TRANSFORMATION OF LONGITUDINAL WAVES INTO TRANSVERSE WAVES IN A WEAKLY TURBULENT PLASMA

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Submitted to JETP editor January 15, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 1732-1739 (June, 1965)

A method is developed for deriving the matrix elements entering into the collision terms of the kinetic equations for waves in a plasma. The total Lagrangian function for a non-collision plasma is the basis of the method. Three-wave processes in which two longitudinal waves merge into single transverse wave or a transverse wave disintegrates into two longitudinal ones are considered. The growth rate of the energy density of the transverse waves produced as the result of the merging of Langmuir oscillations or of Langmuir and ion-sound waves in an isotropic homogeneous weakly turbulent plasma is calculated by employing the matrix elements found by the method under discussion.

1. The study of various relaxational and kinetic processes in an almost ideal plasma (the number of particles in the Debye sphere $N_D \gg 1$) with excited collective degrees of freedom reduces to the study of the nonlinear effects brought about by the interactions of a particle with a wave and of a wave with a wave. The first type of interaction is considered in the framework of quantum theory and leads to the result that the increment (decrement) γ_k which enters into the energy balance equation for the k-th harmonic of the oscillations becomes a functional of the background distribution f_0, $\gamma_k = \gamma_k[f_0]$, while the function f_0 changes slowly with time under the action of the averaged fast oscillations.[1]

In investigations of the interaction of waves with one another, two approaches are essentially used: dynamic, by means of which Sturrock[2] and Drummond and Pines[3] first considered the interaction of one dimensional Langmuir oscillations, or statistical, based on the construction of an infinite chain of equations for the correlation functions obtained from the initial equations of the plasma by averaging over some statistical ensemble.[4-7] Depending on the physical conditions of the specific problem, the infinite chain can be cut off and reduced to a closed system of what are in general rather complicated equations, from which one can extract important information on processes taking place in a turbulent plasma.

As a rule, many waves are excited simultaneously in a turbulent plasma. Usually, in the investigation of collective processes, it is assumed that the essential change in amplitude of the waves takes place more slowly than the disappearance of correlation of their phases. Although there does not exist at the present time any direct experimental verification of this assumption, nevertheless, it can be assumed that it is realized, even in a weakly turbulent plasma. Under such conditions, the excitation of waves reduces to their collisions with one another. By introducing the distribution function of the waves, the collisions between them can be described by the kinetic equation for this function in phase space.[8,9] The matrix elements entering into the collision terms of the kinetic equations for the waves are best found by means of the complete Lagrangian function for the non-collision plasma.

The described approach in the theory of plasma turbulence is illustrated in this work by the example of the transformation of longitudinal waves into transverse waves in an isotropic homogeneous plasma.[1)] Such wave processes play an especially important role in the diagnostics of turbulent plasma and in astrophysics.[10-13]

2. The complete Lagrangian function for a non-collision plasma can be written in the form[14]

$$\mathscr{L} = \sum_\alpha \int\int d\xi d\mathbf{v} f_\alpha(\xi, \mathbf{v}) \Big\{ {}^1/_2 m_\alpha (\mathbf{v} + D^\alpha \mathbf{r}^\alpha)^2 - e_\alpha \varphi_0(\xi + \mathbf{r}^\alpha) - e_\alpha \varphi'(\xi + \mathbf{r}^\alpha) + \frac{e_\alpha}{c}(\mathbf{v} + D^\alpha \mathbf{r}^\alpha [A_0(\xi + \mathbf{r}^\alpha) + A'(\xi + \mathbf{r}^\alpha)] \Big\} + \frac{1}{8\pi}\int dv [(\mathbf{E}_0 + \mathbf{E}')^2 - (\mathbf{H}_0 + \mathbf{H}')^2]. \quad (1)$$

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1)We limit ourselves here to an isotropic homogeneous plasma only for the sake of brevity. The formalism can be generalized to an arbitrary case without any special difficulty.

