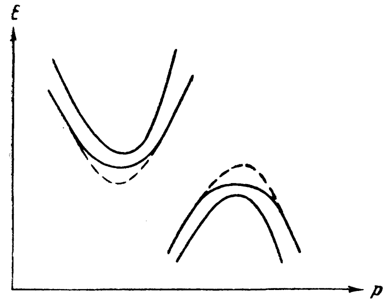

prevented by formation of bound states. Then obviously the dependences of the energy on the quasi-momentum should be deformed in the way indicated in the adjoining figure: near the maximum and the minimum the curves are "flattened," i.e., the effective mass $m^* \simeq (\partial^2 E/\partial p^2)^{-1}$ increases owing to the effect considered here. This impedes the band overlap and the transition to metallic state with low carrier density.

We can assume that metals with low carrier densities can only have structures in which contact or overlapping of the valence and conduction bands are due to degeneracy governed by the crystal symmetry. Removal of the degeneracy, due to the Jahn-Teller effect, occurs with a weak distortion of the degenerate structure and formation of a metal with low carrier densities. Consideration of the crystal structure of known metals with low carrier density (graphite, bismuth, arsenic, antimony) indicates that they all have structures which are weak distortions of more symmetrical structures. The author plans later to publish an analysis of this effect.

The author expresses his deep gratitude to Professor L. F. Vereshchagin, Professor I. M. Lifshitz, and A. I. Likhter for useful discussions.

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1) This formula was given by Mott,[3] but he used the free-electron mass instead of m*.

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1 L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Pergamon, 1958.

2 N. F. Mott, Nuovo cimento Suppl. **7**, 312 (1958).

3 N. F. Mott, Phil. Mag. **6**, 287 (1961).

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CONCERNING ONE POSSIBILITY OF AMPLIFICATION OF LIGHT WAVES

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THE subject of the present note is a discussion of certain possibilities of amplification and generation of light waves in optically transparent crystals, the polarization of which depends in nonlinear fashion on the intensity of the electric field of the propagating wave. A nonlinearity of this type (it can obviously be regarded as the dependence of the dielectric constant on the field) was successfully utilized in several recently described experiments (see [1-3]) on the generation of optical harmonics. Naturally, this does not exhaust the possible nonlinear effects in such crystals. We show below that under certain conditions, in an optically transparent medium whose polarization depends quadratically on the intensity of the electric field, one can obtain parametric amplification of traveling light waves, obtained at the expense of the energy of an intense light wave (the so-called pumping) and that the condition for parametric amplification can be realized in uniaxial crystals.

As is well known (see, for example, [4,5] and also the review [6]), in the region of the fundamental parametric resonance the energy of the intense pumping oscillations of frequency ω_p, carrying out the modulation of the reactive parameters of a resonance circuit or of a transmission line, can be transferred to oscillations at frequencies ω_1 and ω_2, satisfying the condition

$$\omega_p = \omega_1 + \omega_2 \qquad (1)$$

(the particular case when $\omega_1 = \omega_2 = \omega_p/2$ is the so-called "degenerate" parametric interaction). To clarify the features of such an interaction space, it is necessary to consider a semi-bounded medium, the dielectric constant of which varies as1)

$$\varepsilon(t, x, \omega) = \varepsilon_0(\omega)\{1 + m[e^{i(\omega_p t - k_p x)} + e^{-i(\omega_p t - k_p x)}]\} \qquad (2)$$

(the x axis is perpendicular to the separation boundary).

Assume that the waves at frequencies ω_1 and ω_2 have components $E_y = E$; H_x; H; H_z, and assume that their wave vectors make angles θ_1 and θ_2 with the x axis. The electric field in the medium will then be described by the equations

$$\frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = \frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2}, \qquad D = \varepsilon E, \tag{3}$$

and the summary field can be written in the form

$$E = E_1(x)\exp\{i(\omega_1 t - \mathbf{k}_1\mathbf{r})\} + E_2^*(x)\exp\{-i(\omega_2 t - \mathbf{k}_2\mathbf{r})\} + \text{compl. conj.}$$
$$k_i = \omega_i c^{-1}\sqrt{\varepsilon_0(\omega_i)}. \tag{4}$$

