

PRODUCTION OF PROTON-ANTIPROTON PAIRS IN COLLIDING ELECTRON-POSITRON BEAMS

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The process treated is that of production of longitudinally polarized proton-antiproton pairs in the collision of polarized electron and positron beams; effects of the form factors of all the particles involved are taken into account. Expressions are derived for the angular and energy distributions of the resulting pairs. The formulas are used for an analysis of the influences of form factors and spin states of the particles on the angular and energy distributions in the process. Expressions are found which give the angular and energy dependences of the degree of longitudinal polarization of the proton-antiproton pairs produced.

EXPERIMENTS on the production of elementary particles in the collision of beams of high-energy electrons and positrons are a unique way of studying the behavior of the electromagnetic interaction at small distances. The study of such processes may enable us to determine the limits of applicability of quantum electrodynamics at very large energies.

Unlike the experiments of Hofstadter^[1,2] and Wilson,^[3] from which one could get direct evidence about electromagnetic form factors only in the region of spacelike values of the momentum transfer, these processes offer a possibility of studying the form factors in the region of timelike values. As has been pointed out in a paper by Baier,^[4] this is of extremely great interest.

Calculations have been made in^[4,5] on a number of processes of production of strongly interacting particles in the collision of point electrons and positrons. In^[4,5], however, these processes were investigated without regard to the polarization properties of the particles involved.

In the present paper we study the process of production of proton-antiproton pairs in collisions of electrons and positrons,

$$e^- + e^+ = p + \bar{p}, \tag{1}$$

and include effects of the form factors and the longitudinal polarizations of the spins of all the particles involved (although at present there is no experimental indication that there exists a finite size of electrons). The formulas we obtain enable us to examine the effects of the form-factors of the particles on their angular and energy distributions, and also on the longitudinal spin correlation

in this process.

We point out that, just as in the papers we have cited,^[4,5] our treatment here is in lowest order of perturbation theory; that is, we confine ourselves to consideration of one-photon exchange between the electron-positron and proton-antiproton vertices.

In the center-of-mass system of the colliding ultrarelativistic electron and positron the differential cross section for process (1) is given by the formula¹⁾

$$\begin{aligned} \frac{d\sigma}{d(\cos \theta)} = & \frac{\pi\alpha^2 \beta}{32 E^2} \{ |f_1|^2 (1 - s_e - s_{e^+}) [\Phi_0 + s_p s_{\bar{p}} \Phi_1 \\ & + 2s_e - (s_p - s_{\bar{p}}) a_1 \cos \theta] + \gamma_0^2 \mu_a^2 |f_2|^2 (1 + s_e - s_{e^+}) \\ & \times (\Phi_2 + s_p s_{\bar{p}} \Phi_3) - \mu_a (f_1 f_2^+ + f_1^+ f_2) [2a_1 (1 - s_p s_{\bar{p}}) \\ & \times (1 + s_e - s_p \cos \theta) (1 - s_{e^+} s_p \cos \theta) \\ & + a_2 (1 + s_p s_{\bar{p}}) (1 - s_e - s_{e^+} + 2s_e - s_{e^+} \cos^2 \theta)] \}; \tag{2} \end{aligned}$$

$$a_1(E) = |F_1|^2 + \kappa_a^2 |F_2|^2 + \kappa_a (F_1 F_2^+ + F_1^+ F_2),$$

$$a_2(E) = \gamma^{-2} |F_1|^2 + \kappa_a^2 \gamma^2 |F_2|^2 + \kappa_a (F_1 F_2^+ + F_1^+ F_2),$$

$$\Phi_{0,1}(E, \theta) = a_2(E) \sin^2 \theta \pm a_1(E) (1 + \cos^2 \theta),$$

$$\Phi_{2,3}(E, \theta) = a_2(E) \cos^2 \theta \pm a_1(E) \sin^2 \theta, \tag{3}$$

$$\begin{aligned} \beta = \sqrt{\gamma^2 - 1} / \gamma, \quad \gamma = E / M, \quad \gamma_0 = E / m, \quad a = e^2 / 4\pi, \\ \cos \theta = (\mathbf{p}_p \mathbf{p}_{e^-}) / p_p p_{e^-}. \end{aligned}$$

Here the upper sign is for the functions $\Phi_0(E, \theta)$ and $\Phi_2(E, \theta)$, and the lower sign is for $\Phi_1(E, \theta)$ and

¹⁾In this paper we use a system of units with $c = \hbar = 1$.

