INELASTIC SCATTERING OF 2.66-GeV $\pi^-$ MESONS ON NUCLEONS AND CARBON NUCLEI

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Submitted to JETP editor February 2, 1963


The inelastic interaction characteristics of 2.66-GeV pions in a propane bubble chamber were studied. It was found that at this energy the $\pi^-\text{-p}$ inelastic cross section is 24.3 ± 3.0 mb. The multiplicity distribution for secondary charged particles from $\pi^-\text{-p}$ and $\pi^-\text{-n}$ interactions is in agreement with the predictions of statistical theory with allowance for isobar formation. The multiplicity distribution of the secondary relativistic particles is the same for interactions in hydrogen and carbon. It is found that the effective numbers of "quasi-free" protons and neutrons in the carbon nucleus are both close to unity. An attempt was made to study pion resonances in the $(\pi^-\text{-p}) \to (\pi^+, n)$ reaction for neutron energies $T_n \leq 0.2$ GeV.

EXPERIMENTAL METHOD

INTERACTIONS of pions with nucleons and carbon nuclei were studied with a $37 \times 10 \times 10.8$ cm propane bubble chamber without a magnetic field. The chamber was exposed to a $\pi^-$ beam of momentum $p_\theta = 2.8 \pm 0.15$ GeV/c at the proton synchrotron of the Joint Institute of Nuclear Research. The experimental arrangement, the methods of scanning and selection of stars, and the measurements of the spatial coordinates and angles have been described earlier. To study $\pi^-\text{-N}$ interactions we selected 594 stars with no more than one slow proton ($n_\text{h} \leq 1$) in a fiducial region of the chamber 29 cm long. Cases of elastic diffraction scattering of pions on carbon nuclei as a whole, and of diffraction scattering on quasi-free neutrons in carbon nuclei, were separated through the exclusion of one-prong stars with a relativistic particle emitted at an angle less than 15°. Cases of $\pi^-\text{-p}$ elastic scattering have been separated previously. In studying the characteristics of $\pi^-\text{-N}$ inelastic interactions, we did not divide the interactions into interactions with free nucleons and those with quasi-free nucleons (i.e., interactions with carbon nucleons, not accompanied by secondary processes inside the same nucleus). It can be shown that in the great majority of collisions between 2.66-GeV pions and nucleons moving inside the nucleus, the change in energy in the center-of-mass system (c.m.s.) of the colliding particles (in comparison with a collision with a nucleon at rest) is less than 10% and the direction of motion of the c.m.s. in the laboratory system (l.s.) changes by no more than 6°.

Estimates of the energy of the secondary particles and the identification of the slow protons were made from measurements of the range and relative bubble density $g$, where for comparison we always used the track of the primary pion producing the interaction. It was shown that the relative bubble density remains practically unchanged over the length and width of the chamber. We observed, however, that there was a certain change in bubble density (20–30%) over the chamber depth. This was connected with the steady release of heat in the upper part of the chamber upon condensation of the gas phase. To decrease the influence of this effect on the measurements, we chose segments of track with a small difference in height.

The bubble density of secondary-particle tracks was estimated in two steps. In the initial scanning for stars, the interaction products were classified by inspection as black or relativistic tracks without any intermediate categories. Despite its qualitative character, such an approach proved to be sufficiently objective. Disparities in the estimates of the track density by different observers occurred in an insignificant number of cases. The density of the black tracks and part of the relativistic tracks was later estimated more accurately. The bubble density was measured by the gap-length
method\textsuperscript{[4,6]} with a correction for the inclination of the track. It was shown that the visual method permitted correct classification of black tracks \((g \geq 2)\) at angles of inclination \(\alpha \leq 45^\circ\). For \(\alpha > 45^\circ\), about 8\% of the tracks visually classified as black actually had \(g < 1.6\). The correctness of the final classification of the tracks was checked by the flatness of the distribution of the azimuthal angle \(\phi\) for the black tracks \(N(45^\circ \leq \phi \leq 90^\circ)/N(0 \leq \phi \leq 90^\circ) = 0.54 \pm 0.04\).

