matter, within the framework of this theory we cannot decide whether the μ-vacuum is stable against such transitions. As has been mentioned, however, the formulation of quantum field theory is not changed if we assume that the vacuum is actually a μ-vacuum. Therefore it is natural to expect that the local properties of μ-vacuum in relation to transitions to different forms of matter are determined by the same laws as for vacuum, and in this sense it is the lowest state of matter.

It is an extremely interesting question whether there are reasons to suppose that it is possible for transitions to occur between ordinary matter and μ-vacuum in processes other than the annihilation of matter and antimatter. Neglecting departures from isotropy of the momentum flux density, we write the energy-momentum tensor of ordinary matter in the form

$$T_{jk} = (p + \mu) u_j u_k + p g_{jk}, \qquad (4)$$

where u_j is the four-velocity of the comoving reference system, and p and μ are scalars interpreted as pressure and density. Comparison of this expression with (2) shows that ordinary matter goes over into μ-vacuum at a negative pressure:

$$p = -\mu. \qquad (5)$$

The meaning of a negative pressure is that the internal volume forces in the matter are not forces of repulsion (as they are for the media accessible to observation, which consist of particles), but forces of attraction. This implies the assumption that the usual mechanisms which oppose the merging of particles of matter are annulled.

This situation is not utterly unrealistic. An attempt to describe phenomenologically the structure of an elementary charged particle would lead to the conclusion that inside the particle there must be a negative pressure which balances the electrostatic repulsion. This raises the thought that in an ultradense state of matter, with the baryons so compressed that the meson fields which provide the interaction between them (repulsion!) cannot be produced, a continuous medium is formed in which the conditions correspond to an attraction between material elements and are described phenomenologically by a negative pressure. For example, such a state might be reached in gravitational collapse.

It would seem that a negative pressure should lead to an internal instability, and that if there are no volume forces of the type of the electrostatic repulsion it would lead to a contraction without limit. This is not true, however. Let us assume that compression actually leads to a negative pressure. Then at some stage of the compression the increase of the pressure gives way to a decrease—that is, the derivative $\partial p/\partial\mu$ becomes negative. Let us consider this "descending branch" of the equation of state. Substituting (4) in the conservation law $T^k_{j|k} = 0$, we get an equation which locally can be reduced to the form

$$(p + \mu)\, d\mathbf{v}/dt = -\mathrm{grad}\, p \qquad (6)$$

where $\mathbf{v}$ is the 3-velocity of the matter and t is the time. This differs from Newtonian mechanics in that the direction of the acceleration $d\mathbf{v}/dt$ depends on the sign of the sum $p + \mu$, and consequently the redistribution of the matter occurs in the direction of a smoothing out or of an increase of the variations of pressure, depending on the sign not of the derivative $\partial p/\partial\mu$, but on that of the product $(\mu + p)\partial p/\partial\mu$. Therefore the branch $\partial p/\partial\mu < 0$ corresponds to internal instability (increase of the variations of pressure) only as long as $p + \mu > 0$, and accordingly the contraction under the action of forces of negative pressure cannot occur without limit. In the final state achieved in this way $p + \mu \leq 0$. We cannot decide whether a transition is possible to states with $p + \mu < 0$, in which there would be a paradoxical motion of the matter in the direction of increasing pressure, on the basis of Eq. (6), since these states are separated from those with $p + \mu = 0$ by the singular state of the μ-vacuum with $p + \mu = 0$. If, as has been assumed, the vacuum-like state is actually the lowest possible state of matter, a μ-vacuum is the only possible final result of the process of contraction under the action of forces of negative pressure.

The hypothetical process we have considered is interesting in connection with the problem of the final state of matter which has undergone gravitational collapse. According to the ideas developed here this state is a μ-vacuum.

The hypothesis that transitions between ordinary and μ-vacuum states of matter are possible raises questions about the quantum conservation laws, primarily the law of conservation of the baryon number. There are the following two conceivable logical possibilities.

A. The laws of conservation of quantum numbers are due to the mechanism of interaction between the baryons through the meson fields, and are violated in the catastrophic process of transition of ordinary matter into μ-vacuum, so that a μ-vacuum is always characterized by the same quantum numbers as vacuum. In this case the world that arises owing to the excitation of a μ-vacuum would be symmetrical with respect to matter and antimatter.

B. A μ-vacuum can have quantum numbers different from those of a vacuum in the ordinary sense. In this case elementary processes could reveal the asymmetry with respect to matter and antimatter, and this could be the source of violations of symmetry in elementary processes.

In conclusion we indicate the connection of the problem of the μ-vacuum with the logical foundations of the relativity principle. When we regard space as "empty" it is natural to expect that every motion of ordinary matter is only motion relative to other bodies of ordinary matter—that is, that Mach's principle holds. The unceasing publication of papers devoted to this principle is undoubtedly due to its logical cogency.

As we have seen, the μ-vacuum can be described in the framework of the usual formalism of the energy-momentum tensor, and in this sense, its properties are analogous to those of ordinary matter. Therefore there are grounds for regarding the μ-vacuum not as the empty space-time of Mach's principle, but as one of the states of matter. The hypothesis that the actual vacuum is a μ-vacuum narrows the principle of relativity in a natural way into a local principle of the relativity of velocity.

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[2] A. Z. Petrov, Prostranstva Einshteĭna (Einstein Spaces), Fizmatgiz, 1961.

[3] F. A. E. Pirani, Phys. Rev. **105**, 1089 (1957).

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