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DERIVATION OF THE PHOTOPRODUCTION AMPLITUDE FROM DISPERSION RELATIONS

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An expression for the amplitude of photoproduction of pions on nucleons at low energies is derived with the help of integral and differential methods on the basis of one-dimensional dispersion relations. The results obtained by the two methods are compared with each other and with the experiments. The role of the unobservable region is assessed.

1. INTRODUCTION

PHOTOPRODUCTION of pions on nucleons was investigated with the aid of the dispersion relations by many authors [1-5]. In some papers [1,2] approximate account was taken of the nucleon recoil but no account was taken of the $\pi\pi$-interaction contribution. In others [3-5], the Mandelstam representation is used to derive formulas which take into account in principle the nucleon recoil in the $\pi\pi$-interaction. In the most recent papers, relations containing the unobservable region are used. The continuation to this region is made with the aid of the first terms of the Legendre-polynomial expansion. On the whole, the Legendre-polynomial series converges, at least in the large Lehmann ellipse, but we do not know how fast it converges in the unobservable region. If the series converges slowly, then the use of the limited number of terms may lead to errors in the calculation of the dispersion integrals.

The present work is devoted to an evaluation of the corrections for the nucleon recoil and to an analysis of the effect of the unobservable region. We assume here that at the energies under consideration it is sufficient to calculate the S and P waves, and that the higher partial waves for the charged mesons need be accounted for only in the term for the direct photon-pion interaction. In order to obtain the amplitudes of the S and P waves from the one-dispersion relations, we use two different methods. In the first method, as is customary, the amplitude is integrated with respect to the angle (we call this the integral method). The amplitudes then contain explicitly the contribution of the unobservable region. In the second method the amplitude is expanded near the threshold value of the square of the momentum transfer (we call this the differential method). This method has several advantages over the integral method. First, formulas obtained with its aid are much simpler than the formulas obtained with the integral method; second, they do not contain explicitly the unobservable region. The differential method has been used by Chew et al [6] and its advantages when applied to the double dispersion relations were analyzed in detail by Efremov et al [7].

We note that if we neglect the nucleon recoil (we let the mass of the nucleon tend to infinity), then both the differential and the integral method yield identical results.

The calculations made in the present investigation show that when the nucleon recoil is taken into account both methods likewise give results that differ little at low energies. These results are in fair agreement with experiment.

A comparison of the results of both methods enables us to estimate the contribution of the unobservable region to the partial-wave amplitudes.

2. DETERMINATION OF THE PARTIAL AMPLITUDES

We consider the photoproduction of pions on nucleons in the center-of-mass system and use a system of units in which $\hbar = c =$ pion mass $= 1$. The principal symbols are: M—nucleon mass, k—photon energy, ω_q and q—pion energy and momentum, W—total energy, x—cosine of the angle between the photon and pion momenta, $\alpha = \pm, 0$—isotopic variables.

The pion-nucleon photoproduction amplitude can be expressed in terms of the twelve amplitudes $F_i^{(\alpha)}(W, x)$ $(i = 1, 2, 3, 4)$, for which the following one-dimensional dispersion relations exist:

$$\operatorname{Re} F_i^{(\alpha)}(W, x) = F_i^{(\alpha)B}(W, x) + \int_{M+1}^{\infty} dW' \sum_j K_{ij}^{(\alpha)}(W, W', x)$$

$$\times \operatorname{Im} F_j^{(\alpha)}(W', x') = F_i^{(\alpha)B}(W, x) + \delta F_i^{(\alpha)}(W, x); \quad (1)$$

$$k(\omega_q - qx) = k'(\omega_{q'} - q'x') = t. \quad (2)$$

We shall denote the dispersion integrals by $\delta F_i^{(\alpha)}(W, x)$. The explicit form of the Born terms $F_i^{(\alpha)B} = F_{ie}^{(\alpha)B} + F_{i\mu}^{(\alpha)B}$ and of the kernels $K_{ij}^{(\alpha)}$, and also the connection between the functions $F_i^{(\alpha)}$ and the photoproduction cross section, can be found in [1,2].

