ON THRESHOLD ABSORPTION OF SOUND IN A UNIAXIAL ANTIFERROMAGNET

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The frequency and temperature dependence of the coefficient of absorption of sound, resulting from transitions between two branches of the spin-wave spectrum of a uniaxial antiferromagnet, is examined at frequencies near the threshold frequency.

PREVIOUSLY,[1] we have treated the threshold absorption of the energy of an alternating magnetic field by a uniaxial antiferromagnet; we showed that the absorption occurs at frequencies $\omega > \omega_n = 2gH_0$, where H_0 is the external magnetic field, directed along a selected axis of the antiferromagnet. The alternating magnetic field in this case should be polarized parallel to the constant field. It is natural to suppose that, in corresponding fashion, a polarized sound wave whose frequency exceeds some value should also lead to similar transitions. The present article is devoted to an analysis of this phenomenon.

As in the cited work, the absorption considered is caused by the process of transformation of a spin wave of the first sort into a spin wave of the second sort as a result of interaction with a phonon. The energy absorbed in unit time is in this case equal to

$$Q = \frac{V}{(2\pi)^2}\,\omega \int (n_1 - n_2)\,|V_{12}|^2 \delta\,\{\varepsilon_1(\mathbf{k}) + \hbar\omega - \varepsilon^2(\mathbf{k}+\mathbf{f})\}\,d\tau_\kappa, \qquad (1)$$

where $n_{1,2} = n(\epsilon_{1,2})$ [$n(\epsilon)$ is the Bose function]; ω and $\mathbf{f}$ are the frequency and the wave vector of the sound wave; $\omega = sf$ (s = speed of sound); $\epsilon_{1,2}(\mathbf{k})$ is the energy of a spin wave of the first (second) sort; the integration is extended over all values of the wave vectors within the boundaries of a single Brillouin zone; V is the volume of the crystal; V_{12} is the matrix element of the process under consideration.

Our problem is to investigate the dependence of the quantity Q on frequency and on temperature near the threshold value of frequency, $\omega = \omega_n$. As we shall see, to determine the whole form of the curve (i.e., the dependence of the coefficient of absorption Γ, proportional to Q, on frequency at arbitrary frequencies) is possible, in the case considered, only under very special assumptions about the form of the dispersion law of the spin waves.

We shall show first that, except for its dependence on the form of the dispersion law of the spin waves, the quantity Q is always proportional to $\sqrt{\omega - \omega_n}$ at frequencies sufficiently close to ω_n. In fact, because of the presence of the delta function in the expression (1), the integration reduces to an integration over the surface

$$F(\omega, \mathbf{k}) = \varepsilon_1(\mathbf{k}) + \hbar\omega - \varepsilon_2(\mathbf{k} + \mathbf{n}\omega/s) = 0, \quad \mathbf{n} = \mathbf{f}/f. \qquad (2)$$

Let the solution of this equation for the frequency ω (at fixed $\mathbf{n}$) be

$$\omega = \omega(\mathbf{k}). \qquad (3)$$

The existence of a threshold frequency implies that the function $\omega(\mathbf{k})$ has a minimum. We denote by $\mathbf{k}_0$ the point in $\mathbf{k}$-space where $\omega(\mathbf{k})$ attains its minimum value:

$$\omega_n = \min \omega(\mathbf{k}) = \omega(\mathbf{k}_0). \qquad (4)$$

At frequencies $\omega \geq \omega_n$, Eq. (3) can be written in the following form:

$$\omega = \omega_n + \nu(\mathbf{k}), \qquad \nu = \tfrac{1}{2}\alpha_{ik}(k_i - k_{i0})(k_k - k_{k0}),$$
$$\alpha_{ik} = (\partial^2\omega/\partial k_i\,\partial k_k)_{\mathbf{k}=\mathbf{k}_0}. \qquad (5)$$

In other words, at frequencies $\omega \geq \omega_n$ the integration in formula (1) is carried out over the surface of an ellipsoid. Since all quantities except the delta function vary smoothly with frequency, and since by Eqs. (2)—(5)

$$\delta\,[F(\omega, \mathbf{k})] = \frac{1}{|\partial F/\partial\omega|}\,\delta\,[\omega - \omega(\mathbf{k})]$$
$$\approx \frac{1}{|\partial F/\partial\omega|_{\substack{\mathbf{k}=\mathbf{k}_0\\ \omega=\omega_n}}}\,\delta\,[\omega - \omega_n - \nu(\mathbf{k})],$$

therefore

$$Q = \frac{V}{(2\pi)^2}\left\{\frac{\omega(n_1 - n_2)\,|V_{12}|^2}{|\partial F/\partial\omega|}\right\}_{\substack{\mathbf{k}=\mathbf{k}_0\\ \omega=\omega_n}} \int \delta\,[\omega - \omega_n - \nu(\mathbf{k})]\,d\tau_{\mathbf{k}}. \qquad (6)$$