The secondary particle energy was estimated by Bethe's formula for primary ionization. As was shown by Hahn and Hugentobler,\textsuperscript{[6]} this formula describes well the dependence of the bubble density on the particle velocity, including the region beyond minimum ionization in which the ionization loss increases logarithmically. A check on recoil-proton tracks of known energy produced in elastic \(\pi^-\text{-p}\) interactions in our chamber showed that the formula is in satisfactory agreement with the relative bubble density measurements.

**QUASI-ELASTIC \(\pi^-\text{-p}\) INTERACTIONS**

From the obtained material, we selected cases of quasi-elastic scattering of pions on bound protons of carbon accompanied by a small momentum transfer (quasi-elastic diffraction scattering). These cases were separated by means of kinematic criteria based on the assumption that the incident pion collides with a moving target proton. Here the magnitude and direction of the recoil-proton momentum are uniquely determined by initial momentum of the proton inside the nucleus, the momentum transfer during the collision, and the proton momentum loss in overcoming the binding forces upon leaving the nucleus (see, for example,\textsuperscript{[7]})\textsuperscript{[7]}. In the calculation of the kinematics of quasi-elastic collisions, the momentum distribution of the protons in carbon nuclei was approximated by the function

\[
 dW(P) \sim \exp \left( -P^2/2P_0^2 \right) P^dP, \quad P_0 \approx 0.14 \text{ BeV/c}\textsuperscript{[1]}.
\]

It was shown that in the case of quasi-elastic scattering the distribution of the coplanarity parameter \(\delta\) (angle between the direction of the primary proton and the plane of the secondary particles) had a Gaussian distribution:

\[
 dW(\delta) \sim \exp \left( -\sin^2 \delta/2\delta_0^2 \right) d\delta, \quad \delta_0 = \frac{P_0}{\rho_0} = 0.05 = 2.9^\circ.
\]

Two-prong stars with one slow proton were considered as quasi-elastic diffraction scattering if they satisfied the following conditions:

1) \(\delta \leq 2\sqrt{\Delta \delta} + (\Delta \delta)^2 = 6.2^\circ\) (where \(\Delta \delta = 1.25^\circ\) is the rms measurement error of the angle \(\delta\));

2) the event lay inside the region bounded by the kinematic curves connecting the pion and recoil proton emission angles in the momentum interval for bound protons \(0 \leq P \leq 3P_0\);

3) the momentum transferred in the collision was \(\leq 0.7 \text{ BeV/c}\);\textsuperscript{[2]}

4) the energy of the recoil proton was not in contradiction with that permitted by the kinematic relations (with allowance for the energy loss for rupture of the binding); 5) there were no electron-positron pairs or \(V\) events directed toward the point of interaction.

In this way, we found 23 cases which could be regarded as \(\pi^-\text{-p}\) quasi-elastic diffraction interactions, which corresponded to a cross section \(\sigma_Q = 6.0 \pm 1.3 \text{ mb on carbon}\). The distribution of the target mass\textsuperscript{[8]} \(M_t\) calculated from the angle of emission and range of the recoil proton (under the assumption that the scattering occurs on the free proton) is shown in Fig. 1. As expected, they are all grouped close to the value \(M_t = 0\). The angular distribution of the quasi-elasically scattered pions is very similar to the distribution of secondary pions in \(\pi^-\text{-p}\) elastic scattering. If we ignore certain differences between these distributions in the small-angle region, where quasi-elastic scattering does not occur, and take the elastic and quasi-elastic cross sections to be equal, then the effective number of quasi-free protons in the carbon nucleus turns out to be \(n_q = 0.85 \pm 0.20\). This result disagrees with the conclusions of Bayukov et al\textsuperscript{[10]} \(n_q \approx 3\), whose selection criteria for quasi-elastic interactions at the same energy are less strict. The value of \(n_q\) also depends on the experimental conditions, in particular, on the minimum energy of the recoil protons \(T_{p,\text{min}}\) recorded in the chamber; \(T_{p,\text{min}} \approx 6 \text{ MeV}\) in our case while \(T_{p,\text{min}} \approx 12 \text{ MeV}\) in \textsuperscript{[10]}.  