In order to compare theory with experiment at low energies it is sufficient to take into account only the S and P waves for the neutral pions, while for the charged pions it is necessary to take into account in addition to the S and P waves only the higher waves contained in $F_{ie}^{(\alpha)B}$, or more accurately in the part of $F_{ie}^{(\alpha)B}$ which corresponds to the direct photon-pion interaction.

A. Integral Method

We first consider the dispersion integrals, the contributions of which to the amplitudes of the S and P waves have the form [2]

$$\delta E_{0+}(W) = \frac{1}{2}\int_{-1}^{1} dx \Big\{\delta F_1(W, x) - x\delta F_2(W, x) + \frac{1}{2}(1 - x^2)\,\delta F_4(W, x)\Big\},$$

$$\delta E_{1+}(W) = \frac{1}{4}\int_{-1}^{1} dx \Big\{x\delta F_1(W, x) + \frac{1}{2}(1 - 3x^2)\,\delta F_2(W, x) + (1 - x^2)\Big[\frac{1}{2}\delta F_3(W, x) + x\delta F_4(W, x)\Big]\Big\},$$

$$\delta M_{1+}(W) = \frac{1}{4}\int_{-1}^{1} dx \Big\{x\delta F_1(W, x) + \frac{1}{2}(1 - 3x^2)\,\delta F_2(W, x) - \frac{1}{2}(1 - x^2)\,\delta F_3(W, x)\Big\},$$

$$\delta M_{1-}(W) = \frac{1}{2}\int_{-1}^{1} dx \Big\{- x\delta F_1(W, x) + \delta F_2(W, x) + \frac{1}{2}(1 - x^2)\,\delta F_3(W, x)\Big\}, \quad (3)$$

where

$$\delta F_i(W, x) = \int_{M+1}^{\infty} dW' \sum_j K_{ij}(W, W', x)\,\mathrm{Im}\,F_j(W', x'). \quad (4)$$

It is well known that these formulas contain the contribution of the unobservable region. At fixed W and for arbitrary observed x, the cosine of the angle x′ under the integral sign in (14) varies in accordance with (2) in the following manner: as $W' \to \infty$, x′ tends to unity (from below), but as $W' \to M + 1$

$$x' \to \begin{cases} +\infty & t < t_{thr} \\ 0 & t = t_{thr}; \\ -\infty & t > t_{thr} \end{cases} \qquad t_{thr} = \frac{2M+1}{2M+2}. \quad (5)$$

To calculate the dispersion integrals $\delta F_j(W, x)$, one usually expands $\mathrm{Im}\,F_j(W', x')$ in partial waves and this expansion is used to continue $\mathrm{Im}\,F_j(W', x')$ into the unobservable region. It is assumed here that this expansion converges not only in the observable but also in the unobservable region so rapidly that we can confine ourselves to its first few terms (for example, the S and P waves). This expansion can be written symbolically in the form

$$\mathrm{Im}\,F(W', x') = \sum_l \mathrm{Im}\,A_l(W')\,P_l(x'), \quad (6)$$

where P_l is the Legendre polynomial.

From the unitarity condition it follows that when $W' \to M + 1$ ($q' \to 0$) the l-th term of the series decreases as q'^{l+1}. Therefore the series actually converges well near threshold. Matters can become more complicated only away from threshold, when $|x'| > 1$ (but $|x'|$ is finite) at high energies, where we do not know the extent to which the partial amplitudes decrease with increasing l. If the series converges slowly (of course, the series can also diverge outside the Lehmann ellipse), the use of only the first terms of the expansion can lead to an error.

In order to check this circumstance, we use the expansion (6) and confine ourselves in it only to the amplitude of the magnetic-dipole transition in the resonant state (3/2 3/2):

$$\mathrm{Im}\,F_1^{(+)}(W', x') = -2F_1^{(-)}(W', x') = 2\,\mathrm{Im}\,M_{1+\mu}^{(3/2)}(W')\,x',$$

$$\mathrm{Im}\,F_2^{(+)}(W', x') = -2F_2^{(-)}(W', x') = \frac{4}{3}\mathrm{Im}\,M_{1+\mu}^{(3/2)}(W'), \quad (7)$$

$$\mathrm{Im}\,F_3^{(+)}(W', x') = -2F_3^{(-)}(W', x') = -2\,\mathrm{Im}\,M_{1+\mu}^{(3/2)}(W'),$$