$\Phi_3(E, \theta)$; $f_1(q^2)$, $f_2(q^2)$ and $F_1(q^2)$, $F_2(q^2)$ are the form factors, μ_a and κ_a the anomalous magnetic moments, \mathbf{p}_e and \mathbf{p}_p the momenta, and m and M the masses of the electron and proton; E is the total energy of the electron in the c.m.s.; the quantities $s_i = \pm 1$ are the eigenvalues of the projection operator $(\boldsymbol{\sigma}\mathbf{p}_i)/|\mathbf{p}_i|$ which characterizes the longitudinal polarization of the spin of particle i ($i = p, \bar{p}, e^-, e^+$); for $s_i = +1$ we have a right-handed polarized particle, and for $s_i = -1$ a left-handed polarized particle (see the monograph^[6]).

In (2) the terms $\sim s_i s_j$ determine the longitudinal spin correlation between particles i and j ($i \neq j = e^-, e^+, p, \bar{p}$), and the term $\sim s_e s_e s_p s_{\bar{p}}$ gives the correlation between all of the particles.

Averaging over the initial and summing over the final spin states of the particles, and also assuming that $f_1 = 1$, $f_2 = 0$ (point electron without anomalous magnetic moment) we get as a special case of (2) the result of Cabibbo and Gatto.^[5]

Integrating (2) over the angle of emergence of the proton, we get the following expression for the total cross section:

$$\begin{aligned}
 \sigma = & \frac{\pi\alpha^2}{48} \frac{\beta}{E^2} [2|f_1|^2(1 - s_e s_{e^+}) + \gamma_0^2 \mu_a^2 |f_2|^2(1 + s_e s_{e^+}) \\
 & - \mu_a (f_1 f_2^+ + f_1^+ f_2) [1 + s_e s_{e^+} + 2(1 - s_e s_{e^+})]] \\
 & \times (\varphi_0 + s_p s_{\bar{p}} \varphi_1), \quad (4)
 \end{aligned}$$

$$\varphi_{0,1}(E) = a_2(E) \pm 2a_1(E). \quad (5)$$

We see from (4) that the total cross section for process (1) does not depend on the lepton-baryon spin correlations, whereas the expression for the differential cross section contains such terms $\sim s_L s_B$ ($L = e^-, e^+$; $B = p, \bar{p}$).

It can be seen from (2) and (4) that a proton-antiproton pair can be produced by the collision of an electron and positron with longitudinal polarizations of opposite signs ($s_e s_{e^+} = -1$ or -1) or of the same sign ($s_e s_{e^+} = 1$ or -1). Since the part of the effective cross section caused by the anomalous magnetic moment of the electron [terms $\sim f_2(q^2)$] must be much smaller than the other part [terms $\sim f_1(q^2)$], proton-antiproton pairs will be produced mainly in collisions of electrons and positrons with opposite signs of the longitudinal polarization.