Here $\mathbf{r}^\alpha$, φ', and $\mathbf{A}'$ represent the displacement of the particle and the deviations of the scalar and vector potentials from their equilibrium values φ_0 and $\mathbf{A}_0$ in the initial state; these are the variables with respect to which it is necessary to vary the action function $S = \int \mathscr{L} dt$; $f_\alpha(\xi, \mathbf{v})$ is the distribution function of particles of type α in the stationary fields*

$$\mathbf{E}_0 = -\nabla\varphi_0, \qquad \mathbf{H}_0 = \operatorname{rot}\mathbf{A}_0;$$

D^α denotes the operator

$$D^\alpha = \frac{\partial}{\partial t} + (\mathbf{v}\nabla) + \left(\frac{e_\alpha}{m_\alpha}\mathbf{E}_0 + [\mathbf{v}\omega_H{}^\alpha]\right)\nabla_v,$$
$$\omega_H{}^\alpha = eH_0/m_\alpha c. \qquad (2)†$$

The remaining notation is standard.

Expanding in powers of $\mathbf{r}^\alpha$, we write the Lagrangian (1) in the form

$$\mathscr{L} = \sum_n \mathscr{L}_n.$$

$\mathscr{L}_0$ is a functional of the stationary quantities and is not of interest; $\mathscr{L}_1$ vanishes identically. With the help of $\mathscr{L}_2$, which is quadratic in the amplitude of the excitation of the Lagrangian, a system of linear equations is obtained. It is equivalent to the set of Maxwell equations for self-consistent fields and to the linearized Vlasov-Boltzmann equations; consequently, $\mathscr{L}_2$ describes the characteristic oscillations of the plasma. The Lagrangians of higher order will describe the interaction between the characteristic oscillations. [8,9]

3. The three-wave processes in which we are interested—the merging of two longitudinal waves into a single transverse wave, and the decay of a transverse wave into two longitudinal—are described in an isotropic homogeneous plasma by the Lagrangian $\mathscr{L}_3$, for which, in accord with (1), we get

$$\mathscr{L}_3 = \sum_\alpha \int\int d\xi d\mathbf{v} f_\alpha(\mathbf{v}) \Big(-{}^1/_2 e_\alpha r_j{}^\alpha r_l{}^\alpha \nabla_j \nabla_l \varphi'$$
$$+ \frac{e_\alpha}{2c}\mathbf{v} r_j{}^\alpha r_l{}^\alpha \nabla_j\nabla_l \mathbf{A}' + \frac{e_\alpha}{c} D^\alpha \mathbf{r}^\alpha r_j{}^\alpha \nabla_j \mathbf{A}'\Big). \qquad (3)$$

The expression for the operator D^α is easily obtained from Eq. (2). We shall not from the outset specify the type of waves which can take part in the three-wave processes considered. We shall only point out that, inasmuch as the laws of conservation of frequency and wave vector must be satisfied in the transformations given, the possibility of three-wave processes is entirely determined by the wave-dispersion laws of the linear theory.

We select gauge potentials such that the scalar potential in the transverse wave vanishes; then

$$\mathbf{E}^l = -\nabla\varphi, \quad \mathbf{E}^{tr} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \operatorname{rot}\mathbf{A}. \qquad (4)$$

Following Landau and Rumer, [15] we represent the displacement $\mathbf{r}^\alpha$ and the potential φ' in the form

$$\mathbf{r}^\alpha = \mathbf{r}_1{}^\alpha + \mathbf{r}_2{}^\alpha + \mathbf{r}_3{}^\alpha, \qquad \varphi' = \varphi_1' + \varphi_2'.$$

The indices 1, 2 refer to the longitudinal waves, 3 to the transverse wave. We expand the quantities $\mathbf{r}_{1,2}^\alpha$, $\varphi_{1,2}'$, and $\mathbf{A}'$ in a Fourier series:

$$\mathbf{r}_{1,2}{}^\alpha = \sum_{k_{1,2}} \mathbf{r}^\alpha_{k_{1,2}} \exp(i\mathbf{k}_{1,2}\xi), \quad \varphi_{1,2}' = \sum_{k_{1,2}} \varphi'_{k_{1,2}} \exp(i\mathbf{k}_{1,2}\xi),$$
$$\mathbf{A}' = \sum_f \mathbf{A}_f' e^{i\mathbf{f}\xi}.$$