$$\mathrm{Im}\,F_4^{(\pm)}(W', x') = \mathrm{Im}\,F_i^{(0)}(W', x') = 0.$$

These formulas enable us to express all the contributions from the dispersion integrals in terms of a single function $\mathrm{Im}\,M_{1+\mu}^{(3/2)}$. For the amplitude of $M_{1+\mu}^{(3/2)}$ we choose the expression [1]

$$M_{1+\mu}^{(3/2)} = \frac{\mu_p - \mu_n}{2f}\frac{k}{q} f_{33}, \quad (8)$$

where f_{33} is the scattering amplitude:

$$f_{33} = \frac{{}^4/_3 f^2 q^2 / \omega}{1 - \omega/\omega_r - {}^4/_3 i f^2 q^3/\omega}; \quad (9)$$

μ_p and μ_n are the total magnetic moments of the proton and neutron; f is the pion-nucleon coupling constant, $f^2 = 0.0877$ [8]; $\omega = W - M$, $\omega_r = 2.17$.

In view of the narrowness of the (33)-resonance, we put approximately

$$\operatorname{Im} M_{1+\mu}^{(3/2)}(W) = \frac{4.69\pi}{3M} efkq\delta(W - W_r). \quad (10)$$

With the aid of this expression we calculate the amplitudes $\delta E_{0+}^{(+)}$, $\delta E_{1+}^{(+)}$, $\delta M_{1-}^{(+)}$ and $\delta M_{1+}^{(1/2)}$. In order to complete the calculations, it is necessary to add to these quantities the contributions of the Born terms. A direct calculation shows that the Born terms of the partial amplitudes for π^0 mesons are not very sensitive to the method used in their calculation. We have used for these terms the results obtained below by the differential method.

Finally, with the aid of the formulas (16) and (17) below we obtain the differential cross section for the π^0 mesons. The corresponding curves are presented in Fig. 1.

B. The Differential Method

As noted above, a shortcoming of the integral method is a fact that the expansion (7) must be used in the unobservable region. To avoid this, we use the differential method. We consider first, as before, the dispersion integrals (4), which we expand in a Taylor series near $t = t_{\text{thr}}$. We assume that in the physical region it is sufficient to retain only the S and P waves (and take the higher waves into account in the Born terms), so that it is sufficient to retain the first two terms of these series:

$$\delta F_i(W, x) = \delta F_i(W, x_0) + \delta F_i'(W, x_0)(x - x_0), \quad (11)$$

where the prime denotes the derivative with respect to x and

$$x_0 = (k\omega_q - t_{\text{thr}})/kq. \quad (12)$$

Comparing (11) with the expansion in the S and P waves, we obtain

$$\delta E_{0+}(W) = \delta F_1(W, x_0) - \delta F_1'(W, x_0)\, x_0,$$
$$\delta E_{1+}(W) = \tfrac{1}{6}\{\delta F_1'(W, x_0) + \delta F_3(W, x_0)\},$$
$$\delta M_{1+}(W) = \tfrac{1}{6}\{\delta F_1'(W, x_0) - \delta F_3(W, x_0)\},$$
$$\delta M_{1-}(W) = \delta F_2(W, x_0) - \tfrac{1}{3}\{\delta F_1'(W, x_0) - \delta F_3(W, x_0)\}. \quad (13)$$

These formulas replace formulas (3) of the integral method. To obtain them we had to assume that the D wave and the higher waves are negligibly small in the physical region (or else are determined completely by the Born term in the case of charged mesons) at the low energies considered by us (experiment confirms this assumption).

Now, for fixed $x = x_0$ under the integral sign in (4),

$$x' = (k'\omega_{q'} - t_{\text{thr}})/k'q' \quad (14)$$

assumes only observable values, and we can use the partial-wave expansion of $\operatorname{Im} F_i(W', x')$ in the observable region. We note that the nonresonant P-phases are negligibly small and we neglect them in the expansion of the imaginary parts. In the first approximation in $\operatorname{Im} F_i$ we even neglect the S wave (the question of the evaluation of the S phase will be discussed below) and confine ourselves in this expansion only to the amplitude of the magnetic-dipole transition in the resonant state (3/2 3/2), i.e., we use formulas (7) as above. Thus, we have expressed all the contributions for the dispersion integrals in terms of the single function $\operatorname{Im} M_{1+\mu}^{(3/2)}$.