Averaging over the initial spin states, we find from (2) and (4)

$$\begin{aligned}
 \overline{d\sigma/d(\cos\theta)} = & \frac{\pi\alpha^2}{32} \frac{\beta}{E^2} [|f_1|^2(\Phi_0 + s_p s_{\bar{p}} \Phi_1) \\
 & + \gamma_0^2 \mu_a^2 |f_2|^2(\Phi_1 + s_p s_{\bar{p}} \Phi_3) \\
 & - \mu_a (f_1 f_2^+ + f_1^+ f_2)(\varphi_0 + s_p s_{\bar{p}} \varphi_1)], \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 \bar{\sigma} = & \frac{\pi\alpha^2}{48} \frac{\beta}{E^2} [2|f_1|^2 + \gamma_0^2 \mu_a^2 |f_2|^2 - 3\mu_a (f_1 f_2^+ + f_1^+ f_2)] \\
 & \times (\varphi_0 + s_p s_{\bar{p}} \varphi_1). \quad (7)
 \end{aligned}$$

In the production of longitudinally polarized protons and antiprotons there are the following possible correlations between them: they can be either right-handed polarized ($s_p s_{\bar{p}} = 1$, $s_p = s_{\bar{p}} = 1$) or left-handed polarized ($s_p s_{\bar{p}} = 1$, $s_p = s_{\bar{p}} = -1$), or else be of opposite signs of longitudinal polarization ($s_p s_{\bar{p}} = -1$, $s_p = -s_{\bar{p}} = 1$ or -1).

The degree of longitudinal polarization of the pairs produced, corresponding to these two types of spin correlation, can be defined by the formula

$$\mathcal{P}(E, \theta) = \frac{\{d\sigma\}_{s_p s_{\bar{p}}=1} - \{d\sigma\}_{s_p s_{\bar{p}}=-1}}{\{d\sigma\}_{s_p s_{\bar{p}}=1} + \{d\sigma\}_{s_p s_{\bar{p}}=-1}}. \quad (8)$$

From (6) and (8) we get for the angular dependence of the degree of longitudinal polarization of the pairs

$$\mathcal{P}(E, \theta) = \frac{|f_1|^2 \Phi_1 + \gamma_0^2 \mu_a^2 |f_2|^2 \Phi_3 - \mu_a (f_1 f_2^+ + f_1^+ f_2) \varphi_1}{|f_1|^2 \Phi_0 + \gamma_0^2 \mu_a^2 |f_2|^2 \Phi_2 - \mu_a (f_1 f_2^+ + f_1^+ f_2) \varphi_0}. \quad (9)$$

As can be seen from (4), (7), and (8), however, the energy dependence of the degree of longitudinal polarization of the proton-antiproton pairs depends neither on the form factors nor on the polarization states of the electrons and positrons. In fact, for all three cases [$s_e s_{e^+} = 1$, $s_e s_{e^+} = -1$, and Eq. (7)] we have

$$\mathcal{P}(E) = \varphi_1(E) / \varphi_0(E). \quad (10)$$

For a quantitative analysis of the formulas (2)–(10) we examine the case of collisions of point electrons and positrons which have no anomalous magnetic moment ($f_1 = 1$, $f_2 = 0$). Furthermore we use the following approximate expressions for the form factors of the proton, as determined from resonance properties in the region of timelike values of the momentum transfer^[7]:

$$F_1 = 1 - \frac{1.18q^2}{q^2 + 30m_\pi^2}, \quad F_2 = 1 - \frac{1.59q^2}{q^2 + 30m_\pi^2}. \quad (11)$$

Here m_π is the mass of the π^\pm meson, and $q^2 = -4E^2$.

Figure 1 shows the dependence of the differential cross section for process (1) on the angle of emergence θ of the proton for $\gamma = 1.3$, as calculated from (2) and (6). Curves 1–6 correspond to the production of proton-antiproton pairs in the collision of electrons and positrons which have opposite signs of the longitudinal polarization, $s_e s_{e^+} = -1$ (or $s_e s_{e^+} = -1$): a) curves 1, 2 are for the cases $s_p = s_{\bar{p}} = 1$ or -1 ; b) 3, 4 are for

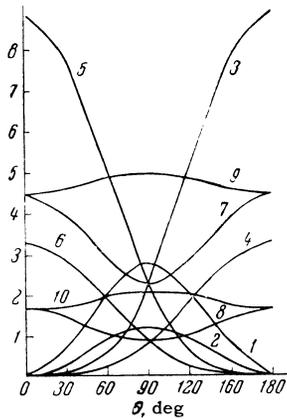
$d\sigma/d(\cos\theta)$ 

FIG. 1.

