PROBLEMS IN THE THEORY OF NONLINEAR OSCILLATIONS OF AN INHOMOGENEOUS PLASMA

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In the first part of this paper we derive equations that describe the interaction of drift waves in a weakly turbulent inhomogeneous plasma. In contrast with a number of other investigations,[4-6] the electric field oscillations are expanded not in plane waves but in the WKB solutions of the linear problem. In the second part of the paper it is shown that a weakly turbulent plasma is unstable against flute perturbations extended along the magnetic field if $T_i/T_e < (mH^2/8\pi nT_eM)^{1/4}$. The strong turbulence that arises under these conditions evidently destroys the plasma boundary in a time $\tau \sim (eH/cT_e)(n/T_e)^2(8\pi nT_eM/H^2m)^{1/4}$.

INTRODUCTION

THE plasma-physics literature contains a large number of papers devoted to investigations of the stability and characteristic oscillations of a spatially inhomogeneous plasma confined by a magnetic field H.[1-3] Primary attention has been given to low-frequency irrotational oscillations ($\mathbf{E} = -\nabla\varphi$) which can exist only in an inhomogeneous plasma; these are the so-called drift waves. It has been shown that drift waves are always unstable in a low-pressure plasma $\beta = 8\pi p/H^2 \ll 1$ with a Maxwellian particle velocity distribution. This universal drift-wave instability is perhaps the basic motivation for the great interest that has been shown in this topic.

A nonlinear theory of drift waves has been developed recently and has been used to estimate the "coefficient of anomalous diffusion" of plasma particles across a confining magnetic field (cf.[4,5]). Without dwelling on certain numerical discrepancies in the results in these papers (a more detailed presentation of the work of Galeev and the author[4] is given in the review in[6]), it is of interest to point out some characteristic features of these results: the statistical approach is manifest in the fact that the phases of waves characterized by different values of the wave vector are assumed to be random in the zeroth approximation in a small parameter that characterizes the strength of the wave-wave interaction.

It is our purpose in the present work to examine certain analyses that have been used earlier.[4,5] Thus, in[4,5] the oscillating fields were not expanded in the characteristic functions of the linear problem, but in plane waves. In Sec. 2 of the present work, in deriving the kinetic equation for waves we use a system of characteristic functions and show that the final equation is not very different from those that have been obtained earlier.[4-6]

It has recently been shown by Vedenov and the author[7] that a nonlinear theory based on the assumption of weak wave-wave coupling gives rise to a number of nonlinear processes such as the instability of turbulent plasma against slow large-scale perturbations. (In[7] the instability of a turbulent plasma in which plasma oscillations are excited was studied. The growth rate for the perturbations was found to be proportional to the amplitude of the plasma oscillations.)

In the third section of the present paper we investigate the stability of the weakly turbulent state of an inhomogeneous plasma. If the ion temperature is much smaller than the electron temperature

$$T_i/T_e \lesssim (mH^2/8\pi nT_eM)^{1/4},$$

it can be shown that the weak turbulence grows rapidly and that the plasma goes into a new turbulent state in which the equations used to describe weak turbulence no longer apply.

1. CHARACTERISTIC OSCILLATIONS AND INSTABILITY OF AN UNSTABLE LAMINAR PLASMA: BASIC EQUATIONS

In this work we shall be interested in low-frequency $\omega \ll eH/Mc$ long-wave $k^2Mc^2T_i/e^2H^2 \ll 1$ oscillations of a plasma in a magnetic field with straight lines of force (ω and $\mathbf{k}$ are the frequency and wave vector of the oscillation; T_i and

T_e are the ion and electron temperatures). It is assumed that the unperturbed pressure of the plasma varies in only one direction (along the x axis) and that it is much smaller than $H^2/8\pi$ and that the magnetic-field variation on the characteristic length scale $L \sim n(dn/dx)^{-1}$ can be neglected.

We shall be concerned with oscillations characterized by irrotational electric fields $\mathbf{E} = -\nabla\varphi$. Mikhaĭlovskiĭ and the author have shown [1] that drift waves do not perturb the magnetic field if their phase velocity along $\mathbf{H}$ is much smaller than $v_A = H(4\pi nM)^{-1/2}$. It will be shown below that an inhomogeneous plasma which becomes turbulent as a result of the drift-wave instability is, in turn, unstable against flute instabilities parallel to the magnetic field. The electric field associated with these perturbations is also irrotational.

We shall use two simplifying assumptions in this work: it will be assumed a) that the ion temperature T_i is much smaller than the electron temperature T_e where T_e = const and b) that the phase velocity of the drift oscillations along $\mathbf{H}$ is large compared with the ion acoustic velocity $c_s = (T_e/M)^{1/2}$; if the latter condition is satisfied the ion motion along the magnetic field can be neglected. With these simplifying assumptions the drift waves and the flute instabilities are described by the following equations:

the ion equation of motion and equation of continuity

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{e}{M}\nabla\varphi + \frac{e}{Mc}[\mathbf{vH}], \tag{1*}$$

$$\frac{\partial n_i}{\partial t} + \operatorname{div} n_i\mathbf{v} = 0; \tag{2}$$

the drift kinetic equation for the electrons

$$\frac{\partial f}{\partial t} + v_z\frac{\partial f}{\partial z} - \frac{c}{H}[\nabla\varphi\mathbf{h}]\nabla f + \frac{e}{m}\frac{\partial\varphi}{\partial z}\frac{\partial f}{\partial v_z} = 0,$$
$$f_0 = \frac{n_0}{u_e}\exp\left(-\frac{mv_z^2}{2T_e}\right) \tag{3}$$

and the neutrality condition

$$n_i = n_e = \int f dv_z; \tag{4}$$

where v_z is the electron velocity component along $\mathbf{H}$ (z axis) $\mathbf{v} = \mathbf{v}\{v_x, v_y, 0\}$ is the ion velocity, $\mathbf{h} = \mathbf{H}/H$, $u_e = (2T_e/m)^{1/2}$.

