RADIATION EMITTED BY FAST PARTICLES IN A NONSTATIONARY INHOMOGENEOUS MEDIUM

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The spectrum and intensity of radiation emitted by fast charged particles in a periodically nonstationary and inhomogeneous medium are considered.

THE study of charged particles in stationary inhomogeneous media has been the subject of many papers[1-3]. For the understanding of the radiation processes in such media it is important that in the course of the radiation the stationary-inhomogeneous medium can acquire the recoil momentum and thus the law of momentum conservation changes during radiation, but not the law of energy conservation. Therefore in the case of radiation in a periodically inhomogeneous stationary medium the conservation laws lead to the relation[3]

$$\omega = n(\mathbf{k}_0\mathbf{v})/(1 - vc^{-1}\sqrt{\varepsilon_0\mu_0}\cos\theta), \tag{1}$$

where n is an integer, ϵ_0 and μ_0 the average values of the dielectric constant and the magnetic permeability, k_0 the "reciprocal lattice vector" characterizing the inhomogeneity ($k_0 = 2\pi/l$, where l is the period of the spatial inhomogeneity), θ the angle between the velocity of the particle and the radiation wave vector, and $\mathbf{v}$ the particle velocity.

In the present paper we consider the singularities of the radiation of a charged particle in a nonstationary medium. Interest in radiation of such media has arisen recently in connection with their use for frequency multiplication[4], for parametric amplification[5], etc. This interest is also due to the fact that a strong electromagnetic wave passing through a medium alters the properties of the medium, and therefore the medium becomes periodically unstable and at the same time inhomogeneous.

Let us see what results can be reached by an account of the periodic nonstationarity of the medium. Let the medium be periodic in space with a period $\mathbf{l}$ and in time with a period T. This means that during the radiation process the medium can acquire a momentum equal to an integral multiple of the quantity

$$h\mathbf{k}_0 = h\frac{2\pi}{l}\frac{\mathbf{l}}{l} \tag{2}$$

and an energy equal to an integral multiple of the quantity

$$h\omega_0 = h2\pi/T, \tag{3}$$

and with the same multiplicity as for $h\mathbf{k}_0$. When the particle radiates a quantum $h\omega$ the laws of energy and momentum conservation are written in the form

$$\Delta E = E_1 - E_2 = h\omega + nh\omega_0,$$
$$\Delta\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2 = \frac{h\omega}{c}\sqrt{\varepsilon_0\mu_0}\,\frac{\mathbf{k}}{k} + nh\mathbf{k}_0. \tag{4}$$

Multiplying the second equation by the velocity of the particle $\mathbf{v}$ and using the relation $\mathbf{v}\Delta\mathbf{p} = \Delta E$, we obtain

$$\omega = n(\mathbf{k}_0\mathbf{v} - \omega_0)/(1 - vc^{-1}\sqrt{\varepsilon_0\mu_0}\cos\theta) \tag{5}$$

This relation gives the radiation spectrum of a fast particle in a periodically inhomogeneous and periodically nonstationary medium. ϵ_0 and μ_0 must be defined in this case as the mean values of these quantities in space and in time.

If the medium is spatially inhomogeneous but periodically nonstationary, then we must put $\mathbf{k}_0 = 0$ in (5). In this case it becomes possible, as indicated in [7], for the presence of the medium to influence the energy conservation law but not the momentum conservation law. The physical meaning lies in the fact that in this case the medium is homogeneous in space but is nonstationary in time.

We note that in the derivation of (5) we have neglected the influence of the inhomogeneity and the nonstationarity of the medium on the properties of the emitted quantum. If the properties of the medium are specified by a dielectric constant in the form

$$\varepsilon(\omega) = \varepsilon_0(\omega) + \varepsilon_1(\omega)\cos(\mathbf{k}_0\mathbf{r} - \omega_0 t), \quad \mu = 1, \quad |\varepsilon| \ll |\varepsilon_0|, \tag{6}$$

then (5) can be regarded as valid if the following inequality is satisfied

$$|\varepsilon_1\omega/2\sqrt{\varepsilon_0}\,(k_0 c\cos\theta-\omega_0\sqrt{\varepsilon_0})|\ll 1. \quad (7)$$

It is easy to write an analogous inequality for a magnetically inhomogeneous medium.

Let us proceed to calculate the radiation intensity. We confine ourselves here to the case when the dielectric constant of the medium is given by (6) and the particle moves in the direction of the vector $\mathbf{k}_0$. The radiation intensity of a fast particle in such a medium can be calculated in the geometrical-optics approximation (by the WKB method). The calculation yields for the decelerating force averaged over the period $T = 2\pi/\omega_0$:

$$\frac{dW}{dx}=-\frac{q^2\,\mathrm{sign}\,(v-u)}{c^2}\sum_{n=-\infty}^{\infty}\int_{\Omega_n}\left[1-\frac{(\omega+n\Omega)^2}{\varepsilon_0\beta^2\omega^2}\right]J_n^2(\Delta)\,\omega d\omega, \quad (8)$$

where $\omega > 0$,

$$\Omega=\kappa_0\,|\,v-u\,|,\qquad u=\omega_0/k_0,$$

$$\Delta=\varepsilon_1\omega/2k_0c\sqrt{\varepsilon_0}\,|\cos\theta-\omega_0/ck_0\sqrt{\varepsilon_0}\,|, \quad (9)$$

and the integration extends over that region of the frequencies Ω_n where condition (5) is satisfied. In particular, when $n = 0$ and $\beta\sqrt{\epsilon_0} > 1$, Eq. (8) gives the energy lost by the charge to Cerenkov radiation, accurate to terms of order Δ^2.

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