CONDITIONS FOR HEATING UP OF A PLASMA BY THE RADIATION FROM AN
OPTICAL GENERATOR

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Various processes accompanying rapid pulsed heating of small volumes of a plasma by radiation from a laser are considered. The power and duration of the radiation pulse required for heating a hydrogen plasma to temperatures of the order of $10^6-10^8$ degrees are estimated.

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1. OPTICAL RADIATION ABSORPTION COEFFICIENT

The plasma is heated when the electrons absorb radiation in the ion field and subsequently transfer the energy to the ions. The radiation absorption coefficient $k$ and the limiting plasma density $n$ can be determined from the dielectric constant $\varepsilon(\omega)$ of the plasma:

$$
\varepsilon(\omega) = 1 - (\omega_0^2/\omega^2) (1 - i\nu/\omega),
$$

where the Langmuir frequency is $\omega_0 = (4\pi ne^2/m)^{1/2}$ and the collision frequency is $\nu = 4\sqrt{2}\pi e^4 n L / 3(kT)^{3/2}m^{1/2}$; $L = \ln(\lambda/\rho)$ is the Coulomb logarithm and $\lambda = (kT/4\pi e^2 n)^{1/2}$ is the Debye length; $\rho$ is the minimal impact parameter: $\rho \approx e^2/kT$ for $e^2/\hbar \gg 1$ (or $\nu \approx h/mv$ for $e^2/\hbar \ll 1$).

The limiting value of the density is determined by the condition $\omega_0 = \omega$ and amounts to $n \approx 3 \times 10^{21}$ cm$^{-3}$ for ruby emission ($\omega = 3 \times 10^{16}$ sec$^{-1}$). In the case when $Re \varepsilon \gg Im \varepsilon$, the absorption coefficient

$$
k \approx 2(\omega/c) Im \varepsilon \quad \text{is equal to}
$$

$$
k = \frac{16\pi \sqrt{2\varepsilon_0 n L}}{3(mkT)^{3/4}(Re \varepsilon)/\omega_0}. 
$$

It is seen from (2) that if $n \approx 3 \times 10^{21}$, so that $Re \varepsilon \approx 1$, and if $T \approx 10^7$ deg, the order of magnitude of the absorption coefficient can reach $10^4$ cm$^{-1}$, from which it follows that the radiation absorption is quite effective up to high temperatures, $\sim 10^7$ deg.

2. ENERGY LOSS IN PLASMA

In the initial stage of the process, when the temperature is sufficiently high, we can use for the plasma the equation of state of an ideal gas. At $T = 10^7$ deg and $n = 3 \times 10^{21}$ cm$^{-3}$ we have for the energy density $u$, assuming practically total
ionization, \( u = 1.4 \times 10^{13} \text{ erg/cm}^3 \), from which it follows that the radiation energy is of the order of 10 J and is sufficient in principle to heat a volume \( V \approx 10^5 \text{ cm}^3 \) of a rather dense plasma to \( 10^7 \) deg. The relative correction to the equation of state, due to the Coulomb interaction, is of the order of \( e^2/\kappa T \), and under these conditions it amounts to \( 10^{-3} \).

The heating of the plasma, as indicated above, occurs as a result of absorption of radiation by the electrons. On the other hand, the ion gas is heated by electron-ion collisions. The rate of electron and ion temperature equalization has an order of magnitude \( \nu \kappa \), where \( M \) is the ion mass. The corresponding relaxation time \( \tau_0 \) is equal to

\[
\tau = 3M(kT)^{3/2}/16V^{1/2}ne^4L. \tag{3}
\]

If the heating time \( \tau \) exceeds \( \tau_0 \), then the plasma heating can be regarded as isothermal; in the opposite case the ions are heated inefficiently. When \( T = 10^7 \) deg and \( n = 3 \times 10^{21} \text{ cm}^{-3} \), the relaxation time is \( \tau = 1.5 \times 10^{-10} \text{ sec} \), i.e., quite short.