If we exploit the fact that the left side of Eq. (1) is small compared with $\omega_H \mathbf{v}\times\mathbf{h} = e\mathbf{v}\times\mathbf{H}/Mc$ and express $\mathbf{v}$ in terms of $\nabla\varphi$ and substitute $\mathbf{v}$ in Eq. (2), (1) and (2) are replaced by

$$\frac{\partial n}{\partial t} - \operatorname{div} n\left\{\frac{c}{H}[\nabla\varphi\mathbf{h}] + \frac{c}{H\omega_H}\frac{\partial}{\partial t}\nabla\varphi - \frac{c^2}{H^2\omega_H}([\nabla\varphi\mathbf{h}]\nabla)\nabla\varphi\right\} = 0. \tag{5}$$

We first find the spectrum of characteristic small oscillations described by (3)—(5) in the linear approximation in the amplitude φ. The solutions of the linearized system (3)—(5) are written in the form

$$\varphi(\mathbf{r}, t) = \sum_\varkappa \varphi_\varkappa(x)\exp\{-i\omega_\varkappa t + i\varkappa\mathbf{r}\}, \qquad \varkappa = \varkappa\{0, k_y, k_z\}.$$

Omitting some simple calculations (these can be found in [1,2]) we present the final equation for the case $\omega/k_z \ll u_e$:

$$\rho_H^2\frac{d}{dx}n\frac{d\varphi_\varkappa}{dx} - n\left[k_y^2\rho_H^2 + \left(1 - \frac{k_y v_d}{\omega_\varkappa}\right)\left(1 + i\sqrt{\pi}\frac{\omega_\varkappa}{|k_z|u_e}\right)\right]\varphi_\varkappa = 0;$$
$$\rho_H^2 = \frac{Mc^2T_e}{e^2H^2}, \qquad v_d = -\frac{cT_e}{eH}\frac{1}{n}\frac{dn}{dx}. \tag{6}$$

The theory of equations such as this has been fully developed. In the semiclassical limit $|(1/k_x n)(dn/dx)| \ll 1$ the solution of (6) in the region $a < x < b$ is given by

$$\varphi_\varkappa = \sum_l \varphi_{\varkappa l},$$

$$\varphi_{\varkappa l} = \frac{C_{\varkappa l}}{|k_x n|^{1/2}}\left[\exp\left(i\int_a^x k_x dx\right) + \exp\left(-i\int_a^x k_x dx\right)\right], \tag{7}$$

$$k_x = \frac{1}{\rho_H}\left[-k_y^2\rho_H^2 - \left(1 - \frac{k_y v_d}{\omega_\varkappa}\right)\left(1 + i\sqrt{\pi}\frac{\omega_\varkappa}{|k_z|u_e}\right)\right]^{1/2}. \tag{8}$$

The spectrum of characteristic frequencies $\omega_\kappa(l)$ is determined from the Bohr quantization rule

$$\int_a^b k_x dx = \pi l, \tag{9}$$

where l is a large positive integer and $a(\kappa, l)$, $b(\kappa, l)$ are the turning points (the points at which $\operatorname{Re} k_x = 0$). We now wish to note a property of the characteristic functions (7) which is important for our purposes; these functions are orthogonal with respect to the weighting factor:

$$\frac{dn}{dx} = -\frac{\omega_\varkappa}{k_y}\frac{eHn}{cT_e}(1 + k_x^2\rho_H^2 + k_y^2\rho_H^2).$$

For any given distribution $n(x)$, using (9) it is possible to find in explicit form the dependence of the oscillation frequency $\operatorname{Re}\omega_\kappa$ and the growth rate $\gamma_\kappa = \operatorname{Im}\omega_\kappa$ on κ and l. However, the sign of γ_κ can be established for arbitrary $n(x)$. For

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.

this purpose we exploit the smallness of the quantities $\omega/k_z u_e$ and γ/ω and expand the radical in (9). Separating real and imaginary parts we find

$$\pi l \rho_H = \int_a^b \left(\frac{k_y v_d}{\omega_\varkappa} - 1 - k_y^2 \rho_H^2 \right)^{1/2} dx, \tag{10}$$

$$\gamma_\varkappa = \sqrt{\pi} \frac{\omega_\varkappa^2}{|k_z| u_e} \int_a^b \rho_H^2 (k_x^2 + k_y^2) \frac{dx}{k_x} \Big/ \int_a^b \Big[1 + (k_x^2 + k_y^2) \rho_H^2 \Big] \frac{dx}{k_x}. \tag{11}$$

2. STATISTICAL THEORY OF NONLINEAR INTERACTION OF DRIFT WAVES

Because of nonlinear terms in (1)—(5) the amplitudes $\varphi_{\kappa l}$ and phases $\alpha_{\kappa l}$ of the characteristic oscillations do not, in general, vary independently. In a weakly turbulent plasma $\gamma \ll \omega$ the nonlinear interaction which leads to a limitation on the growth rates of the unstable oscillations becomes important at relatively low amplitudes $\varphi_{\kappa l}$. For this reason we may assume that the phases of the characteristic oscillations $\alpha_{\kappa l}$ are coupled weakly. Thus, if a turbulent state arises in an unstable inhomogeneous plasma it can be represented by an ensemble of a large number of weakly interacting oscillations. We shall show in the following section that this weakly turbulent state is not always stable itself.

The nonlinear terms in (1)—(5) can be regarded as small corrections. The principal nonlinearity in the drift oscillations is due to ion motion. Using the completeness property of the system of characteristic functions (6) we now write the solution of the system of equations consisting of (2), (3), and (5) in the form

$$\varphi(\mathbf{r}, t) = \sum_{\varkappa l} \varphi_{\varkappa l}(x, t) \exp\{-i\omega_\varkappa t + i\varkappa \mathbf{r}\},$$

$$\varphi_{\varkappa l} = \frac{C_{\varkappa l}(t)}{|k_x n|^{1/2}} \Big[\exp\Big(i \int_a^x k_x dx \Big) + \exp\Big(-i \int_a^x k_x dx \Big) \Big],$$

$$k_x = \frac{1}{\rho_H} \left(\frac{k_y v_d}{\omega_{\varkappa l}} - 1 - k_y^2 \rho_H^2 \right)^{1/2}, \quad \varkappa = \varkappa\{0, k_y, k_z\}. \tag{12}$$

To ensure that the function $\varphi(\mathbf{r}, t)$ is real we impose the following conditions on $\omega_{\kappa l}$ and $C_{\kappa l}$:

$$\omega_{-\varkappa, l} = -\omega_{\varkappa l}, \qquad C^*_{-\varkappa, l} = C_{\varkappa l}. \tag{13}$$