Substituting (4) in (3), taking (2) and (1) into account, and collecting the components with frequencies ω_1 and ω_2, we can obtain the differential equations that characterize the spatial variations of the amplitudes $E_{1,2}$. It turns out here that the presence of the term ϵE in equation (3) appreciably influences the behavior of the amplitudes $E_{1,2}$ and can lead to amplification only if the following relation (momentum conservation law) is satisfied:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_H. \tag{5}$$

For small m it is natural to assume that

$$d^2E_i/dx^2 \ll k_i dE_i/dx \qquad (i = 1, 2).$$

Then the equations for $E_{1,2}$ assume the form

$$\frac{dE_1}{dx} = -\frac{im_1k_1}{2\cos\theta_1}E_2^*, \qquad \frac{dE_2^*}{dx} = \frac{im_2k_2}{2\cos\theta_2}E_1$$

and consequently

$$\frac{d^2E_1}{dx^2} = \frac{m_1m_2k_1k_2}{4\cos\theta_1\cos\theta_2}E_1 \quad (m_i = m(\omega_i)). \tag{6}$$

It is seen from (6) that in the medium under consideration exponentially growing waves are possible, with a build-up factor

$$\alpha = \frac{1}{2}[m_1m_2k_1k_2/\cos\theta_1\cos\theta_2]^{1/2}.$$

Assuming that $E_1 = E_0$ and $E_2 = 0$ when $x = 0$ (oscillations of "difference" frequency ω_2, necessary for amplification, occur in the nonlinear medium, the angle θ_1 is determined by the incidence conditions and by the properties of the medium, while the angle θ_2 is automatically established in accordance with condition (5)[2)]), and using the boundary conditions, we obtain

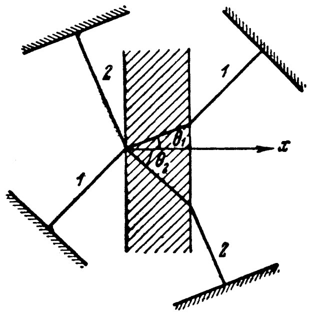
$$E_1 = E_0\,\mathrm{ch}\,\alpha x,$$
$$E_2 = iE_0\sqrt{m_2k_2\cos\theta_1/m_1k_1\cos\theta_2}\,\mathrm{sh}\,\alpha x. \tag{7*}$$

To make more specific the requirements imposed on the dispersion properties of the nonlinear medium (5), let us consider for simplicity the case $\omega_1 \approx \omega_2 \approx \omega$. It then follows from (5) that to obtain amplification the phase velocity of the pumping (frequency $\sim 2\omega$) should exceed the phase velocity of the wave at the frequency ω, or, for the refractive indices

$$n(\omega) > n(2\omega). \tag{8}$$

The condition (8) can be satisfied, for example, in a uniaxial negative crystal, if the frequency ω (signal) excites the ordinary wave and the frequency 2ω (pumping) the extraordinary wave (see also [2]). The foregoing denotes that in such a crystal the conditions of the problem investigated above can be realized.

The amplification mechanism considered above can be used to construct coherent optical generators of adjustable frequency. One of the possible schemes of such a generator is shown in the figure.



A crystal bounded on two sides is pressed between two pairs of parallel mirrors. In this case the dielectric constant has the form

$$\varepsilon = \varepsilon_0[1 + m_f\cos(\omega_p t - k_p x) + m_b\cos(\omega_p t + k_p x)].$$

Assume that the amplification conditions for the frequencies ω_1 and ω_2 are satisfied in the directions θ_1 and θ_2 respectively. Then, by establishing the planes of the mirrors normal to rays 1 and 2, we can excite parametric oscillations at frequencies close to ω_1 and ω_2. The condition of self-excitation of the generator has the form $m \geq 1/\sqrt{Q_1Q_2}$, where $Q_{1,2}$ are the figures of merit of the optical resonators formed by the parallel mirrors.[3)]

A factor limiting the self-oscillation amplitude is the reaction of this oscillation on the pumping field. This means that the efficiency of the generator under consideration must be sufficiently high. Thus, a crystal with nonlinear polarizability excited by an intense light wave can serve as a tunable light amplifier or generator of appreciable efficiency. By varying the anisotropy parameters of the crystal by means of the external field it is possible to modify the conditions of energy exchange between the waves and consequently modulate the amplified or generated oscillations.

