NEW POSSIBILITIES IN THE STUDY OF THE ELECTROMAGNETIC PROPERTIES OF THE ELECTRON AND MUON

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It is suggested that conversion decays of strongly interacting vector mesons (ω^0, η^0, ρ) and of the hypothetical σ^0 meson can be used to study the electromagnetic properties of the electron and μ meson at small distances.

On the basis of the existing theoretical notions, the electron and the muon, which have different masses, can under certain conditions interact in different manners with other particles. No such differences in electron and muon interactions were observed so far. In this connection, great interest is attached to an investigation of the electromagnetic properties of electrons and muons at small distances (large momentum transfers at the γee and $\gamma\mu\mu$ vertices). Such investigations can yield important results but, unfortunately, the corresponding experiments are very difficult to set up.

In the present paper we wish to call attention to new possibilities of checking the electrodynamics at small distances, possibilities which arise in connection with the discovery of the strongly-interacting vector mesons (ω^0 meson [1,2] with mass $m_\omega = 787$ MeV and isotopic spin 0, η^0 meson [3] with mass $m_\eta = 550$ MeV and isotopic spin 0, and ρ meson [1,4] with mass $m_\rho = 750$ MeV and isotopic spin 1). Kobzarev and Okun' [5] have shown that the small widths of the pion decays of the ω^0 and η^0 mesons bring about a situation wherein radiative decays (with participation of photons) can become significant. One might think that for the same reason, conversion decays

$$\omega(\eta) \to l^+ + l^-, \qquad \omega(\eta) \to \pi^0 + l^+ + l^-,$$

where $l = e$ or μ, can be sufficiently effective. These decays, and the decays

$$\sigma^0 \to \gamma + l^+ + l^-$$

of hypothetical neutral pseudoscalar σ^0 mesons, if such mesons were to be observed experimentally, are proposed here for use in the investigation of the electromagnetic properties of the electrons and muons.

We estimate the order of magnitude of the constants contained in the matrix element by perturbation theory. In spite of the uncertainty of the estimates given below for the probabilities of these decays, it follows that the experimental observation and study of such decays is quite feasible.

1. Let us consider the conversion decays of the ω^0, η^0, and ρ^0 mesons (we shall denote them by V) into (e^+e^-) and $(\mu^+\mu^-)$ pairs

$$V \to e^+ + e^-, \tag{1}$$

$$V \to \mu^+ + \mu^-. \tag{2}$$

These decays are remarkable in that the lepton energy in their rest system is equal to the mass of the V meson. To obtain such high energies in the machinery, it is necessary to create opposing electron beams with energy ~ 400 MeV. The matrix element corresponding to such decays has the form

$$M = (4\pi)^{3/2} f_V \alpha \chi_\mu \bar{u} \gamma_\mu v. \tag{3}$$

The vertex of the $V \to \gamma$ transition, shown in Fig. 1a, corresponds here, if we separate the electromagnetic-interaction constant $\alpha = 1/137$, to a quantity $\sqrt{\alpha}\, f_V 4\pi q^2 \chi_\mu$, where f_V is an unknown dimensionless constant, and χ_μ and m_V are the wave function and mass of the V meson. Using (3), we obtain for the total probability of the de-

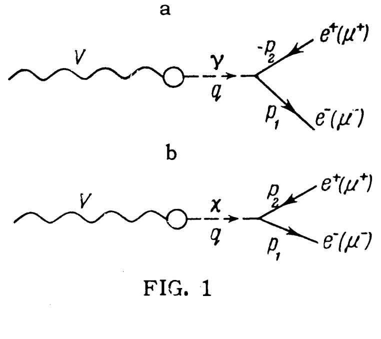

FIG. 1

cays (1) and (2) the expression

$$W = \frac{16\pi^2}{3}\alpha^2 f_V^2 m_V \left(1 + \frac{2m^2}{m_V^2}\right)\sqrt{1 - \frac{4m^2}{m_V^2}}, \quad (4)$$

where m will designate from now on the mass of either the electron or the muon. The ratio of the total probabilities does not contain the unknown quantity f_V and is completely determined by the electromagnetic properties of the electrons and muons. From (4) we obtain