The time dependence of $C_{\kappa l}(t)$ is related to the nonlinear wave interaction. Hence we expect that

$$(1/\omega_{\varkappa l})(d \ln C_{\varkappa l}/dt) \ll 1.$$

In the linear approximation in φ we have from (3)

$$\delta n = \sum_{\varkappa l} n \frac{e\varphi_{\varkappa l}}{T_e} \left(1 + i\sqrt{\pi} \frac{\omega_{\varkappa l}}{|k_z| u_e} \right). \tag{14}$$

Substituting (12) and (14) in (5) we obtain the single nonlinear equation

$$\sum_l \Big(\frac{dC_{\varkappa l}}{dt} - \gamma_{\varkappa l} C_{\varkappa l} \frac{(1 + k^2\rho_H^2)}{|k_x n|^{1/2}} \exp\Big(i \int_a^x k_x dx - i\omega_{\varkappa l} t \Big)$$
$$= -\frac{c}{2H} \rho_H^2 \sum_{\varkappa' l', \varkappa'' l''} \frac{[\mathbf{k}'\mathbf{k}'']_z (k'^2 - k''^2)}{|k_x' k_x''|^{1/2}} \delta_{\varkappa, \varkappa' + \varkappa''} C_{\varkappa' l'} C_{\varkappa'' l''}$$
$$\times \exp\Big\{ -i(\omega_{\varkappa' l'} + \omega_{\varkappa'' l''}) t + i \int_{a(\varkappa' l')}^x k_x' dx + i \int_{a(\varkappa'', l'')}^x k_x'' dx \Big\};$$
$$\mathbf{k} = \{k_x(\varkappa, l, x), k_y, k_z\}, \qquad k^2 = k_x^2 + k_y^2. \tag{15}$$

In (15) we have only taken account of the principal nonlinear terms [in the parameters $(dn/dx)/nk_y$, $(dn/dx)/nk_x$]; $\delta_{\kappa,\kappa'+\kappa''}$ is the Kronecker delta. Here and everywhere below the x-component of k assumes the two values $\pm(k_y v_d/\omega_{\kappa l} - 1 - k_y^2 \rho_H^2)^{1/2}$ in the summation over l. In order to save space below whenever possible we will not write the lower limits $a(\kappa, l)$ on the integrals nor the subscripts on ω, γ and C and the summation indices:

$$\int^x k_x dx \equiv \int_{a(\varkappa, l)}^x k_x dx, \qquad \omega \equiv \omega_{\varkappa l}, \qquad \omega' \equiv \omega_{\varkappa' l'},$$

$$C \equiv C_{\varkappa l} \quad \text{and so on.}$$

It should be recalled that the summation is taken over all suppressed indices.

We now integrate (15) with respect to x over the region L, in which $dn/dx \neq 0$, first multiplying by

$$\frac{1}{|k_x n|^{1/2}} \exp\Big(-i \int^x k_x dx \Big).$$

As a result we find

$$\frac{dC_{\varkappa l}}{dt} - \gamma C_{\varkappa l} = -\sum V e^{-i(\omega' + \omega'' - \omega)t} \delta_{\varkappa, \varkappa' + \varkappa''} C' C'',$$

$$V = \frac{c\rho_H^2}{2Ha} \int_L \frac{[\mathbf{k}'\mathbf{k}'']_z (k'^2 - k''^2)}{|k_x k_x' k_x'' n|^{1/2} \omega}$$
$$\times \exp\Big[i \int^x (k_x' + k_x'' - k_x) dx \Big] dx,$$

$$a \equiv a_{\varkappa l} = \int_L \frac{1 + k^2 \rho_H^2}{|k_x| \omega} dx. \tag{16}$$

The growth rate γ is given by (11).

The quantity V contains an integral of the product of three rapidly oscillating WKB functions. This integral is easily computed by the method of steepest decent:

$$V=\left\{\sqrt{\pi}\frac{c\rho_H^2}{2Ha}\left(1+i\,\mathrm{sign}\frac{d(k_x'+k_x''-k_x)}{dx}\right)\right.$$
$$\left.\times\left|\frac{d(k_x'+k_x''-k_x)}{dx}\right|^{-1/2}e^{i\Psi(x)}\frac{[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{|k_xk_x'k_x''n|^{1/2}}\right\}_{x=x_0}; \tag{17}$$

where x_0 is the point at which $k_x' + k_x'' - k_x = 0$; $\Psi(\kappa l, \kappa' l', x_0)$ is the sum of advances of the phases at the point x_0 for the three WKB functions.

The solution of the nonlinear equation (16) can be found by successive approximations. We write C in the form

$$C(t) = C^{(0)} + C^{(1)} + \ldots.$$

The quantity $C^{(1)}(t)$ satisfies (16) with C on the right side replaced by $C^{(0)}$ and so on. We shall omit these simple calculations which can be found, for example, in [6]. We might point out, however, that in computing the quantity $d|C|^2/dt$ it is important to make use of the assumption that the phases $\alpha_{\kappa l}$ of the oscillations $C_{\kappa l}$ are random in the turbulent state so that

$$\langle C_{\kappa l}^{(0)}C_{\kappa' l'}^{(0)}\rangle = |C_{\kappa l}^{(0)}|^2\,\delta_{-\kappa',\kappa}\delta_{l'l};$$

where the angle brackets denote averages over a large number of random phase ensembles $\alpha_{\kappa l}$. We present the result

$$\frac{d}{dt}|C_{\kappa l}|^2 = 2\gamma|C_{\kappa l}|^2 + 4\pi\sum\{|V_{\kappa\kappa'\kappa''}|^2|C'|^2|C''|^2$$
$$-2V_{\kappa\kappa'\kappa''}V_{\kappa'',-\kappa',\kappa}|C|^2|C'|^2\}\,\delta_{\kappa,\kappa'+\kappa''}\,\delta(\omega-\omega'-\omega''). \tag{18}$$

This equation can be written in symmetric form by introducing the "wave occupation number" $N_{\kappa l} \equiv N = e^2\alpha|C|^2/2T_e$. The expression $\alpha_{\kappa l}V_{\kappa\kappa'\kappa''}$ does not change upon permutation of the subscripts since

$$\frac{1}{\omega}[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)\,|_{x=x_0} = \frac{v_d}{\rho_H^2}[\mathbf{k}'\mathbf{k}'']_z\frac{k_y'\omega''-k_y''\omega'}{\omega\omega'\omega''}\bigg|_{x=x_0}$$
$$=\frac{v_d}{\rho_H^2}[\mathbf{k}'\,\mathbf{k}]_z\frac{k_y'\omega-k_y\omega'}{\omega\omega'\omega''}\bigg|_{x=x_0} \tag{19}$$