Substituting these expressions in (3) and taking into account the conservation of wave vector in the interaction, we get

$$\mathscr{L}_3 = \sum_\alpha e_\alpha \sum_{k_1k_2f} \int d\mathbf{v} f_\alpha(v) \Big\{ (\mathbf{r}_{k_2}{}^\alpha \mathbf{k}_1)(\mathbf{r}_f{}^\alpha \mathbf{k}_1)\varphi_{k_1}'$$
$$+ (\mathbf{r}_{k_1}{}^\alpha\mathbf{k}_2)(\mathbf{r}_f{}^\alpha\mathbf{k}_2)\varphi_{k_2}' - \frac{1}{c}[(\mathbf{r}_{k_1}{}^\alpha\mathbf{f})(\mathbf{r}_{k_2}{}^\alpha\mathbf{f})(\mathbf{A}_f'\mathbf{v})$$
$$- i(\mathbf{r}_{k_2}{}^\alpha\mathbf{f})(\mathbf{B}_{k_1}{}^\alpha\mathbf{A}_f') - i(\mathbf{r}_{k_1}{}^\alpha\mathbf{f})(\mathbf{B}_{k_1}{}^\alpha\mathbf{A}_f')]\Big\}$$
$$\times \Delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{f}), \qquad (5)$$

where $\mathbf{B}_k^\alpha = (D^\alpha \mathbf{r}^\alpha)_k$,

$$\Delta(x) = \begin{cases} 1, & x = 0, \\ 0, & x \neq 0. \end{cases} \qquad (6)$$

We express $\mathbf{B}_{k_{1,2}}^\alpha$ and $\mathbf{r}_{k_{1,2}}^\alpha$ in terms of $\varphi_{k_{1,2}}^\alpha$ and $\mathbf{r}_f^\alpha$ in terms of $\mathbf{A}_f'$. For this purpose, use is made of an equation which connects the displacement of the particle with the field of the waves; this equation is obtained by variation of $S = \int \mathscr{L}_2 dt$ with respect to $\mathbf{r}^\alpha$, and has the form

$$D^{\alpha^2}\mathbf{r}^\alpha = \frac{e_\alpha}{m_\alpha}\left(\mathbf{E}' + \frac{1}{c}[\mathbf{v}\mathbf{H}']\right). \qquad (7)$$

It then follows that

$$\mathbf{r}_{k_{1,2}}^\alpha = i\frac{e_\alpha}{m_\alpha}\frac{\mathbf{k}_{1,2}}{(\Omega_{k_{1,2}} - \mathbf{k}_{1,2}\mathbf{v})^2}\varphi_{k_{1,2}}';$$

$$\mathbf{r}_f{}^\alpha = -i\frac{e_\alpha}{cm_\alpha}\frac{\varkappa_f}{(\omega_f - \mathbf{f}\mathbf{v})^2}A_f', \qquad \mathbf{A}_f' = \varkappa_f A_f';$$

$$\mathbf{B}_{k_{1,2}}^\alpha = \frac{e_\alpha}{m_\alpha}\frac{\mathbf{k}_{1,2}}{(\Omega_{k_{1,2}} - \mathbf{k}_{1,2}\mathbf{v})}\varphi_{k_{1,2}}'. \qquad (8)$$

We normalize the Fourier components of the potentials φ_k' and $\mathbf{A}_f'$ according to the equations

* rot ≡ curl.