Hence we easily find

$$Q = \frac{V}{(2\pi)^2}\left\{\frac{\omega(n_1 - n_2)\,|V_{12}|^2}{|\partial F/\partial\omega|}\right\}_{\substack{\mathbf{k}=\mathbf{k}_0\\ \omega=\omega_n}} \sqrt{ABC}\,(\omega - \omega_n)^{1/2}.$$

Here 1/A, 1/B, and 1/C are the principal values of the tensor α_{ik}. The temperature dependence of the effect is determined by the factor $(n_1 - n_2)$ at $\mathbf{k} = \mathbf{k}_0$, $\omega = \omega_n$. To estimate this factor, it is necessary to investigate in what region of $\mathbf{k}$-space the point $\mathbf{k}_0$, at which the function $\omega(\mathbf{k})$ reaches its minimum, is located.

If for this purpose we use the approximate (phenomenological) dispersion law, valid for $ak \ll 1$ (the notation is the same as in [1] and [2]),

$$\varepsilon_{1,2} = \sqrt{\varepsilon_0^2 + (\Theta_c ak)^2} \mp \mu H_0,$$

then it turns out that the minimum of the function $\omega(\mathbf{k})$ is reached at infinitely large values of the wave vector;[1] then $\omega_n = 2gH_0/(1 + \Theta_c/\Theta_d)$, where $\Theta_d = s\hbar/a$ coincides in order of magnitude with the Debye temperature. This means, on the one hand, that in order to obtain exact quantitative results we should use a dispersion law that takes account of the discrete structure of the magnet;[2] and on the other, that $\epsilon_1 = \epsilon_1(k_0) \approx \Theta_c \gg T$ (the last inequality is the condition for applicability of the spin-wave concept).

If $\hbar\omega_n \ll T$, then

$$n_1 - n_2 \approx -\frac{\partial n}{\partial \varepsilon}\hbar\omega_n \approx \frac{\hbar\omega_n}{T}\exp\left[-\varepsilon_1(\mathbf{k}_0)/T\right].$$

If, however, $\hbar\omega_n \gg T$, then

$$n_1 - n_2 \approx n_1 \approx \exp\left[-\varepsilon_1(k_0)/T\right].$$

The magnitude of the threshold frequency can be estimated in the following way. Taking into account that $f \ll k$, we expand the function $\epsilon(\mathbf{k} + \mathbf{n}\omega/s)$ in a series. Then from the expression (2) we get

$$F \approx \hbar\omega - 2\mu H_0 - \frac{\omega}{s}\cos\vartheta_{fk}\frac{\partial\varepsilon}{\partial k}$$

(ϑ_{fk} is the angle between the vectors $\mathbf{f}$ and $\mathbf{k}$).

Hence we shall have

$$\hbar\omega_n = 2\mu H_0 \Big/ \left\{1 - \frac{1}{\hbar s}\left(\cos\vartheta_{fk}\frac{\partial\varepsilon}{\partial k}\right)_n\right\},$$

where the values of $\cos\vartheta_{fk}$ and $\partial\epsilon/\partial k$ are taken at the point at which $\omega(\vartheta, k)$ reaches a minimum. The dispersion law of the spin waves can be taken in the form[3]

$$\varepsilon = \varepsilon_{max}\{1 - b_1 a^2(\mathbf{k} - \mathbf{k}_0)^2 + b_2 a^4(\mathbf{k} - \mathbf{k}_0)^4\},$$

where b_1 and b_2 are multiplicative constants of order unity. Then for the threshold frequency we get the following value:

$$\hbar\omega_n = \frac{2\mu H_0}{1 + c\Theta_c/\Theta_d} \qquad (c \sim 1). \tag{7}$$

From (7) it is clear that the threshold frequency is lower in the case of sound that in the case of an alternating magnetic field.