$$R_V = \frac{W(V \to \mu^+ + \mu^-)}{W(V \to e^+ + e^-)} \approx 1.$$

We emphasize that the value of R_V is obtained under the assumption that the vertices γee and $\gamma\mu\mu$ have identical properties at c.m.s. lepton energies $\sim m_V$. If, for example, the muon were to have an anomalous interaction [6], then the matrix element for the decay (2) would contain an additional term, corresponding to the diagram of Fig. 1b. In this case the probability ratio for the decays (1) and (2) would have the form

$$R_V \approx \left(1 + \frac{f_A^2}{\alpha}\frac{m_V^2}{m_V^2 - m_\chi^2}\right)^2,$$

where m_χ is the mass of the χ meson which brings about the anomalous interaction of the μ meson, and f_A is the constant of the anomalous μ-χ interaction. Since experiment[1)] yields $(g/2)_e = 1.001162 \pm 0.000005$, where g is the gyromagnetic ratio for the muon, we obtain when $m_\chi \gg m_\omega$, for example, $R_\omega \approx 0.41$ (i.e., the number of muon decay events will amount to only 40 per cent of the number of electron decays). On the other hand, if $m_\chi \approx m_\omega$, then R_ω becomes large (the number of cases of muon decays becomes larger than the number of electron decays). For example, if $m_\chi = m_\omega$ (in which case we introduce a χ meson which interacts "strongly" with the nucleons and anomalously with the muons), then $R_\omega \approx 9$. We note also that the equality $m_\chi = m_\omega$ does not contradict the data on the scattering of muons on protons, from which it follows that the contribution made by the anomalous interaction to the angular distribution of μ at momentum transfers $q \approx 300$ MeV/c may amount to less than 20 per cent.

A rough estimate of f_V is obtained by assuming that when q^2 is small the electromagnetic form factors of the proton and neutron are determined essentially by the isovector ρ^0 meson and the isoscalar ω^0 and η^0 mesons. Then

$$1 + \frac{1}{6}\langle r^2\rangle_p q^2 = 1 + \xi_V \frac{4\pi g_{VNN}}{m_V^2} q^2,$$

where $\langle r^2\rangle_p = 0.88 \times 10^{-13}$ cm [7] is the mean square radius of the proton charge, g_{VNN} is the V meson-nucleon interaction constant, and ξ_V is a quantity that estimates the contribution of the V meson to the electromagnetic form factor of the proton. This condition denotes that

$$\xi_V g_{VNN} f_V = m_V^2 \langle r^2\rangle_p / 24\pi.$$

To estimate ξ_ρ, $g_{\rho NN}$, and $g_{\omega NN}$ we can use the following. Since $\langle r^2\rangle = 0$ for the neutron, we have in our approximation $\xi_\rho \approx 2$ and, if the ρ meson interacts with the isovector current, then $g_{\rho NN} = g_{\rho\pi\pi}/2$. But $g_{\rho\pi\pi}$ can be determined by using the experimental probability of the $\rho \to 2\pi$ decay. Then

$$g_{\rho NN}^2 = \frac{3W(\rho \to 2\pi)}{m_\rho}\left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{-3/2}.$$

For $W(\rho \to 2\pi) \approx 100$ MeV we get $g_{\rho NN} \approx 0.7$ and $\xi_\rho g_{\rho NN} = 1.4$. If unitary symmetry holds true (see [8]), then $g_{\omega NN} = \sqrt{3}\, g_{\rho NN}$. Putting $\xi_\eta \approx 1$, $\xi_\omega \approx 1$, $g_{\eta NN} \approx 1$, and using the foregoing estimates for ξ_ρ, $g_{\rho NN}$, and $g_{\omega NN}$, we obtain

$$f_\omega = 0.14, \qquad f_\eta = 0.084, \qquad f_\rho = 0.11. \quad (5)$$

The total probabilities are

$$W(\omega^0 \to e^+ + e^-) \approx W(\omega^0 \to \mu^+ + \mu^-) = 0{,}043 \text{ MeV},$$
$$W(\eta^0 \to e^+ + e^-) \approx W(\eta^0 \to \mu^+ + \mu^-) = 0{,}011 \text{ MeV},$$
$$W(\rho^0 \to e^+ + e^-) \approx W(\rho^0 \to \mu^+ + \mu^-) = 0{,}025 \text{ MeV}. \quad (6)$$