(x_0 is the point at which $k_x = k_x' + k_x''$, $k_y'' = k_y - k_y'$, $\omega'' = \omega - \omega'$, $k^2\rho_H^2 = k_yv_d/\omega - 1$). Thus, (16) can be replaced by

$$\frac{dN_{\kappa l}}{dt} = 2\gamma N_{\kappa l} + \sum\widetilde{W}(N'N''-NN'-NN'')$$
$$\times\,\delta_{k_x,k_x'+k_x''}\,\delta_{\kappa,\kappa'+\kappa''}\,\delta(\omega-\omega'-\omega''),$$

$$\widetilde{W}=\left\{\frac{4\pi^2\rho_H^6}{Mn}\frac{[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{|k_xk_x'k_x''|^{1/2}\omega^2aa'a''}\left|\frac{d(k_x'+k_x''-k_x)}{dx}\right|^{-1}\right\}_{x=x_0} \tag{20}$$

It is convenient to use another form of this equation

$$\frac{dN_{\kappa l}}{dt} = 2\gamma N_{\kappa l} + \sum_L\int dx\widetilde{W}(x)\left|\frac{d(k_x'+k_x''-k_x)}{dx}\right|$$
$$\times(N'N''-NN'-NN'')\,\delta_{\kappa,\kappa'+\kappa''}\,\delta(k_x'+k_x''-k_x)$$
$$\times\,\delta(\omega-\omega'-\omega''). \tag{21}$$

The relation (21) may turn out to be more convenient for calculation if the explicit form of $n(x)$ is unknown. In this case the quantity $\omega = \omega(k_y l)$ is also unknown and it is impossible to solve the equation $\omega = \omega' + \omega''$ with respect to one of the numbers l' and l'', and to carry out one of the summations. In the integral over x in (21) it is convenient to convert from summation to integration with respect to κ', κ'' and l', l'', making the substitution of variables $l' \to dk_x'$ and $l'' \to dk_x''$. At any given point x the quantity $k_x = k_x(k_y, l, x)$ is a single valued and monotonic function of x. In order to demonstrate this feature we differentiate the following inequality with respect to l:

$$\pi l = \int_{a(\kappa l)}^{b(\kappa l)} k_x d_x, \qquad \rho_H^2k_x^2 = \frac{k_yv_d}{\omega} - 1 - k_y^2\rho_H^2.$$

Whence

$$\frac{\partial k_x}{\partial l} = \pi\frac{k_yv_d}{\omega k_x}\left(\int_a^b\frac{k_yv_d}{\omega k_x}dx\right)^{-1} = \pi\frac{1+k^2\rho_H^2}{\omega a|k_x|} > 0.$$

Making the substitution

$$\sum_{\kappa}\to\int dk_y\,dk_z, \qquad \sum_l\to\int dl\to\int\frac{dl}{dk_x}dk_x,$$
$$\delta_{\kappa,\kappa'+\kappa''}\to\delta(\kappa-\kappa'-\kappa''),$$

we rewrite (21) in the form

$$\frac{dN_{\kappa l}}{dt} = 2\gamma N_{\kappa l} + \int_L dx\sum_{k_x',k_x''\gtrless 0}\int d\mathbf{k}'d\mathbf{k}''W_{\kappa l k'k''}(x)$$
$$\times(N_{k'}N_{k''}-N_{\kappa l}N_{k'}-N_{\kappa l}N_{k''})\,\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')$$
$$\times\,\delta(\omega_{\kappa l}-\omega_{k'}-\omega_{k''}),$$

$$W_{\kappa l k'k''}(x) = \frac{4\rho_H^6}{Mn}\frac{[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)\,\omega_{k'}\omega_{k''}}{a_{\kappa l}\omega_{\kappa l}^2|k_x|(1+k'^2\rho_H^2)(1+k''^2\rho_H^2)}. \tag{22}$$

Here

$$\omega_k = \frac{k_yv_d}{1+k_y^2\rho_H^2}, \qquad N_k = \frac{L'^2}{(2\pi)^2}N_{\kappa l}$$

is the spectral density and L' is the normalized length. This equation is very similar to the kinetic equation for $N_k(x)$ obtained earlier [4-6] where the field $\varphi(\mathbf{r}, t)$ was expanded in traveling plane waves, which are not, however, characteristic functions of the problem. There is one essential difference: in (22) we have taken account of the fact that the phases of waves with the same k_z, k_y, and l but different k_x are not random as in [4-6], but are coupled by the boundary condition; furthermore, the occupation number in (22) is actually an integral of the local occupation number $N_k(x)$ from [4-6].

In principle, the "decay conditions"

$$\mathbf{k} = \mathbf{k}' + \mathbf{k}'', \quad \omega_k = \omega_{k'} + \omega_{k''}$$

can be solved with respect to k'' and k'_x and the integration can be carried out over k'' and k'_x in (22). This procedure leaves the integration over k'_y and k'_z and the resulting equation can be solved more simply than (22).

In the nonlinear wave interaction described by the collision term in (20) the total wave energy and momentum

$$\sum_{\varkappa l} N\omega, \quad \sum_{\varkappa l} N\boldsymbol{\varkappa}$$

are conserved. This feature can be demonstrated using the expression

$$\sum_{\varkappa l, \varkappa' l', \varkappa'' l''} \binom{\omega}{\varkappa} \widetilde{W}(N'N'' - NN' - NN'') \times \delta_{\varkappa, \varkappa'+\varkappa''} \delta_{k_x, k_x'+k_x''} \delta(\omega - \omega' - \omega'')$$

in which we carry out a cyclic permutation of the suppressed indices and use the symmetry of the quantity $\widetilde{W}$.