†$[\mathbf{v}\mathbf{w}] \equiv \mathbf{v}\times\mathbf{w}$, $(\mathbf{v}\nabla) \equiv \mathbf{v}\cdot\nabla$.

$$(4\pi)^{-1}\sum_k\left[\frac{\partial(\omega\varepsilon^l)}{\partial\omega}\right]_{\omega=\Omega_k}|\mathbf{k}|^2\varphi_k'^+\varphi_k'=\sum_k N_k\hbar\Omega_k,$$

$$(4\pi c^2)^{-1}\sum_f\omega_f^2\left[\frac{\partial(\omega\varepsilon^{tr})}{\partial\omega}+\varepsilon^{tr}\right]_{\omega=\omega_f}\varkappa_f^2A_f'^+A_f'$$

$$=\sum_f n_f\hbar\omega_f, \quad (9)$$

where ϵ^l and ϵ^{tr} are the longitudinal and transverse permittivities of the plasma, N_k is the number of longitudinal waves with wave vector **k** and frequency Ω_k, and n_f is the number of transverse waves with wave vector **f** and frequency ω_f.

It is convenient to introduce the following notation:

$$\varphi_k'=\frac{1}{k}\left\{\frac{4\pi\hbar\Omega_k}{[\partial(\omega\varepsilon^l)/\partial\omega]_{\omega=\Omega_k}}\right\}^{1/2}a_k,$$

$$A_f'=\left\{\frac{4\pi\hbar c^2}{\omega_f[\partial(\omega\varepsilon^{tr})/\partial\omega+\varepsilon^{tr}]_{\omega=\omega_f}\varkappa_f^2}\right\}^{1/2}b_f. \quad (10)$$

Then the normalization conditions (9) take the form

$$\sum_k\hbar\Omega_k a_k^+a_k=\sum_k N_k\hbar\Omega_k,$$

$$\sum_f\hbar\omega_f b_f^+b_f=\sum_f n_f\hbar\omega_f.$$

Thus a_k^+, a_k, b_f^+, and b_f can be treated as creation (a_k^+, b_f^+) and annihilation (a_k, b_f) operators corresponding to longitudinal and transverse waves with wave vectors **k** and **f** and frequencies Ω_k and ω_f, respectively. The non-zero matrix elements of these operators have the form

$$(N_k-1|a_k|N_k)=\sqrt{N_k}\exp(-i\Omega_k t),$$

$$(N_k+1|a_k^+|N_k)=\sqrt{N_k+1}\exp(i\Omega_k t),$$

$$(n_f-1|b_f|n_f)=\sqrt{n_f}\exp(-i\omega_f t),$$

$$(n_f+1|b_f^+|n_f)=\sqrt{n_f+1}\exp(i\omega_f t). \quad (11)$$

With the help of Eqs. (8) and (10), the Lagrangian (5) is written in the form

$$\mathcal{L}_3=\sum_{k_1k_2f}\Phi_{f;\,k_1k_2}b_f^+a_{k_1}a_{k_2}\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{f})+\text{h.c.} \quad (12)$$

We shall not write out here the general complicated expression for the matrix element $\Phi_{f;\,k_1k_2}$ which determines the probability of the three-wave processes described above, and proceed immediately to consider specific examples.

4. As the first example, we consider the transformation of a Langmuir oscillation into a transverse wave in a weakly turbulent, isotropic, homogeneous plasma. The conservation laws for frequencies and wave vectors of the waves participating in the process have the form

$$\Omega_{k_1}+\Omega_{k_2}=\omega_f,\qquad \mathbf{k}_1+\mathbf{k}_2=\mathbf{f}. \quad (13)$$

The effect of spatial dispersion on the spectrum of transverse oscillations is a small correction in a nonrelativistic plasma, and can be neglected. Furthermore, it makes sense to consider only a weakly damped Langmuir oscillation, the wavelength of which far exceeds the Debye radius $R_D = (T_e/4\pi e^2 n)^{1/2}$. Under such conditions, we can use the following expressions for the dispersion laws of the longitudinal and transverse waves:

$$\Omega_k=\Omega_0(1+{}^3/_2k^2R_D^2),\qquad \omega_f=(\Omega_0^2+f^2c^2)^{1/2}, \quad (14)$$

where $\Omega_0 = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency.