Estimation of the matrix element of the transition shows that the coefficient of absorption Γ has the following order of magnitude:[4]

$$\frac{\Gamma}{\omega_n} \sim \begin{cases} \left(\frac{\Theta_c}{\rho s^2 a^3}\right)^2 \frac{\hbar\omega_n}{T}\frac{\omega_n}{\tilde{\omega}^{3/2}} e^{-\varepsilon_1(\mathbf{k}_0)/T}(\omega - \omega_n)^{1/2} & (T \gg \hbar\omega_n) \\ \left(\frac{\Theta_c}{\rho s^2 a^3}\right)^2 \frac{\omega_n}{\tilde{\omega}^{3/2}} e^{-\varepsilon_1(\mathbf{k}_0)/T}(\omega - \omega_n)^{1/2} & (T \ll \hbar\omega_n) \end{cases}, \tag{8}$$

where the notation $\tilde{\omega}^{-3/2} = a^3\sqrt{ABC}$ has been introduced ($\tilde{\omega} \sim \omega_n$).

Far from the threshold frequency, the macroscopic model is correct. At the threshold frequency for absorption in an alternating magnetic field, $\omega = 2gH_0$, the spin waves that take part in the process are those for which $ak \ll 1$, if $\Theta_d \gg \mu H_0$. At such frequencies, the absorption coefficient no longer contains an exponential factor. Calculation of the absorption coefficient Γ at frequency $\omega = 2gH_0$ shows that $\Gamma \sim T^5$:

$$\frac{\Gamma}{\omega} \sim \frac{\Theta_c}{\Theta_d}\left(\frac{\mu M_0}{\rho s^2 a^3}\right)^2\left(\frac{T}{\Theta_c}\right)^5 \sin^2\vartheta\,(1 + \cos^2\vartheta) \quad (\omega \approx 2gH_0), \tag{9}$$

but it should be remarked that in the formula there are numerical factors of order 10^2, which we have omitted.

The absorption coefficient, as we see, is in this case characterized by a pronounced anisotropy.

Comparison of formulas (8) and (9) shows that the absorption coefficient Γ increases sharply in a narrow frequency interval.

One more remark: since $\epsilon_{1,2}(\mathbf{k})$ is a bounded periodic function, the function $\omega(\mathbf{k})$ [cf. the definition (3)] certainly has a maximum value ω_{max}. This means that the width of the absorption line being considered is finite (for $\omega > \omega_{max}$, absorption is impossible). Depending on the relation between Θ_c and Θ_d, the absorption limit will lie either at very high frequencies, $\omega_{max} \approx \Theta_c/\hbar$ ($c'\Theta_c \geq \Theta_d$, $c' \approx 1$), or in the low-frequency

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[1]Inclusion in the dispersion law of terms due to dipole-dipole interaction does not change anything substantially; in all cases of interest, they are negligibly small.

[2]When account is taken of the periodic structure of the magnet, $\epsilon(\mathbf{k})$ is a periodic function of the wave vector. The last circumstance guarantees attainment of the minimum at finite values of the wave vector.

[3]This form of the dispersion law corresponds to the assumption that the minimum is attained near the maximum of $\epsilon(\mathbf{k})$.

[4]We do not quote here the expression for the interaction Hamiltonian between spin waves and phonons. A detailed investigation of the interaction described by this Hamiltonian will be the subject of a separate communication (I. E. Chupis).

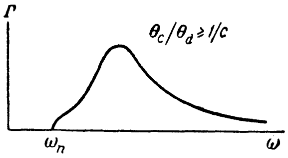

FIG. 1

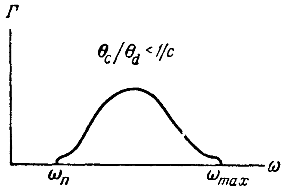

FIG. 2

region, $\omega_{max} \approx 2gH_0/(1 - c'\Theta_c/\Theta_d)$ $(c'\Theta_c < \Theta_d)$. In the first case, the absorption coefficient, after going through a maximum, decreases smoothly with increase of frequency (Fig. 1); in the second case, it has a sharp limit, with $\Gamma \sim \sqrt{\omega_{max} - \omega}$ when $\omega \lesssim \omega_{max}$ (Fig. 2).

In closing, we take this opportunity to express our thanks to V. M. Tsukernik for helpful discussions.

[1] M. I. Kaganov and I. E. Chupis, JETP **44**, 1695 (1963), Soviet Phys. JETP **17**, 1141 (1963).

[2] Akhiezer, Bar'yakhtar, and Kaganov, UFN **71**, 533 (1960) and **72**, 3 (1960), Soviet Phys. Uspekhi **3**, 567 and 661 (1961).

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