If one neglects the interaction of drift waves with the average background (in (20) this is the linear Landau excitation (damping) due to resonance electrons) the wave system must have the properties of an isolated statistical system: the interaction between waves and quasiparticles increases the entropy of the system

$$S = \sum_{\varkappa l} \ln N_{\varkappa l},$$

while the distribution function for N approaches the distribution $N = \Theta/\omega$, for which the entropy is a maximum. We note that (20) exhibits these properties of a statistical system comprising a large number of waves. To show this, we divide (20) (without the $2\gamma N$ term) by N and sum over all values of κ and l. If one now makes a cyclic permutation of the suppressed indices in the summation and uses the symmetry of the quantity $\widetilde{W}$ it is easy to obtain the following relation

$$\frac{dS}{dt} = \frac{d}{dt}\sum_{\varkappa l} \ln N = \frac{1}{3}\sum \widetilde{W} NN'N'' \left(\frac{1}{N} - \frac{1}{N'} - \frac{1}{N''}\right)^2 \times \delta_{\varkappa, \varkappa'+\varkappa''} \delta_{k_x, k_x'+k_x''} \delta(\omega - \omega' - \omega'').$$

Since $\widetilde{W}NN'N'' > 0$, in accordance with the considerations above we have $dS/dt \geq 0$; the equality holds when $N = N'N''/(N' + N'')$ for any κ and l related by the conditions $\kappa = \kappa' + \kappa''$, $\omega = \omega' + \omega''$. The distribution function $N = \Theta/\omega$, where Θ is independent of κ and l, satisfies this condition.

At this point it might be appropriate to comment on the wave energy $N\omega$ and momentum $N\kappa$. The determination of these quantities naturally follows from the kinetic equation (20) since $\Sigma N\omega$ and $\Sigma N\kappa$ are conserved in the interaction. Thus, the wave energy may be defined as the work which is dissipated in the excitation of the waves. The drift waves are excited by resonance electrons; in unit time these electrons do an amount of work given by

$$\int_L E_z j_z dx = -ie \int_L \sum_{\varkappa l} \omega_{\varkappa l} \varphi_{-\varkappa l} \delta n_{\varkappa l} dx = \sum_{\varkappa l} 2\gamma_{\varkappa l} \omega_{\varkappa l} N_{\varkappa l} = \frac{d}{dt}\sum_{\varkappa l} N\omega$$

[$\delta n_{\kappa l}$ and $\gamma_{\kappa l}$ are defined above, cf. (11), and (14)].

The mechanical momentum of the resonance electrons $mv\delta n$ is negligibly small but the generalized momentum of the particles must be conserved in the magnetic field

$$\mathbf{P} = mn\mathbf{v} - \frac{e}{c}\mathbf{A}n \approx -\frac{e}{2c}[\mathbf{Hr}]n.$$

Hence the increment in wave momentum must be equal to the reduction in the generalized momentum of the resonance particles and

$$-\frac{d}{dt}\int \mathbf{P}\, dx = \frac{e}{c}\int_L [\mathbf{H}\dot{\mathbf{r}}]\, \delta n dx = e\int_L \mathbf{E}\, \delta n dx = \sum_{\varkappa l} 2\gamma_{\varkappa l} N_{\varkappa l} \boldsymbol{\varkappa} = \frac{d}{dt}\sum_{\varkappa l} N\boldsymbol{\varkappa}.$$

The energy and momentum obtained from physical considerations are seen to agree with those obtained from (20).

In principle (22) can be used to find N_k which can then be used to find the anomalous flow of particles across the confining magnetic field nv_x. We have shown above that the expansion of the field $\varphi(r, t)$ in terms of plane waves (rather the WKB functions of the problem) does not lead to any great error. Hence the reader interested in estimating the flow nv_x is referred to the detailed review in [6]. In the following section we shall estimate the energy of the oscillations in the stationary turbulent state of the plasma. An estimate for $\int N_k \omega_k dk$, which will be low, can be obtained by equating the first and third terms of (22):

$$\int N_k\omega_k d\mathbf{k} = \int dk \int_L dx \frac{1+k^2\rho_H^2}{\rho_H^2} nM\frac{c^2}{H^2}|\varphi_k|^2 \approx Mn\frac{\gamma\omega}{k^2}L. \quad (23)$$

3. INSTABILITY OF A WEAKLY TURBULENT INHOMOGENEOUS PLASMA

We show in this section that a weakly turbulent inhomogeneous plasma can be unstable against perturbations of the flute type (along **H**) if $T_i/T_e < (m/M\beta)^{1/4}$. A feature of the flute instability is the fact that a laminar plasma exhibits neutral stability to this instability when $T_i = 0$. Hence, even small nonlinear effects can give rise to an instability.

First of all we shall obtain the equation for a flute perturbation φ_q in a weakly turbulent plasma in the approximation linear in φ_q. For this purpose we integrate the continuity equations for the ions (5) and the electrons with respect to the z coordinate and subtract one from the other. As a result, for quasineutral perturbations we have

$$\int_{-\infty}^{\infty} dz\, \mathrm{div}\, n(\mathbf{v}_i - \mathbf{v}_e) = -\int_{-\infty}^{\infty} dz\, \mathrm{div} \frac{c}{H\omega_H}\left\{\frac{\partial}{dt}\nabla\varphi - \frac{c}{H}([\nabla\varphi\mathbf{h}]\nabla)\nabla\varphi\right\} = 0. \quad (24)$$

The plasma potential $\varphi(\mathbf{r}, t)$ is written in the form

$$\varphi(\mathbf{r},t) = \sum_{q_y}\varphi_q(x,t)e^{iq_y y} + \sum_{\varkappa l}\frac{C_{\varkappa l}(t)}{|k_x n|^{1/2}} 2\cos\left(\int_a^x k_x dx\right)\exp(-i\omega_{\varkappa l}t + i\boldsymbol{\varkappa}\mathbf{r}), \quad (25)$$

where $C_{\kappa l}$ is the amplitude of the drift wave; the quantity $\omega_{\kappa l}$ and the two-dimensional vector $\boldsymbol{\kappa} = \boldsymbol{\kappa}\{0, k_y, k_z\}$, $a(k_y, l)$ and k_x are defined above [(cf. (7) and the following text)] and are computed for $\varphi_q = 0$; $\omega_{-\kappa l} = -\omega_{\kappa l}$, $C^*_{-\kappa l} = C_{\kappa l}$. We substitute (25) in (24) and then linearize the latter with respect to φ_q. Neglecting small terms in the parameters $(dn/dx)/kn$ and $(dn/dx)/qn$ and using (24) we have

$$n\frac{\partial}{\partial t}\left(\frac{\partial^2\varphi_q}{\partial x^2} - q_y^2\varphi_q\right) = \frac{c}{H}\sum\frac{[\mathbf{k}'\mathbf{k}'']_z(k'^2 - k''^2)}{|k_x'k_x''|^{1/2}} \times \exp\left\{-i(\omega'+\omega'')t + i\int^x (k_x'+k_x'')dx\right\} \times \delta_{k_z', -k_z''}\delta_{k_y, k_y'+k_y''}C''\delta C', \quad (26)$$

where k_x', (k_x'') assume the following two values in the summation over κ' and l': $\pm(k_y' v_d/\omega' - 1 - k_y'^2\rho_H^2)^{1/2}$; $C' \equiv C_{\kappa' l'}$, $C'' \equiv C_{\kappa'' l''}$, $\omega' \equiv \omega_{\kappa' l'}$ etc. and the summation is taken over all suppressed indices.