**MULTIPLE PRODUCTION OF PIONS IN \(\pi^-\text{-p}\) AND \(\pi^-\text{-n}\) COLLISIONS**

Since the cross sections for \(\pi^-\text{-p}\) and \(\pi^-\text{-n}\) interactions at our energies,\textsuperscript{[11]} are practically

\[
 \text{FIG. 1. Distribution of target mass } M_t \text{ for cases of quasi-elastic } \pi^-\text{-p scattering (M is the proton mass).}
\]
the same, the number of collisions of pions with bound protons and neutrons is also expected to be the same. Therefore the number of collisions between pions and free protons is equal to the difference between the number of stars \((n_h = 1)\) with even and odd numbers of prongs. As a result, we have found that the cross section for inelastic \(\pi^\pm-p\) interactions at 2.66 GeV is\(^1\)

\[\sigma_e = 24.3 \pm 3.0 \text{ mb}.\]

In the calculation of the cross section, we introduce a correction for one-prong inelastic interactions with a relativistic particle emitted at an angle \(\delta < 15^\circ\) and a correction for the contribution from quasi-elastic \(\pi^\pm-p\) scattering. We also used the value of the \(\pi^\pm-p\) total cross section \([\text{Ref. } 12]\) at 2.66 GeV:

\[\sigma_t = 33.03 \pm 0.32 \text{ mb} \quad [11].\]

We also estimated the contribution from stars with \(n_h = 1\) produced in collisions between pions and carbon nuclei accompanied by secondary processes inside the nucleus. Such a procedure is necessary for comparison of the secondary-particle multiplicity distribution in \(\pi^-\mathrm{N}\) interactions with the predictions of the statistical model. Here we base ourselves on the characteristics of stars with several slow protons \((n_h > 2)\) produced in interactions between pions and carbon nuclei. Analysis of these stars showed that the fraction of black protons emitted in the backward hemisphere is \((37 \pm 1)\%\). This result does not depend on the number of black and relativistic protons which gives us a right to extend it also to stars with \(n_h = 1\). Since we found a total of 45 stars with black tracks directed toward the backward hemisphere, we can conclude that 45/0.37 = 123 events with \(n_h = 1\) correspond to interactions between pions and carbon nuclei. This correction, as should be expected, is the same for \(\pi^-\mathrm{p}\) and \(\pi^-\mathrm{n}\) interactions and constitutes half of all stars with \(n_h = 1\). As a result, the fraction of pion-nucleon interactions accompanied by the emission of a slow proton \((T_p \leq 0.2 \text{ GeV})\) proved to be \(\sim 20\%\), which is close to the results of Birger et al\([12]\) for 6.8-GeV/c \(\pi^-\) mesons and indicates that the contribution from peripheral collisions changes very little with the energy.

From the number of \(\pi^-\mathrm{N}\) collisions with an odd number of protons \((\pi^-\mathrm{n}\) interactions) we can estimate the number of quasi-free neutrons in the carbon nucleus, which turns out to be \(1.5 \pm 0.3\).

This value is somewhat greater than that obtained in an analysis of quasi-elastic interactions, which is, perhaps, connected with the inclusion of stars produced as a result of collisions between \(\pi^-\) mesons and carbon nuclei not accompanied by visible tracks of slow secondary protons.