The Born partial-amplitude terms corresponding to the neutral pions are not very sensitive to the computation method. We use for these terms expansions at the point $x_0 = 0$:

$$E_{0+}^B(W) = F_1^B(W, 0),$$
$$E_{1+}^B(W) = \tfrac{1}{6}\{F_1^{B'}(W, 0) + F_3^B(W, 0)\},$$
$$M_{1+}^B(W) = \tfrac{1}{6}\{F_1^{B'}(W, 0) - F_3^B(W, 0)\},$$
$$M_{1-}^B(W) = F_2^B(W, 0) - \tfrac{1}{3}\{F_1^{B'}(W, 0) - F_3^B(W, 0)\}. \quad (15)$$

For charged pions, this remains true of those Born terms which depend on the magnetic moments. Adding expressions (15) to the corresponding expressions in (13) we obtain the complete expressions for the partial neutral-pion photoproduction waves. For the amplitude $M_{1+\mu}^{(3/2)}$ we obtain (together with the unitarity condition) an inhomogeneous linear integral equation. The numerical calculations show that expression (8) together with the approximation (10) satisfies well this equation, at least near threshold. Therefore, as before, we choose expression (8) for $M_{1+\mu}^{(3/2)}$, and use the approximation (10) for the calculation of the nonresonant amplitudes.

3. RESULTS OF THE CALCULATIONS

Using the formulas of Sec. 2 we can calculate the cross sections and compare the results of both methods. For the process

$$\gamma + p \to \pi^0 + p$$

the differential cross section has the form

$$d\sigma/d\Omega = A(W) + B(W)\,x + C(W)\,x^2, \quad (16)$$

where

$$A = \frac{q}{k}\{(E_{0+})^2 + \tfrac{5}{2}(M_{1+})^2 + (M_{1-})^2 + M_{1+}M_{1-} + \tfrac{9}{2}(E_{1+})^2$$
$$- 3E_{1+}(M_{1-} - M_{1+}) + \tfrac{10}{9}(\operatorname{Im} M_{1+\mu}^{(3/2)})^2\},$$

$$B = \frac{2q}{k} E_{0+}\{M_{1+} + 3E_{1+} - M_{1-}\},$$

$$C = \frac{q}{k}\{-\tfrac{3}{2}(M_{1+})^2 + \tfrac{9}{2}(E_{1+})^2 - 3M_{1+}M_{1-} + 9E_{1+}(M_{1+} - M_{1-}) - \tfrac{2}{3}(\mathrm{Im}\, M^{(3/2)}_{1+\mu})^2\}. \quad (17)$$

Figure 1 shows plots of the quantities A, B, and C, obtained with the aid of the integral and differential methods. From the figure it is seen that both methods give almost identical results at low energies. At energies above resonance, the results begin to diverge, the difference increasing with the energy. Nonetheless, the results of both methods agree sufficiently well with experiment. Figure 2 shows plots of the angular distributions of the π^0 mesons for different energies, calculated by means of the differential method.

For the photoproduction of charged mesons we have considered only the case when x = 0 (angle 90°) and used only the differential method. For the process $\gamma + p \rightarrow \pi^+ + n$ we have here

$$\begin{aligned}
\mathrm{Re}\, F_1(W,0) &= F_1^B(W,0) + \delta E_{0+}^{(-)}(W),\\
\mathrm{Re}\, F_2(W,0) &= F_2^B(W,0) + \delta M_{1-}^{(-)}(W) + 2\delta M_{1+}^{(-)},\\
\mathrm{Re}\, F_3(W,0) &= F_3^B(W,0) + 3\delta E_{1+}^{(-)} - \delta M_{1+}^{(-)},\\
\mathrm{Re}\, F_4(W,0) &= F_4^B(W,0),
\end{aligned} \quad (18)$$

where F_i and F_i^B correspond to π^+ mesons. The values of δE and δM were calculated by formulas (13).