The change in the amplitude of the drift waves $\delta C'$ caused by the flute instability is a nonlinear effect but since the quantity φ_q is small we find that $\delta C'/C' \ll 1$. In flute perturbations

$$\delta n_e = \sum_{q_y}\delta n_q = -\sum_{q_y}\xi_x\frac{dn}{dx}. \quad (27)$$

The displacement ξ is defined by the following expression:

$$\dot{\boldsymbol{\xi}} = -\frac{c}{H}[\nabla\varphi_q\mathbf{h}]. \quad (28)$$

The density perturbation δn_q leads to a perturbation $\delta C'$ in accordance with the equation

$$\omega' a'\frac{dC'}{dt} = -i\int_L dx v_d\sum\frac{[\mathbf{k}\nabla\xi_x]_z}{|k_x k_x'|^{1/2}} \times \exp i\int(k_x - k_x')dx - i(\omega-\omega')t\Big\}\delta_{q_y, k_y'-k_y}\delta_{k_z, k_z'}C,$$

$$a' = \int_L dx\frac{1+k'^2\rho_H^2}{|k_x'|\omega'}. \quad (29)$$

The derivation of this equation is similar to the derivation of (16) except that $\delta n = \delta n_q$.

Equations (26)—(29) and the equation $\mathrm{div}\,\boldsymbol{\xi} = 0$ comprise a complete system of equations for the flute perturbation in a weakly turbulent plasma. We now reduce this system to a single equation. For this purpose, we use the equation

$$\mathrm{div}\,\boldsymbol{\xi} = iq_y\xi_y + d\xi_x/dx = 0$$

to express ξ_y and substitute in (28); (28) and the solution of (29) are then substituted in (26). As a result we obtain a single linear integro-differential equation for $\xi_x(x, t)$:

$$n\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2\xi_x}{\partial x^2} - q_y^2\xi_x\right) = -\frac{c^2}{H^2}\sum\frac{[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{|k_x'k_x''|^{1/2}a'\omega'} \times \exp\left\{i\int^x(k_x'+k_x'')dx - i(\omega'+\omega'')t\right\} \times \delta_{q_y, k_y'+k_y''}\delta_{k_z', -k_z''}\delta_{k_y, -k_y''}\delta_{k_z, -k_z''}\int\frac{dx q_y v_d}{|k_x k_x'|^{1/2}} \times\left[\mathbf{k}\nabla\int_0^t\xi_x e^{i(\omega'-\omega)t}dt\right]\exp\left\{-i\int^x(k_x'-k_x)dx\right\}CC''. \quad (30)$$

Instead of solving (30) we use the random-phase property of $\alpha_{\kappa l}$ for the drift waves in a turbulent plasma and average the right side over a large number of arbitrary phases $\alpha_{\kappa l}$. As a result of the averaging of the sum over $\boldsymbol{\kappa}$ and l we are left with terms for which $\boldsymbol{\kappa} = \boldsymbol{\kappa}''$ and $l = -l''$. In this way we obtain a simpler equation but one which only determines the most probable dependence of ξ_x on time. The solution is written in the form of a superposition of WKB functions (7) of Eq. (6) for $k_y = q_y$:

$$\xi_x = \sum_{p'} e^{-i\Omega_{p'}t} \frac{C_{p'}}{|q_x n|^{1/2}} 2\cos\left(\int_a^x q_x dx\right),$$

$$q_x \equiv k_x = \frac{1}{\rho_H}\left(\frac{q_y v_d}{\omega_{q_y, p'}} - 1 - q_y^2\rho_H^2\right)^{1/2}. \quad (31)$$

The function ξ_x satisfies the boundary conditions since it is bounded and vanishes when the drift oscillations vanish.

We substitute (31) in (30), multiply the latter by

$$q^{-2}(1+q^2\rho_H^2)\,|nq_x|^{-1/2}\exp\left(-i\int^x q_x dx\right)$$

and integrate with respect to x:

$$\Omega_p^2 \int_L \frac{1+q^2\rho_H^2}{|q_x|} dx\, C_p + \frac{c^2}{H^2}\sum\int_L \frac{(1+q^2\rho_H^2)[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{q^2|q_x k_x' k_x'' n|^{1/2} a'\omega'}$$
$$\times \exp\left(i\int^x (k_x'+k_x''-q_x)dx\right)$$
$$\times C_{p'}|C''|^2 \delta_{q_y, k_y'+k_y''}\delta_{k_z', -k_z''}$$
$$\times \int_L dx q_y v_d \frac{[\mathbf{k}''\mathbf{q}]_z}{(\Omega_{p'}-\omega'-\omega'')|k_x'k_x''q_x n|^{1/2}}. \quad (32)$$