The probabilities of the main decays for the ω^0 meson ($\omega^0 \to 3\pi$, $\omega^0 \to \pi^0 + \gamma$) are

$$W(\omega^0 \to 3\pi) \approx W(\omega^0 \to \pi + \gamma) \approx 0.5 \text{ MeV}, \quad (7)$$

and for the η^0 meson ($\eta^0 \to \pi^0 + \gamma$)

$$W(\eta^0 \to \pi^0 + \gamma) = 0.2 \text{ MeV} \quad (8)$$

(We take into account the fact that $W(\eta^0 \to \pi^0 + \gamma)/W(\eta^0 \to 3\pi) = 25$ [5]). We use here the estimates made in [5] with an effective dimension $1/m_\pi$. The ratios of the probabilities (6), (7), and (8) are

$$\frac{W(\eta^0 \to e^+ + e^-)}{W(\eta^0 \to \pi^0 + \gamma)} \approx \frac{W(\eta^0 \to \mu^+ + \mu^-)}{W(\eta^0 \to \pi^0 + \gamma)} \approx 5.5\%,$$

$$\frac{W(\omega^0 \to e^+ + e^-)}{W(\omega^0 \to 3\pi)} \approx \frac{W(\omega^0 \to \mu^+ + \mu^-)}{W(\omega^0 \to 3\pi)} \approx 8.6\%.$$

The analogous ratios for the ρ^0 mesons are considerably smaller (the larger width is ~ 100 MeV):

$$\frac{W(\rho^0 \to e^+ + e^-)}{W(\rho^0 \to 2\pi)} \approx \frac{W(\rho^0 \to \mu^+ + \mu^-)}{W(\rho_0 \to 2\pi)} \approx 0.025\%.$$

Apparently the estimates of [5] are somewhat too high. If we calculate f_V by perturbation theory, for a $V \to \gamma$ transition via a nucleon-antinucleon pair, we obtain, by discarding the gauge-

1) I. S. Shapiro, private communication.

invariant quadratic divergence,

$$f_V = \frac{g_{VNN}}{12\pi^2} \ln \Lambda^2/m_N^2,$$

where m_N is the nucleon mass and Λ is the cut-off constant. For $\ln(\Lambda^2/m_N^2) \approx 1$ and the values of g_{VNN} assumed above we have

$$f_\omega \approx 0.01, \quad f_\eta \approx 0.0084, \quad f_\rho = 0.0059;$$

$$W(\omega^0 \to e^+ + e^-) \approx W(\omega^0 \to \mu^+ + \mu^-) \approx 2.21 \cdot 10^{-4} \text{ MeV},$$

$$W(\eta^0 \to e^+ + e^-) \approx W(\eta^0 \sim \mu^+ + \mu^-) = 1.1 \cdot 10^{-4} \text{ MeV},$$

$$\frac{W(\eta^0 \to e^+ + e^-)}{W(\eta^0 \to \pi^0 + \gamma)} \approx \frac{W(\eta^0 \to \mu^+ + \mu^-)}{W(\eta^0 \to \pi^0 + \gamma)} \approx 0.055\,\%,$$

$$\frac{W(\omega^0 \to e^+ + e^-)}{W(\omega^0 \to 3\pi)} \approx \frac{W(\omega^0 \to \mu^+ + \mu^-)}{W(\omega^0 \to 3\pi)} \approx 0.044\,\%.$$

In spite of their uncertainty, these estimates allow us to conclude that the ratio of conversion decays (1) and (2) for ω^0 and η^0 mesons to the main decays is on the order of one percent.[2)]