The differential cross section of the π^+ mesons emitted at 90° has the form

$$\frac{d\sigma(\pi^+)}{d\Omega}\Big|_{\theta=90^\circ} = \frac{2q}{k}\Big\{(\mathrm{Re}\, F_1(W,0))^2 + (\mathrm{Re}\, F_2(W,0))^2 + \frac{1}{2}(\mathrm{Re}\, F_3(W,0))^2 + \frac{1}{2}(\mathrm{Re}\, F_4(W,0))^2 + \mathrm{Re}\, F_2(W,0) \times \mathrm{Re}\, F_3(W,0) + \mathrm{Re}\, F_1(W,0)\, \mathrm{Re}\, F_4(W,0) + \frac{5}{18}(\mathrm{Im}\, M^{(3/2)}_{1+\mu})^2\Big\}. \quad (19)$$

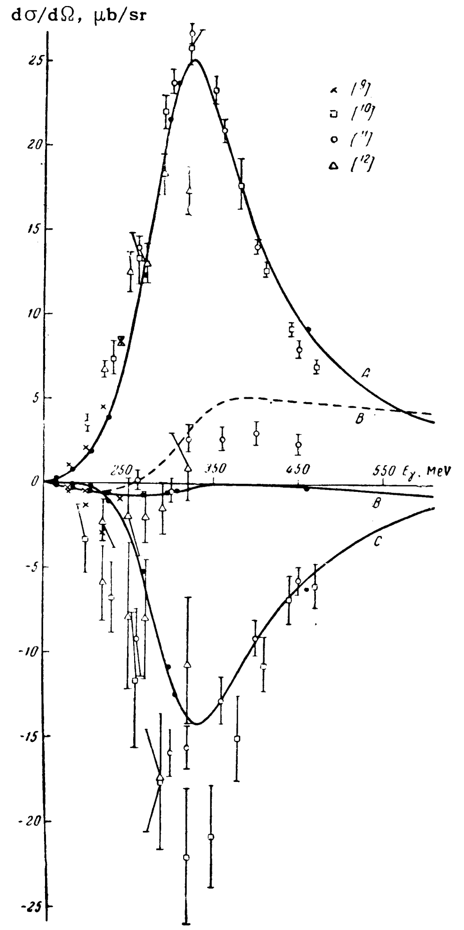

FIG. 1. Plot of the coefficients A, B, and C of the differential cross section for the photoproduction of neutral pions vs. the laboratory-system energy of the photon. The continuous curves show the results of the calculation by the differential method without account of the imaginary part of the S wave, while the dashed curve accounts for the imaginary part of the S wave. The black dots represent the results of the calculation by the integral method (without account of the imaginary part of the S wave).

It is shown in Fig. 3. Figure 4 is a plot of quantity

$$a_0^+ = \frac{d\sigma(\pi^+)}{d\Omega}\Big|_{\theta=90^\circ} \Big/ q\omega_q\left(1 + \frac{k}{M}\right)^{-2}. \quad (20)$$

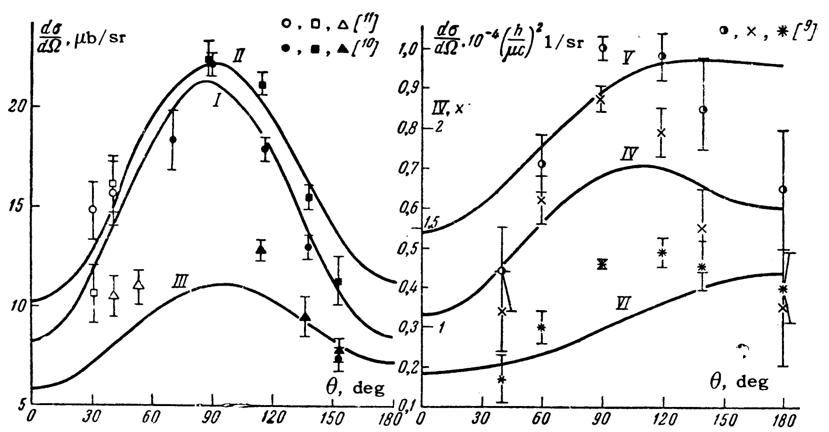

FIG. 2. Dependence of the differential cross section of the photoproduction of neutral pions on the angle in the c.m.s. Curve I (○, ●) – for E_γ = 360 MeV, curve II (□, ■) – for E_γ = 300 MeV, curve III (△, ▲) – for E_γ = 260 MeV, curve IV (×) – for E_γ = 220 MeV, curve V (○) – for E_γ = 200 MeV, curve VI (*) – for E_γ = 180 MeV.