The integrals with respect to x in (32) are easily computed by the method of steepest decent (cf. Sec. 2). For fixed $\varkappa' l'$, $\varkappa'' l''$, $p' q_y$ we have

$$\int_L A(x)\exp\left(i\int^x (k_x'+k_x''-q_x)dx\right)dx$$
$$= \sqrt{\pi}\left[\left(1+i\,\mathrm{sign}\frac{d(k_x'+k_x''-q_x)}{dx}\right)\right.$$
$$\left.\times e^{i\Psi(x)}\left[\frac{d(k_x'+k_x''-q_x)}{dx}\right|^{-1/2} A(x)\right]_{x=x_0}, \quad (33)$$

where

$$\Psi(x_0, q_y p', \varkappa' l', \varkappa'' l'') = \int^{x_0} (k_x'+k_x''-q_x)dx,$$

while the point x_0 is determined from the equation $q_x = k_x' + k_x''$. If (33) is now substituted in (32) and a summation is carried out, say over l'', because of the phase factor $\exp\{i\Psi(q_x) - i\Psi(q_x')\}$ in the sum over p' only terms for which $q_x' = q_x$ remain. Hence Eq. (32) can be written in the form

$$\Omega_p^2 = -\frac{2\pi\rho_H^2}{M}\sum\left[\frac{a_p(x)}{a_p q^2}\frac{q_y v_d[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{|k_x'k_x''n|\,|d(k_x'+k_x''-q_x)/dx|}\right]_{x=x_0}$$
$$\times \frac{N'\omega'-N''\omega''}{(\Omega_p-\omega'-\omega'')a'\omega'a''\omega''}\delta_{q_y, k_y'+k_y''}\delta_{k_z', -k_z''}\delta_{q_x, k_x'+k_x''},$$

$$a_p = \int_L a_p(x)dx = \int_L \frac{1+q^2\rho_H^2}{|q_x|\omega_{pq_y}}dx \quad (34)$$

or, somewhat differently,

$$\Omega_p^2 = -\frac{2\pi\rho_H^2}{M}\int_L dx\frac{a_p(x)}{q^2 a_p}\sum\frac{q_y v_d[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{|k_x'k_x''n|}$$
$$\times\frac{N'\omega'-N''\omega''}{(\Omega_p-\omega'-\omega'')a'\omega'a''\omega''}\delta_{q_y, k_y'+k_y''}\delta_{k_z', -k_z''}$$
$$\times\delta(q_x-k_x'-k_x''), \quad (35)$$

where $N\omega = e^2\alpha\omega|C^2|/2T_e$ is the energy of the drift waves.

Proceeding as in Section 2, we convert from a summation over l' and l'', $\varkappa'$ and $\varkappa''$ to integration over k_x', k_x'', $\varkappa'$, $\varkappa''$ in the integration over x. As a result we find

$$\Omega_p^2 = -\frac{2\rho_H^2}{\pi M}\int_L dx\frac{a_p(x)}{nq^2 a_p}q_y v_d\sum_{k_x', k_x''\gtrless 0}\int d\mathbf{k}'d\mathbf{k}''$$
$$\times\frac{[\mathbf{k}'\mathbf{k}'']_z(k'^2-k''^2)}{(1+k'^2\rho_H^2)(1+k''^2\rho_H^2)}\frac{N_k'\omega_k'-N_k''\omega_k''}{\Omega-\omega_{k'}-\omega_{k''}}$$
$$\times\delta(\mathbf{q}-\mathbf{k}'-\mathbf{k}''), \quad (36)$$

where $N_k = L'^2 N_{\varkappa l}/(2\pi)^2$ is the spectral density of the wave energy.

The sign of Ω_p^2 can be established uniquely if $q \ll k'$ for the k' which gives the basic contribution to the integral over k' in Eq. (36). In this case (36) can be written in the form

$$\Omega_p^2 = -\frac{4\rho_H^2}{\pi M}\int_L dx\frac{a_p(x)}{nq^2 a_p}q_y v_d$$
$$\times\sum_{k_x\gtrless 0}\int d\mathbf{k}\frac{[\mathbf{qk}]_z^2(\mathbf{qk})}{(1+k^2\rho_H^2)}\frac{\mathbf{q}\partial N_k\omega_k/\partial\mathbf{k}}{\Omega-\mathbf{q}\partial\omega/\partial\mathbf{k}}. \quad (37)$$

If we neglect Ω compared with $(\mathbf{q}\partial\omega/\partial\mathbf{k})$ and integrate (37) with respect to $\mathbf{k}$ by parts, we obtain finally

$$\Omega_p^2 = -\frac{4\rho_H^2}{\pi M}\int_L dx\frac{a_p(x)}{na_p}$$
$$\times\sum_{k_x\gtrless 0}\int dk\frac{[\mathbf{qk}]_z^2\omega_k^2}{(1+k^2\rho_H^2)(\mathbf{q}\partial\omega/\partial\mathbf{k})^2}\frac{q_y^2}{k_y^2}N_k\omega_k$$
$$= -\frac{2}{\pi}\frac{c^2}{H^2}\int_L dx\frac{a_p(x)}{a_p}\sum_{k_x\gtrless 0}\int dk\frac{[\mathbf{qk}]_z^2\omega_k^2}{(\mathbf{q}\partial\omega_k/\partial\mathbf{k})^2}\frac{q_y^2}{k_y^2}|\varphi_k|^2. \quad (38)$$

Thus, the flute perturbations in an inhomogeneous turbulent plasma are always unstable. If we use the estimate (23) for $\int N_k\omega_k dk$ then $\Omega_p^2 \approx -q^2(\gamma\omega/k^2)_{max}$ and at the limit of applicability of (37) for $q \lesssim k$ we have

$$\Omega_{max}^2 \approx -(\gamma\omega)_{max} \approx -\left(\frac{C_s}{L}\right)^2\left(\frac{m}{M\beta}\right)^{1/2}. \qquad (39)$$

The growth rate for the flute instability Ω satisfies the inequality $\gamma \ll \Omega \ll \omega$. This inequality validates the solutions of Eqs. (29) and (30) found under the assumption that the quantities $C^{(0)}$ are independent of time. Actually, when $\Omega \gg \gamma$ the change in the amplitude of C due to the instability of the drift waves and the nonlinear interaction described by Eq. (22) can be neglected.