Inasmuch as $kR_D \ll 1$, it follows from the conservation laws (13) that the transverse wave which is formed as the result of the merging of two Langmuir oscillations has a frequency $\omega_f \approx 2\Omega_0$ and a wave number $\sqrt{3}\Omega_0/c$; the latter is smaller by a factor v/c than the mean wave number of the Langmuir oscillations existing in a slightly turbulent plasma or one in thermodynamic equilibrium. Therefore, the processes of merging take place in practice only under conditions for which $\mathbf{k}_1 = -\mathbf{k}_2$.

The kinetic equation for the distribution function of transverse waves is constructed by means of the Lagrangian (12) according to the usual scheme. Keeping only the quadratic term in the number of longitudinal waves in the collision integral, we get

$$\frac{\partial n_f}{\partial t}=\frac{2\pi}{\hbar^2}\sum_{k_1}|\Phi_{f;\,k_1f-k_1}|^2N_{k_1}N_{f-k_1}\delta(\omega_f-2\Omega_0). \quad (15)$$

In the case under consideration, Eqs. (10) take the form

$$\varphi_k'=\frac{1}{k}(2\pi\hbar\Omega_k)^{1/2}a_k,\qquad A_f'=c\left(\frac{2\pi\hbar}{\omega_f}\right)^{1/2}b_f. \quad (16)$$

We assume the distribution function for the particles to be Maxwellian. Taking it into account that $kv \ll \Omega_k \approx \Omega_0$, and substituting in (15)

$$N_k=|E_k|^2/2\pi\hbar\Omega_0,\qquad n_f=|E_f|^2/2\pi\hbar\omega_f,$$

we get

$$\frac{\partial}{\partial t}|E_f|^2\approx\pi\left(\frac{e}{m\Omega_0}\right)^2\sum_k f^2\cos^2\vartheta I_{-kk}\delta(\omega_f-2\Omega_0),$$

where ϑ is the angle between the vectors **k** and **f**.

We integrate the latter equation with respect to df; as a result, we get for the rate of increase of energy of the transverse oscillations

$$\frac{\partial W}{\partial t}\approx 8\sqrt{3}\frac{\pi^2}{c^5}\left(\frac{e\Omega_0}{m}\right)^2\sum_k I_{-kk},$$

$$W=\int|\mathbf{E}_f|^2df,\qquad I_{-kk}=|\mathbf{E}_k|^2|\mathbf{E}_{-k}|^2. \quad (17)$$

We assume that there is in the plasma a packet of Langmuir oscillations of width $\Delta \mathbf{k} = \Delta k_x \Delta k_y \Delta k_z$ and that the energy density of the wave vectors is constant in this interval. Here we can write

$$\frac{1}{2\pi} \sum_k |\mathbf{E}_k|^2 = \frac{1}{2\pi} |\mathbf{E}^l|^2 \Delta \mathbf{k} = \mathscr{E}_l nT, \qquad (18)$$

where n is the plasma density. By means of this relation, (17) is rewritten in the form

$$\frac{\partial W}{\partial t} \approx 8\sqrt{3}\pi^3 \left(\frac{v_{T_e}}{c}\right)^5 \mathscr{E}_l^2 \Omega_0 \frac{nT}{R_D^3 \Delta \mathbf{k}}. \qquad (19)$$

Let us consider Langmuir oscillations at thermodynamic equilibrium. In this case, the waves have a Rayleigh-Jeans distribution, while the wave vector must be cut off from above at the value R_D^{-1}. Then the energy density of the oscillations will be:

$$\frac{1}{(2\pi)^3} \int N_k \hbar \Omega_k d\mathbf{k} = \frac{1}{2\pi^2} \frac{nT}{N_D}.$$

Thus, in the case of thermodynamic equilibrium, $\mathscr{E}_l = (1/2)\pi^2 N_0$ and $\Delta \mathbf{k} = 1/R_D^3$; we then get from Eq. (19)

$$\frac{\partial W}{\partial t} \approx \left(\frac{v_{T_e}}{c}\right)^5 \frac{\Omega_0}{N_D^2} nT. \qquad (20)$$