The multiplicity distribution of secondary charged particles in pion-nucleon interactions (Fig. 2) is in good agreement with the predictions of statistical theory with allowance for isobar production.\(^2\) The mean multiplicities are 2.22 \(\pm 0.06\) and 2.46 \(\pm 0.12\), respectively, for stars with odd and even numbers of secondary prongs. The mean charged-particle multiplicities at 2.66 GeV calculated from statistical theory with allowance for isobar formation are 2.26 and 2.27 for \(\pi^-\mathrm{p}\) and \(\pi^-\mathrm{n}\) collisions, respectively.\([13]\)

\[\text{FIG. 2. Secondary charged-particle multiplicity distribution for pion-nucleon interactions. The histogram represents the result calculated from statistical theory with allowance for isobar formation.}\]

In a study of the angular distribution of the \(\pi^-\mathrm{N}\) interaction products, we noticed an increase in the mean emission angle of relativistic particles with increasing multiplicity. This phenomenon is evidently directly related to the decrease in the asymmetry of the angular distribution of the secondary pions in the c.m.s. with increasing multiplicity (see, for example,\([18]\)).

INTERACTIONS BETWEEN \(\pi^-\) MESONS AND CARBON NUCLEI

For a comparison of the characteristics of interactions of \(\pi^-\) mesons with nucleons and carbon nuclei we separated the stars into two classes. All stars with \(n_h = 2\) and those stars with \(n_h = 1\) which were inconsistent with the kinematics of inelastic \(\pi^-\mathrm{N}\) interactions were regarded as inelastic interactions between pions and carbon nuclei. All stars with \(n_h = 0\) and also part of the stars

\(^{0}\text{We include here cases of elastic nondiffraction scattering}\([1]\), \(\sigma_e = 1.3 \pm 0.5 \text{ mb}\).

\(^{2}\text{In this energy region, the pion resonance interaction has little effect on the secondary-particle multiplicity distribution in pion-nucleon interactions (cf.} \([11]\) \text{and} \([14]\)).\)
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Comparison of the characteristics of interactions of 2.66-GeV $\pi^-$ mesons with nucleons and carbon nuclei (l.s.)

<table>
<thead>
<tr>
<th>Interactions of pions with carbon nuclei</th>
<th>$\pi^- - N$ interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_h = 1$</td>
<td>2</td>
</tr>
<tr>
<td>Mean multiplicity of relativistic particles</td>
<td>1.94±0.15</td>
</tr>
<tr>
<td>Mean projected emission angle in plane of observation, deg</td>
<td>34.5±2.3</td>
</tr>
<tr>
<td>Ratio $N_{\text{back}}/N_{\text{tot}}$, %</td>
<td>10±3</td>
</tr>
</tbody>
</table>

FIG. 3. a) Charged relativistic particle multiplicity distribution in interactions of pions with nucleons and carbon nuclei; b) distribution of the charged relativistic particle emission angle projected on the plane of observation in the l.s. in interactions of pions with nucleons and carbon nuclei. The solid line gives the histogram for $\pi^- - N$ interactions (496 stars, 984 relativistic particles) and the dashed line represents $\pi^- - C$ interactions (360 stars, 716 relativistic particles).

with $n_h = 1$ were regarded as $\pi^- - N$ interactions. In the latter case, it was necessary that the energy and angle of the slow proton be within the kinetically allowed limits determined by the "effective mass" method, which was calculated from the formula

$$(m/\mu)^2 = (n_s + n_0) + \sum_{i<j} (1 - \cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j \cos (\varphi_i - \varphi_j)).$$

Here $n_s$ is the number of secondary relativistic particles, $n_0$ is the number of $\pi^0$ mesons in the star, $\theta$ and $\varphi$ are the polar and azimuthal angles of emission of the relativistic particle, $m$ is the effective mass, and $\mu$ is the pion mass. We have assumed here that the momenta of the secondary relativistic particles are equal to the pion mass $p_i = \mu c$. We also assumed that one $\pi^0$ meson was produced in the star when we observed an electron-positron pair directed toward the star vertex or when a secondary prong had a direction which clearly indicated that momentum conservation was not satisfied if we took into account only the charged particles.

To select the cases, we constructed the multiplicity distribution of relativistic particles and the distribution of the emission angles projected on the plane of observation (Fig. 3). Some characteristics of these distributions are shown in the table. As is seen from the table and Fig. 3a, the multiplicity distributions of relativistic particles from pion interactions with nucleons and carbon nuclei are practically the same and the mean values of the multiplicity for both classes of interactions agree.