FIG. 1. Dependence of the differential cross section (in units $\pi\alpha^2/6M^2$) on the angle of emergence of the proton; curves 1, 3, 5, 7, and 9 are for point protons, and curves 2, 4, 6, 8, and 10 for extended protons.

FIG. 2. Dependence of the total cross section (in units $\pi\alpha^2/6M^2$) on the energy of the electron; curves 1, 3, and 5 are for point protons, and curves 2, 4, and 6 are for extended protons.

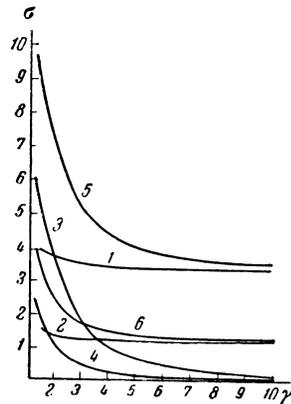


FIG. 2.

$s_p = -s_{\bar{p}} = -1$ ($s_p = -s_{\bar{p}} = 1$); c) 5, 6 are for $s_p = -s_{\bar{p}} = 1$ ($s_p = -s_{\bar{p}} = -1$); d) 7, 8 are for the cases $s_p = -s_{\bar{p}} = 1$ or -1 ; e) 9, 10 are for the production of unpolarized protons and antiprotons. Curves 7–10 correspond to the production of proton-antiproton pairs in the collision of unpolarized electron and positron beams.

Figure 2 shows the dependence of the total cross section for the production of proton-antiproton pairs on the energy of the electron in the c.m.s.: curves 1, 2 are for case a); 3 and 4 are for cases b), c), d); and 5 and 6 are for case e).

As can be seen from Figs. 1 and 2, the angular and total cross sections for the processes of production of either polarized or unpolarized proton-antiproton pairs are extremely sensitive to the form-factors of the proton, and in all of the cases considered above the form-factors (11) for the proton decidedly decrease the values of the differential and total cross sections (curves with even numbers) as compared with the corresponding values for the point proton (curves with odd numbers).

The angular dependence of the degree of longitudinal polarization of the proton-antiproton pairs, as calculated from (9) for $\gamma = 1.3$, shows that at

angles $0^\circ \leq \theta \leq 15^\circ$ almost all of the proton-antiproton pairs produced have the spin correlation $s_p s_{\bar{p}} = -1$ ($s_p = -s_{\bar{p}} = 1$ or -1). As the angle increases there is an increase of the fraction of pairs with the spin correlation $s_p s_{\bar{p}} = 1$ ($s_p = s_{\bar{p}} = 1$ or -1); at a certain value of the angle (in the case of the point proton at 71° , and in that of the extended proton at 65° there is a reversal of the spin of the proton or antiproton in each pair. As the angle increases farther (up to 90°) the number of proton-antiproton pairs with the spin correlation $s_p s_{\bar{p}} = 1$ predominates over the number of pairs with the spin correlation $s_p s_{\bar{p}} = -1$. Since the function $\mathcal{P}(E, \theta)$ is symmetrical relative to the angle 90° , all that has been said applies also to the second half of the range ($90^\circ \leq \theta \leq 180^\circ$).

The energy dependence of the degree of longitudinal polarization $\mathcal{P}(E)$ shows that as the energy of the electrons increases there is an increase of the fraction of the proton-antiproton pairs with the longitudinal spin correlation $s_p s_{\bar{p}} = 1$. This dependence is extremely sensitive to the form-factor of the proton, so that the value of $\mathcal{P}(E)$ for the case of the extended proton is everywhere larger than the value for the point proton.

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