The mechanism considered for the transfer of energy to the transverse branch of oscillations is especially effective in the presence of epithermal noise in the plasma. Let the noise be formed in an unstable system of plasma-beam as the result of the development of quasilinear relaxation. The volume of the spectrum of excited oscillations in wave-vector space is

$$\pi (\Delta k_\perp)^2 \Delta k_\parallel \approx \pi \frac{\Omega_0^3}{u_b^3} \frac{\Delta v_\parallel}{u_b} (kR_D)^2. \qquad (21)$$

Here u_b is the velocity of the beam, and $\Delta v_\parallel$ is the scatter of velocities in the beam along the direction of its motion. Drummond has found [17] that the quasilinear relaxation leads to an increase in the energy density of the thermal noise by a factor

$$\frac{8\pi^2 R_1}{(kR_D)^5} \left(\frac{\gamma}{\Omega_0}\right) N_D,$$

where R_1 is the product of the spectrum growth time by the increment, while the remaining notation is standard. We note that this factor can take on gigantic values ($\sim 10^{10}$).

Taking (21) into account, Eq. (19) can be written in the form

$$\frac{\partial W}{\partial t} \approx \mu \left(\frac{v_{T_e}}{c}\right)^5 \frac{\Omega_0}{N_D^2} nT, \qquad (22)$$

$$\mu = 10^3 \pi^6 \frac{R_1}{(kR_D)^{10}} \left(\frac{\gamma}{\Omega_0}\right)^2 \left(\frac{u_b}{v_{T_e}}\right)^3 \left(\frac{u_b}{\Delta v_\parallel}\right) N_D^2. \qquad (23)$$

5. In addition to Langmuir oscillations, ion-sound waves also exist in an isotropic homogeneous plasma. (It is assumed that the condition for existence of ion sound is satisfied—the temperature of the electrons greatly exceeds the temperature of the ions $T_e \gg T_i$.) In connection with this fact, there is a possibility of the appearance of transverse oscillations as the result of the merging of Langmuir and ion-sound waves. The laws of conservation which should be satisfied in the merging process have the form

$$\Omega_k + \omega_q = \omega_f, \qquad \mathbf{k} + \mathbf{q} = \mathbf{f}. \qquad (24)$$

Here ω_q is the frequency of the ion sound, while q is its wave vector. We limit ourselves here to long-wave ion sound and use in what follows the dispersion law $\omega_q = c_s q$, where $c_s = (T_e/M)^{1/2}$.

Inasmuch as $\omega_q \ll \Omega_0$, the transverse wave which arises as the result of the merging of the ion sound with the Langmuir wave has, according to the conservation laws (24), a frequency which differs slightly from the Langmuir frequency $\omega_f \approx \Omega_0$, while the wave number $f \approx (2\omega_q/\Omega_0)^{1/2}\Omega_0/c$ is much less than the characteristic wave numbers of both the Langmuir and ion-sound oscillation existing in thermodynamic equilibrium or in a weakly turbulent plasma. Therefore the processes of merging take place in practice only if $\mathbf{k} = -\mathbf{q}$.

Using the expression for the permittivity of the plasma in the limit of low frequencies, the first formula in Eq. (10) can be rewritten in the form

$$\varphi_q' = R_D (2\pi\hbar\omega_q)^{1/2} a_q. \qquad (25)$$

We substitute in the kinetic equation (15)

$$\mathbf{k} = \mathbf{k}_2, \qquad N_{k_1} = N_q = \frac{|\mathbf{E}_q|^2}{2\pi\hbar\omega_q} (qR_D)^{-2}.$$

The equation for the matrix element $\Phi_{f;kq}$ is found by the method described above. Omitting further simple calculations, we find from (15) for the rate of transfer of energy to the transverse oscillations:

$$\frac{\partial W}{\partial t} \approx \frac{2\sqrt{2}}{3} \pi^2 \frac{e^2}{c^3} \Omega_0^{7/2} \left\langle \frac{1}{m(\mathbf{nv})^2} \right\rangle \sum_q \frac{\omega_q^{1/2}}{q^2} I_{-qq}, \qquad (26)$$

where the angle brackets denote averaging over the particle distribution function.