The mean projected angle of emission of the relativistic particles and the fraction of particles emitted in the backward hemisphere in the l.s. are somewhat greater for interactions of pions with carbon nuclei. This difference can be ascribed to the scattering of part of the secondary pions in elastic collisions with bound nucleons.

The difference in the angular distributions will obviously be still less marked if we consider all interactions with carbon nuclei, including interac-
tions with quasi-free nucleons. The obtained data indicate that almost no secondary interactions accompanied by the production of relativistic pions occur in the carbon nucleus.

SEARCH FOR PION RESONANCES IN THE \( \pi^- + p \rightarrow \pi^- + \pi^+ + n \) REACTION

We analyzed 183 two-prong stars with relativistic particles and selected interactions which corresponded to the kinematics of the reaction

\[
\pi^- + p \rightarrow \pi^- + \pi^+ + n \quad (1)
\]

with a neutron energy \( T_n \leq 0.2 \text{ GeV} \). Such cases correspond to events with small angles between the plane of the secondary pions and the direction of the primary pions \((\delta < 13^\circ)\). It is readily shown that \( \sin \delta = \sin \alpha \sin \varphi \), where \( \alpha \) is the angle between the resultant momentum of the secondary pions \( p_{\pi\pi} \) and the momentum of the primary pion \( p_\pi \), and \( \varphi \) is the angle between the plane of the secondary pions and the plane of the primary pion and neutron. For this reason, it can be assumed, with a probable error of 14%, that \( \sin \alpha \approx \sin \delta \) and the direction of the resultant momentum of the secondary pions coincides with the projection of \( p_\pi \) on the plane determined by them.

It can be shown that for pions produced in this reaction with \( T_n \leq 0.2 \text{ GeV} \) the momentum \( p_{\pi\pi} \) is practically uniquely related (to an accuracy of at least 4%) to the total energy of the two pions \( M_{\pi\pi} \) in their own c.m.s. This permits the calculation of \( M_{\pi\pi} \) from the angles of emission of the pions relative to the direction \( p_{\pi\pi} \).

As a result of this selection, we eliminated two-prong stars for which the value of \( M_{\pi\pi} \) was forbidden by the kinematics of the investigated reaction. We also discarded those cases in which the secondary-pion tracks lay on one side of the projection of \( p_\pi \) on their plane. We also discarded cases of elastic nondiffraction \( \pi^- - p \) scattering accompanied by a large momentum transfer to the proton which could have been erroneously identified with the investigated reaction. Such cases are characterized by a small coplanarity parameter \( \delta \leq 2\Delta\delta \leq 2.5^\circ \) and are grouped in a narrow band about the kinematic curve for the angles of the elastically scattered pion and recoil proton. The number and the angular distribution of these cases are in good agreement with the data in the literature.\(^{(2,14)}\)

The \( M_{\pi\pi} \) spectrum of the 67 cases selected is shown in Fig. 4. A \( \chi^2 \) test showed that the experimental distribution is in satisfactory agreement with the phase-space curve calculated from statistical theory. Nevertheless, the grouping of cases about the value \( M_{\pi\pi} = 5 - 6 \mu \) (\( \mu \) is the pion mass) is in agreement with the conclusions of Shalamov and Grashin\(^{(17)}\) that part of reaction (1) proceeds through the intermediate \( \rho^0 \) meson with a mass 765 MeV.

The authors are grateful to Professor A. I. Alikhanyan for his support of the present work and for discussion of results and E. P. Kuznetsov and A. M. Gal'per for aid in carrying out the experiment. The authors express their gratitude to V. M. Maksimenko for carrying out calculations used in the comparison of the experimental data with the predictions of the statistical theory. The authors thank R. A. Latypova and É. G. Novikova for calculations on the electronic computer and T. G. Chernysheva and G. A. Ignatova for aid in the reduction of the experimental data. We take this opportunity to thank the proton-synchrotron crew for the faultless operation of the machine and for their careful work.

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