[1)]The modulation coefficient m can amount to $\sim 10^{-4}$–10^{-5} if modern coherent light generators are used (see [7]).

[2)]Of course, if (5) can be satisfied at all.

[3)]We note that inasmuch as Q_1 and Q_2 are quite large in the optical band, self-excitation of the oscillations is possible also in the case when the mirrors are installed only in one of the directions.

*sh = sinh; ch = cosh.

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[3] Maker, Terhune, Nisenoff, and Savage, ibid. **8**, 21 (1962).

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[5] P. Tien, ibid. **29**, 1347 (1959).

[6] M. D. Karasev, Usp. Fiz. Nauk **69**, 217 (1959), Soviet Phys. Uspekhi **2**, 719 (1960).

[7] Franken, Hill, Peters, and Weinreich, Phys. Rev. Lett. **7**, 118 (1961).

Translated by J. G. Adashko
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POWER INCREASE IN A PULSED RUBY LASER BY MEANS OF MODULATION OF RESONATOR Q

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MODULATION of the Q of the resonator permits an increase in the power radiated by a quantum mechanical generator by the accumulation of active particles during the time the pump is in action, when the Q of the resonator has its minimum value. By means of a rapid increase in the resonator Q to its maximum value, an "emission" of particles results. The time of this "emission" is determined by the lesser of two times, the time of growth of Q or the time of establishment of vibrations in the resonator. Ideas of this kind have been discussed, in particular, by Hellwarth.[1]

We briefly describe below a generator in which the resonator Q is modulated by means of a rapidly rotating disk, and present some characteristics of

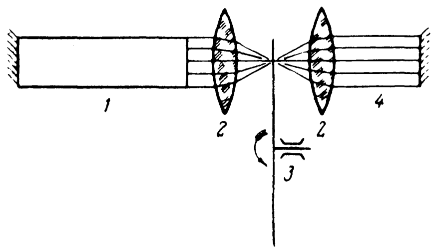

FIG. 1. 1 – ruby; 2 – objectives; 3 – rotating disk with aperture; 4 – external mirror.

such a generator. The basic plan of the generator is shown in Fig. 1. As can be seen from this figure, a decrease in the on-time of resonator Q is achieved by interrupting the emitted beam in the focus of the objectives 2, 2. Such an arrangement allows the simultaneous decrease in the number of the various modes that can be excited in the system. The on-time of maximum Q was 10^{-6} sec. In the experiments the power of a generator operating with constant Q was compared with the power of a generator in which the Q was modulated.

In Figs. 2 and 3 are shown oscillograms of the radiation obtained in these two cases; the amplitude of the oscillograms in Fig. 3 was reduced by 2×10^3 times relative to Fig. 2 by means of neutral filters on the photomultipliers. The majority of the experiments were performed on crystals whose total energies were ~ 1 J. The maximum Q of the resonator was turned on from 0.3 to 0.5 μsec after the exciting lamp was turned on. The

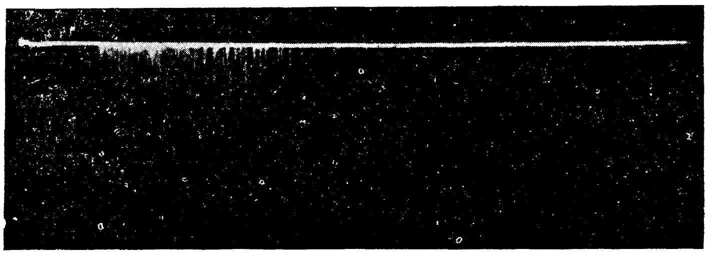

FIG. 2. Emission of an optical generator with constant Q. Length of scan – 1 μsec.

FIG. 3. Emission of an optical generator with modulated Q. Length of scan – 100 μsec. The second trace records the approximate position of the aperture at the moment of pulse generation.