We consider the more interesting case when the increase in energy of transverse oscillations in a plasma takes place as the result of interaction of the Langmuir packets and ion-sound waves. Let the effective width of the packets be the same: $\Delta \mathbf{k} = \Delta \mathbf{q}$, and let the energy density in the interval $\mathbf{k}_0 - \Delta \mathbf{k}_0/2$, $\mathbf{k}_0 + \Delta \mathbf{k}_0/2$ be constant. We can write down the following expression for the ion sound waves:

$$\frac{1}{2\pi}\sum_q |\mathbf{E}_q|^2(qR_D)^{-2} = \frac{1}{2\pi}|\mathbf{E}^s|^2(\Delta q) = \mathscr{E}_s nT_e. \quad (27)$$

With the aid of the expressions (18) and (27), we can write Eq. (26) in the form

$$\frac{\partial W}{\partial t} \approx \pi^3\left(\frac{m}{M}\right)^{1/4}\left(\frac{v_{T_e}}{c}\right)^3 \frac{(k_0\cdot R_D)^{1/2}}{(\Delta k_0\cdot R_D^3)} \mathscr{E}_l \mathscr{E}_s \Omega_0 nT_e. \quad (28)$$

In the case of thermodynamic-equilibrium noise, we then get in order of magnitude

$$\frac{\partial W}{\partial t} \approx 10^{-2}\left(\frac{v_{T_e}}{c}\right)^3 \frac{\Omega_0}{N_D^2} nT_e. \quad (29)$$

It follows from Eqs. (18), (19), (27), and (28) that the rate of increase of the energy of transverse oscillations from the merging of the ion-sound with the Langmuir wave exceeds the rate of increase from the merging of two Langmuir waves if

$$(v_{T_e}/c)^2 < 10^{-2}|\mathbf{E}^s|^2/|\mathbf{E}^l|^2. \quad (30)$$

This condition takes the following form for thermal fluctuations:

$$(v_{T_e}/c)^2 < 10^{-2}. \quad (31)$$

Equation (28) can be used to estimate the power of the radiation generated in the plasma in which there are epithermal longitudinal waves. For example, let us consider the case in which the Langmuir oscillations are excited in a system composed of a plasma and two rather fast, oppositely directed beams, as the result of the development of quasilinear relaxation. Inasmuch as the beams are rather fast, one can assume that the ion-sound waves are equilibrium waves. We used the well-known relations[1,13]

$$\Delta k_z \approx \Omega_0\frac{\Delta v_z}{u_b^2}, \qquad (\Delta k_\perp)^2 \approx 2\pi\Omega_0^2\frac{(\Delta v_\perp)^2}{u_b^4},$$

$$\mathscr{E}_l \approx \frac{n_b m u_b \Delta v_z}{nT_e}, \qquad \mathscr{E}_s \approx \frac{1}{2\pi^2 N_D}.$$

Here u_b and n_b are the velocity and density, respectively, of the beam ($u_b > v_{Te}$), and Δv is the thermal velocity scatter in the beam.

For the mean wave number k_0, we can put $k_0 \approx \Omega_0/u_b$. We then have from Eq. (28)

$$\frac{\partial W}{\partial t} \approx 10^{-1}\left(\frac{u_b}{c}\right)^3\left(\frac{v_{T_e}}{u_b}\right)^{1/2}\left(\frac{u_b}{\Delta v_\perp}\right)^2\frac{\Omega_0}{N_D} n_b m u_b^2. \quad (32)$$

It follows from this equation that the maximum effect is obtained if one uses beams of high energy.

I express my sincere gratitude to A. A. Vedenov for his attention and for a number of discussions.

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Translated by R. T. Beyer
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