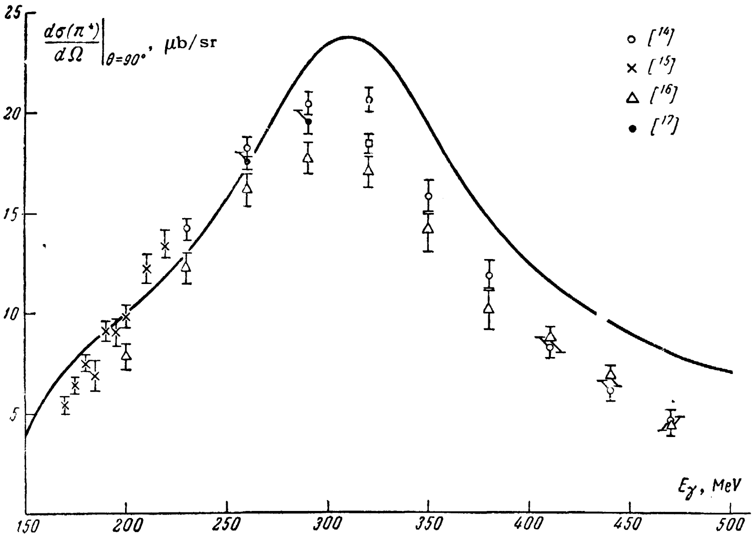

FIG. 3. Differential cross section of photoproduction of positive pions at 90° in the c.m.s. as a function of the laboratory-system energy of the photon.

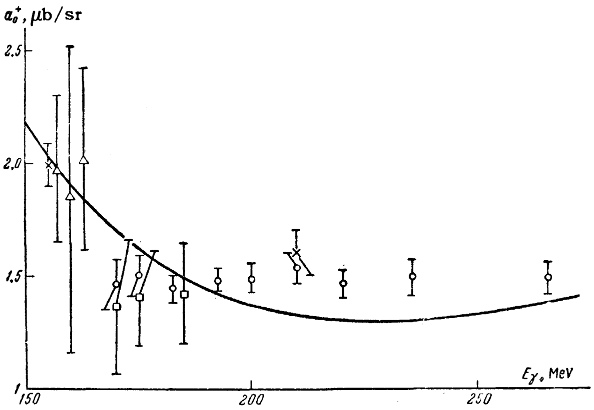

FIG. 4. Value of Eq. (20) for the photoproduction of positive pions as a function of the laboratory photon energy; experimental data: o —[15], □ —[18], × —[19], △ —[20].

Formulas (18) and (19) can be used for the photoproduction of negative pions ($\gamma + n \rightarrow \pi^- + p$), but F_i and F_i^B should correspond to the π^- mesons, and the signs of the δ-terms in (18) must be reversed. Finally, Fig. 5 shows a plot of the ratio

$$\{[d\sigma(\pi^-)/d\Omega]/[d\sigma(\pi^+)/d\Omega]\}_{\theta=90°}.$$

4. DISCUSSION OF THE CONTRIBUTION OF THE UNOBSERVABLE REGION TO THE PARTIAL WAVES [24]

In order to note the differences between the integral and differential methods, we have compared the contributions of the dispersion integrals to the nonresonant amplitudes, (i.e., the quantities $\delta E_{0+}^{(+)}$, $\delta M_{1-}^{(+)}$, and $\delta M_{1+}^{(1/2)}$), calculated by both methods. The difference between them is caused by the following:

1) Neglect of the higher-order partial waves in the calculations by the differential method; this

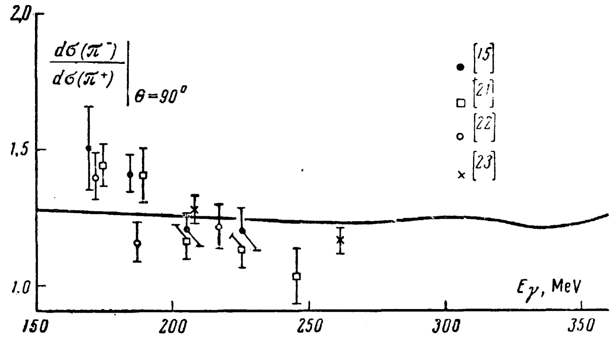

FIG. 5. Ratio of differential photoproduction cross sections of negative and positive pions at 90° in the c.m.s. as a function of the photon laboratory energy.

circumstance can, generally speaking, manifest itself at all energies.