The degree of plasma ionization can be determined from the known Saha formula \( [8] \), which, however, is valid under conditions of thermodynamic equilibrium. Inasmuch as under our conditions the plasma is practically completely transparent to the hard ("thermal") radiation (see below), there is no equilibrium with the radiation. However, if the process of ionization by electron impact is more efficient than the process of photoionization under equilibrium conditions, then Saha's formula can be used, but the temperature in this formula is that of the electrons. The photoionization probability is \( \omega_{ph} = \omega n_0 kT \), where \( n_0 \) is the thermodynamic-equilibrium density of the neutral atoms and \( \omega \) is the recombination radiative power, given by \( (7) \). The probability of ionization by electron impact is \( \omega_e = n_0 \sigma_i \), where \( \sigma_i \) is the ionization cross section and \( n_0 \) the electron density. Under the conditions of interest to us \( (T = 10^7 \text{ deg}, n = 3 \times 10^{21} \text{ cm}^{-3}, \sigma_i \approx 10^{-18} \text{ cm}^2) \) we have \( \omega_e \gg \omega_{ph} \) so that Saha's formula is applicable.

The plasma energy loss is due to the electronic and radiative thermal conductivities. The coefficient of electronic thermal conductivity \( K_e \) is, in accordance with \( [7,6] \)

\[
K_e = 40V^2 k_\nu T^4/\pi^2 ne^4L = 1.24 \times 10^{-4} T_{10^7}^4 \quad (L=15), \tag{4}
\]

where the coefficient \( \delta \) corrects for the electron-electron collisions \( [6] \) and for the secondary electric field. The coefficient of radiative thermal conductivity \( K_{rad} \) is \( [8] \)

\[
K_{rad} = \frac{16}{3} \sigma T^3, \tag{5}
\]

where \( \ell \) is the photon range as averaged by Rosse-land \( [8] \), and \( \sigma \) is the Stefan-Boltzmann constant. At densities on the order of \( 3 \times 10^{21} \text{ cm}^{-3} \) in a hydrogen plasma, \( \ell \) becomes comparable with the dimensions of the system \( (~ 10^{-2} \text{ cm}) \) at temperatures lower than approximately \( 10^5 \) deg, and reaches \( \approx 10^2 \text{ at } T = 10^7 \text{ deg} \).

Thus, at \( T = 10^7 \) deg the plasma is practically transparent and the radiation energy loss is determined by the bremsstrahlung and recombination radiation, \( [6,8,10] \), the powers of which per unit volume are

\[
Q_b = 1.57 \times 10^{-21} T_{10^7}^{3/2}, \quad Q_i = 1.08 \times 10^{-21} n_0 T_{10^7}^{-1/2}, \tag{6,7}
\]

The recombination radiation becomes appreciable at \( T \approx 10^5 \text{ deg} \); at higher temperatures, formula \( (7) \) cannot be used, but the contribution of recombination radiation is small in this region.

On the basis of \( (6) \), the power loss of a plasma with \( n = 3 \times 10^{21} \text{ cm}^{-3} \) and \( V = 10^5 \text{ cm}^3 \) is \( Q_{rad} = Q_b + Q_i = 1.4 \times 10^{11} T_{10^7}^{1/2} \text{ erg/sec} \). The maximum energy loss due to thermal conductivity is \( [2] \)

\[
Q_e = 8\pi k_\nu T^4/4 \approx 3.5 \times 10^{-8} T_{10^7}^{2/2} \text{ erg/sec}. \tag{8}
\]

At \( T = 1.6 \times 10^6 \text{ deg} \) we have \( Q_e = Q_{rad} \). At \( T = 10^7 \text{ deg} \) the losses due to thermal conductivity prevail and amount to \( Q_e \approx 1.3 \times 10^{11} \text{ erg/sec} \) \( (Q_{rad} \approx 4.5 \times 10^{14} \text{ erg/sec}) \). The question of possible thermal insulation of the plasma thus becomes important here. The dependence of the maximum attainable temperature, determined by the thermal conductivity, on the laser power \( Q \) is of the form \( T \sim Q^{2/7} \).