If the ion temperature is comparable with the electron temperature the flute instability cannot be excited because of finite Larmor radius stabilization (the Rosenbluth effect). When $T_i \ll T_e$ the effect of the thermal motion of the ions can be taken into account by introducing a magnetic-viscosity term [1] in (24). Some simple calculations lead to the following criterion for stability:

$$\frac{1}{4}\left(\frac{T_i}{T_e}\right)^2 q_y^2 v_d^2 > |\Omega^2| \quad \text{or} \quad \frac{T_i}{T_e} > \left(\frac{m}{M\beta}\right)^{1/4}. \qquad (40)$$

The dispersion equation in (38) can be obtained in another way which sheds light on the physical nature of this instability. Let us consider the particular case in which the plasma density $n(x)$ varies in accordance with the relation $n(x) = ne^{-x/L}$ between dielectric walls at $x = \pm L/2$. Under these conditions

$$k_x = \frac{1}{\rho_H}\left(\frac{k_y v_d}{\omega} - 1 - k_y^2\rho_H^2\right)^{1/2}$$

is independent of x and the WKB solutions (7) of Eq. (6) become standing waves in the x-direction with nodes at $x = \pm L/2$ and oscillation frequencies

$$\omega_l = k_y \frac{cT_e}{eHL}\frac{1}{1 + k_y^2\rho_H^2 + k_x^2\rho_H^2}, \quad |k_x| = \frac{\pi(l+1)}{L}.$$

Suppose that we excite drift oscillations for which $(k_x^2 + k_y^2)\rho_H^2 \ll 1$ in such a plasma. The flute displacement

$$\xi(\mathbf{r}, t) = \sum_{q_x \gtrless 0} \xi \exp(-i\Omega t + iq_y y + iq_x x), \; q_x = \frac{\pi(p+1)}{L}$$

changes the plasma density distribution by an amount

$$\delta n_q = -\xi_x \frac{dn}{dx} = \frac{\xi_x}{L} n.$$

In the perturbed plasma the long-wave drift oscillations $k^2\rho_H^2 \ll 1$ are governed by the equation

$$\frac{\partial\varphi}{\partial t} - \frac{cT_e}{neH}[\nabla(n + \delta n_q)\cdot\nabla\varphi]_z = 0. \qquad (41)$$

The solution of this equation is written in the form

$$\varphi(\mathbf{r}, t) = \sum_{k, k_x \gtrless 0} \varphi_k \exp\left\{-i\frac{cT_e}{eHL}k_y t + i\mathbf{kr} + is_k(\mathbf{r}, t)\right\},$$

$$\operatorname{Im} s_k = 0, \quad \mathbf{k} = \{\pi(l+1)/L, k_y, k_z\},$$

$$\omega_{-k} = -\omega_k, \quad \varphi_{-k} = \varphi_k^*.$$

If $\delta n_q = 0$ then $s_k = 0$. In the linear approximation in δn_q the eikonal s_k satisfies the equation

$$\frac{\partial s_k}{\partial y} = [\nabla\xi_x(\mathbf{r}, t)\,\mathbf{k}]_z = i\sum_{q_x \gtrless 0}[\mathbf{qk}]_z \xi_x e^{-i\Omega t + i\mathbf{qr}}, \qquad (42)$$

since $\partial s_k/\partial t \sim \Omega s_k$ is much smaller than

$$\frac{cT_e}{eHL}\frac{\partial s_k}{\partial x} \sim \omega s_k \frac{q_x}{k_y}.$$

Thus, in the particular case considered here the flute perturbation causes the wave vector of the drift oscillations to change by an amount

$$\delta\mathbf{k} = \nabla s_k = \sum_{q_x \gtrless 0} \delta\mathbf{k}_q e^{-i\Omega t + i\mathbf{qr}} = i\sum_{q_x \gtrless 0}\frac{\mathbf{q}}{q_y}[\mathbf{qk}]_z \xi_x e^{-i\Omega t + i\mathbf{qr}} \qquad (43)$$

and does not change the amplitude φ_k.

In turn the drift waves effect the flute instability through the high-frequency force $\delta\langle\mathbf{F}\rangle$:

$$\delta\langle\mathbf{F}\rangle = -Mn\delta\langle(\mathbf{v}\nabla)\mathbf{v}\rangle = Mn\frac{c^2}{H^2}\langle([\nabla\varphi\mathbf{h}]\nabla)[\nabla\varphi\mathbf{h}]\rangle$$

$$= Mn\frac{c^2}{H^2}\sum_{k, k'}(-i)[\mathbf{qk}]_z[\delta\mathbf{k}_q\mathbf{h}]|\varphi_k|^2$$

$$\times \exp\{-i\Omega t + i\mathbf{qr} + i(k_x - k_x')x\}\,\delta_{ll'}\delta_{k_y, -k_y'}\delta_{k_z, -k_z'}$$

$$= Mn\frac{c^2}{H^2}\sum_{k;\, k_x,\, q_x \gtrless 0}[\mathbf{qk}]_z^2|\varphi_k|^2\xi\exp\{-i\Omega t + i\mathbf{qr}\}$$

$$= Mn\frac{c^2}{H^2}\sum_{k, k_x \gtrless 0}[\mathbf{qk}]_z^2|\varphi_k|^2\xi(\mathbf{r}, t), \qquad (44)$$

which is always directed along the displacement. The angle brackets denote an average over phases of the amplitudes φ_k (cf. above). It is assumed that $|\varphi_k|^2$ is a smooth function of k so that

$$\sum_{k, k'}[\mathbf{qk}]_z[\mathbf{qk}']_z|\varphi_k|^2 e^{i(k_x - k_x')x} = \sum_k[\mathbf{qk}]_z^2|\varphi_k|^2.$$

Equating $\delta\langle\mathbf{F}\rangle$ and

$$Mn\frac{\partial^2\xi}{\partial t^2} = -Mn\Omega^2\xi(\mathbf{r}, t)$$

(for the chosen sign of q_x) we find

$$\Omega^2 = -\frac{c^2}{H^2}\sum_{k, k_x \gtrless 0}[\mathbf{qk}]_z^2|\varphi_k|^2$$

$$= -2\frac{c^2}{H^2}\sum_k(q_x^2k_y^2 + q_y^2k_x^2)|\varphi_k|^2,$$

in agreement with Eq. (38).

If (40) is not satisfied the flute instability leads to a situation in which density fluctuations $\delta n_q \sim n$ arise in the plasma. This strong turbulence is not described by Eq. (22). Such large scale ($q \sim n^{-1} dn/dx$) fluctuations of the flute type can lead to a destruction of the plasma boundary in a characteristic time τ:

$$\tau \sim \frac{1}{\Omega} \sim \frac{eHL^2}{cT_e}\left(\beta \frac{M}{m}\right)^{1/4},$$

that is to say,

$$\frac{L}{\rho_H}\left(\frac{M}{m}\beta\right)^{3/4}\frac{T_e}{T_i}$$

times faster than that due to the turbulent diffusion caused by the drift waves.[6]

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