2) Continuation into the unobservable region with the aid of a finite number of Legendre polynomials in the calculation by the integral method; this circumstance, by virtue of the approximation (10), can manifest itself only at energies above resonance, $\omega > 2.17$.

Calculation shows that the difference between the results of both methods increases with increasing energy, but even when $\omega = 2.77$ ($E\gamma = 465$ MeV) it amounts to 2.5% for $\delta E_{0+}^{(+)}$, 21% for $\delta M_{1-}^{(+)}$, and 25% for $\delta M_{1+}^{(1/2)}$. At resonant energy, the differences (due only to the failure to account for the high-order partial waves) amount to 1, 13, and 8% respectively. We can therefore conclude that the continuation into the unobservable region with the aid of a finite number of Legendre polynomials leads to negligible errors for energies below resonance, and for energies from resonance up to 460 MeV it leads to errors not exceeding 1—2% for the contributions to the dispersion integrals by the S-wave amplitudes and 10—20% for contributions of the P-wave amplitudes to the dispersion intervals. For the partial amplitudes as a whole, this error is smaller or larger, depending on whether the signs of the corresponding Born terms and of the dispersion contributions are the same or opposite.

It would be interesting to ascertain the error that arises if the contribution of the unobservable region is completely disregarded in the dispersion integrals when using the integral method.

The unobservable region in (4) corresponds in the c.m.s. to an integral with respect to W′ from M + 1 to the value determined by the equation $x' = (k'\omega_{q'} - t)/k'q'$ with $x' = 1$ ($t < t_{thr}$) or $x' = -1$ ($t > t_{thr}$).

In the approximation (10) this means that the contribution of the unobservable region is equal to zero if $W \leq W_r$, and if $W > W_r$ this contribution occurs when

$$x > x_+ \text{ and } x < x_-, \quad (21)$$

where

$$x_\pm = (k\omega_q - k_r\omega_{q_r} \pm k_r q_r)/kq. \quad (22)$$

Thus, in our approximation the contribution of the unobservable region to the nonresonant partial amplitudes (3) is equal to zero when $W < W_r$. In order to discard this contribution when $W > W_r$, it is necessary to integrate in (3) not from −1 to +1, but from x_- to x_+. Since we are now interested only in qualitative deductions, it is perfectly sufficient to consider the dispersion relations in the static approximation, $M \to \infty$.

In this approximation the results of the integral and differential methods coincide and the contribution of the dispersion integral to the S wave is

$$\delta E_{0+}^{(+)} = ef\frac{4{,}69}{3M}\frac{4}{3}\omega. \quad (23)$$

If we discard, on the other hand, the unobservable region [and determine the partial amplitudes with the aid of the projection formulas (3)], then we obfor $\delta E_{0+}^{(+)}$ an expression that coincides with (23) when $\omega < \omega_r$ and has for $\omega > \omega_r$ the form

$$\overline{\delta E}_{0+}^{(+)} = 0. \quad (24)$$

Thus, when $\omega > \omega_r$ the entire contribution to (23) is produced by the unobservable region.

For other nonresonant amplitudes, the contribution of the physical region is not equal to zero when $\omega > \omega_r$. Nonetheless, the contribution of the unphysical region is in this case quite large and is at least comparable with the contribution of the physical region.

In the approximation (10) the quantities $\overline{\delta E}_{0+}^{(+)}$ and $\overline{\delta M}_{1-}^{(+)}$ experience at the point $\omega = \omega_r$ discontinuities due to the discontinuity of the amplitudes with isotopic spin 3/2. If we replace the δ-function (10) by a continuous function then, of course, these discontinuities are smoothed out. However, owing to the existence of the narrow (33) resonance, this change does not modify the deduction that the contribution of the unobservable region is quite small when $\omega < \omega_r$ and, to the contrary, it is quite large when $\omega > \omega_r$.