3. GAS DYNAMIC EXPANSION OF A BOUNDED PLASMA

Let us estimate the role of the gas–dynamic expansion during the heating of a plasma, on the basis of the approximate model of free plasma with spherical volume \( V = 4\pi r^3/3 \), described by the average values of the density and temperature. The energy conservation equation (see \([11]\)), integrated over the entire volume of the plasma, yields an equation that describes the time variation of the kinetic and internal energy \( E \) of the plasma:

\[
\frac{d}{dt} \left( \frac{2}{5} \mu + E \right) = Q, \tag{8}
\]

where \( w \) is the average rate of gas–dynamic expansion, \( G \) the mass, and \( Q \) the power of the radiated heat. The equation of motion can be attained from (8) by putting \( Q = 0 \) and \( dE = -pdV \), where \( p \) is
the average pressure. Thus, the system of equations describing the expansion and heating of the plasma assumes the form

$$G \frac{dw}{dt} - 4 \pi r^2 p = 0, \quad \frac{d}{dt} \left( \frac{w}{2} + E \right) = Q. \tag{9}$$

For the case of an ideal gas, when $E = \frac{3NkT}{2}$ (N is the total number of particles in the system), the solution of (9) is of the form

$$r^2 = r_0^2 + \frac{4}{G} \int_0^t \int_0^{t'} dt'' \int_0^{t''} Q(t'') dt''' \tag{10}$$

and in the simple case when

$$Q = \left\{ \begin{array}{ll} \text{const.}, & t > 0 \\ 0, & t < 0 \end{array} \right. \tag{11}$$

we have

$$r^2 = r_0^2 + \frac{2}{3} \frac{Qt^2}{G}, \quad T = \frac{Qt}{s_N Nk} \frac{QpG + r_0^2}{2QpG + r_0^2}. \tag{11}$$

We see from (6) that as $t \to 0$ we have $T \sim 2Qt/3Nk$, which corresponds to a weak influence of the gas-dynamic expansion at the initial instant of time. It is important however, that when $t \gg (3r_0^2G/Q)^{1/3}$ the temperature also varies linearly with time: $T \sim Qt/6Nk$. This circumstance corresponds to the fact that asymptotically the ratio of the thermal energy of the plasma to the energy of gas-dynamic motion is constant. In this case, asymptotically, one quarter of the laser radiation energy goes to heating of the plasma and the rest goes to dynamic expansion. An asymptotic ratio exists apparently also in the real case, but the specific value of this ratio may be somewhat different.

Thus, for the same temperatures to be attained, gas-dynamic considerations lead to a need for increasing the laser power. The characteristic time parameter within which allowance for expansion becomes important is $t_0 = (3r_0^2G/Q)^{1/3}$, and amounts to $t_0 = 2 \times 10^{-9}$ for a laser power $Q = 10^9$ W, $V = 10^{-8}$ cm$^3$, and $G = 10^{-7}$ g. This estimate shows that the gas-dynamic expansion plays an important role and calls for small time durations, on the order of the length of the radiation pulse.

From the foregoing analysis as a whole it follows that the most important role in the plasma heating process is played by the energy loss due to thermal conductivity and to gas-dynamic expansion of the hot plasma. At a laser power on the order of $10^9$ W and a pulse duration of about $10^{-8}$ sec, it is apparently possible to heat a hydrogen plasma of density $3 \times 10^{21}$ cm$^{-3}$ to a temperature somewhat lower than $10^7$ deg, although a specific method of realizing such an experiment still needs further analysis. The possibility of attaining such temperatures depends in our opinion essentially on the progress in the techniques of generating optical radiation.

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2 L. A. Artsimovich, Upravlyaemye termoyadernye reaktsii (Controlled Thermonuclear Reactions), Fizmatgiz, 1961.