5. DISCUSSION OF THE RESULTS AND CONCLUSIONS

It is seen from Fig. 1 that the integral and differential methods give results which are hardly distinguishable at low energies; the difference increases with increasing energy. The calculated values of the coefficients A and C of the differential cross section of the neutral mesons (16) agree well with experiment, and for the small coefficient B (which characterizes the forward-backward asymmetry) there is a small discrepancy at large energies. If we take into account the imaginary part of the S-wave amplitude, i.e., if we add in the coefficient B the term $\mathrm{Im}\,E_{0+}^{(+)}\,\mathrm{Im}\,M_{1+\mu}^{(3/2)}$, then the behavior of this coefficient changes radically at high energies.

The coefficient B with account of the imaginary part of the S wave (which we have taken from the paper by Chew et al [1] in the form $\mathrm{Im}\,E_{0+}^{(+)} = efF_S \cdot 2/3\,(\delta_1 - \delta_3)$ with the phases taken from [8]) is plotted in Fig. 1 (dashed curve). The term $\mathrm{Im}\,E_{0+}^{(+)}\,\mathrm{Im}\,M_{1+\mu}^{(3/2)}$ makes an appreciable contribution to the coefficient B at high energies. At low energies this term is negligibly small. As regards the contribution of the imaginary part of the S wave [the term $(\mathrm{Im}\,E_{0+}^{(+)})^2$] to the coefficient A, it amounts only to several per cent at high energies. The relative contribution of this term increases with decreasing energy: it reaches 10 and 20% at E = 180 and 160 MeV, respectively.

Figure 2 shows directly the angular distributions [1)] obtained by the differential method with account of the imaginary part of the S waves. We see that for all the energies under consideration the angular distributions of the π^0 mesons are in fair agreement with experiment. The differences caused by the use of the integral method or when the imaginary part of the S wave is included can be estimated from Fig. 1.

For the charged mesons, we see from Figs. 3—5

[1)]For energies 160, 180, 200, and 220 MeV the plotted points were obtained from data for the coefficients A, B, and C, which in turn were obtained from the experimental data for three angles.

that the agreement with experiment is much worse. At low energies the discrepancy is most noticeable for the ratio $d\sigma(\pi^-)/d\sigma(\pi^+)$. It can be assumed that this discrepancy is due to the crudeness of the approximation used here (failure to take into account the $\pi\pi$-interaction and the small scattering phase shifts, as well as the approximate evaluation of the integrals). It must be noted that we have considered only the 90° angle for the charged mesons. The relative contribution of the imaginary part of the S-wave amplitude is then negligibly small.

Let us summarize the comparison of the integral and differential methods:

1. In the integral method, owing to the existence of the narrow resonance, the contribution of the unobservable region to the partial amplitudes is negligibly small at energies below resonance. At energies above resonance this contribution is quite large and is comparable with the total contribution of the dispersion integral.

2. A continuation into the unobservable region with the aid of a finite number of Legendre polynomials does not lead to noticeable errors in the partial amplitudes at energies below resonance. At energies above resonance, the error increases with increasing energy, but even for 460 MeV it does not exceed 1 or 2% for the contributions of the dispersion integrals to the S-wave amplitudes and 10—20% for the contribution of the dispersion integrals to the P-wave amplitudes.

3. In the differential method, the assumption that the amplitudes of the D, F, and higher-order waves are identically equal to zero in the observable region leads to a small error, on the order of 1% for contributions of the dispersion integrals to the amplitudes of the S-waves and on the order of 10% for the contributions of the dispersion intervals to the P-wave amplitudes.

It must be emphasized that we have discarded in the dispersion integral the imaginary part of all the amplitudes except the resonant one. All the deductions are based on this approximation. We do not know what error is introduced by this approximation itself.

From a comparison of the calculations with experiment we can draw the following conclusions:

1. The calculations performed can serve as a basis for further refinement of the results.

2. In order to refine the coefficient B for neutral mesons (and also the corresponding quantities for charged mesons), it is necessary to take accurate account of the S-wave amplitudes.

3. The greatest discrepancy at low energies is obtained for the ratio of the cross sections of the negative and positive pions, i.e., for a quantity which is most sensitive to the influence of the $\pi\pi$-interaction[3-5]. Consequently any further research must take the $\pi\pi$-interaction into account.

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48