2. The decays (1) and (2) considered above contain information on the electromagnetic properties of the electron and muon only for three fixed values of the energy (787, 550, and 750 MeV in the lepton center of mass). For this reason, the fact that the experimental ratios R_V differ from unity still does not tell us unambiguously for which lepton the deviation from quantum electrodynamics takes place and what the value of this deviation is for each individual lepton. By precisely the same token, the equality $R_V = 1$ still would not mean the absence of such deviations (they could be the same for e and μ for these values of q^2). We therefore consider the conversion decays

$$V \to \pi + e^+ + e^-, \qquad V \to \pi + \mu^+ + \mu^-. \tag{9}$$

These make it possible to obtain information, for both electrons and muons, over a whole range of values of energy in the lepton center of mass. The matrix is written in the form

$$M = (4\pi)^2 m_V^{-1} q^{-2} \alpha g \beta_V(q^2)\, \varepsilon_{\alpha\beta\gamma\delta} \chi_\alpha k_\beta p_\gamma \bar{u} \gamma_\delta v. \tag{11}$$

The vertex $V \to \pi + \gamma$ (Fig. 2) corresponds to the expression

$$(4\pi)^{3/2} m_V^{-1} \sqrt{\alpha} g \beta_V(q^2)\, \varepsilon_{\alpha\beta\gamma\delta} \chi_\alpha k_\beta p_\gamma \bar{u} \gamma_\delta v,$$

where $\beta_V(q^2)$ is a dimensionless function of q^2 or, what is the same, of the pion energy ($q^2 = m_V^2 + m_\pi^2 - 2E_\pi m_V$), and $g = \sqrt{14.5}$ is the strong-interaction constant. If we assume that the function $\beta_V(q^2)$ is constant in the considered interval

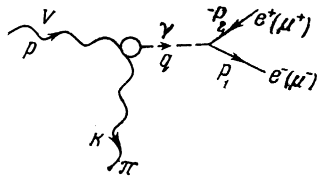

FIG. 2

of q^2, we obtain from (11) the values of the total probabilities of decay into electrons and muons

$$W(\omega^0 \to \pi^0 + e^+ + e^-) \approx W(\rho \to \pi + e^+ + e^-) \approx 10.4 W_0,$$

$$W(\eta^0 \to \pi^0 + e^+ + e^-) \approx 9.5 W_0,$$

$$W(\omega^0 \to \pi^0 + \mu^+ + \mu^-) \approx W(\rho \to \pi + \mu^+ + \mu^-) \approx 0.59 W_0,$$

$$W(\eta^0 \to \pi^0 + \mu^+ + \mu^-) \approx 0.19 W_0, \tag{12}$$

where $W_0 = \frac{2}{9}\pi\alpha^2 g^2 \beta_V^2 m_V$. The probability ratios do not contain the unknown quantity β_V and turn out to be

$$\frac{W(\omega^0 \to \pi^0 + e^+ + e^-)}{W(\omega^0 \to \pi^0 + \mu^+ + \mu^-)} \approx \frac{W(\rho \to \pi + e^+ + e^-)}{W(\rho \to \pi + \mu^+ + \mu^-)} \approx 18,$$

$$\frac{W(\eta^0 \to \pi^0 + e^+ + e^-)}{W(\eta^0 \to \pi^0 + \mu^+ + \mu^-)} \approx 50. \tag{13}$$

A rough estimate for the probabilities (12) can be obtained by taking for β_V the value obtained by perturbation theory for the $V \to \pi + \gamma$ decay [5]:

$$\beta_V = (g_{VNN}/4\pi^2)\, m_V/m_N,$$

where g_{VNN} is the constant for the coupling between the V meson and the nucleon. For $g_{\omega NN} \approx 1.2$, $g_{\rho NN} \approx 0.7$ (see above), and $g_{\eta NN} = 1$, we obtain

$$\beta_\omega = 2.5 \cdot 10^{-2}, \quad \beta_\eta = 1.5 \cdot 10^{-2}, \quad \beta_\rho = 1.5 \cdot 10^{-2} \tag{14}$$

and further

$$W(\omega^0 \to \pi^0 + e^+ + e^-) = 2.6 \cdot 10^{-3} \text{ MeV},$$

$$W(\omega^0 \to \pi^0 + \mu^+ + \mu^-) = 1.4 \cdot 10^{-4} \text{ MeV},$$

$$W(\eta^0 \to \pi^0 + e^+ + e^-) = 6 \cdot 10^{-4} \text{ MeV},$$

$$W(\eta^0 \to \pi^0 + \mu^+ + \mu^-) = 1.3 \cdot 10^{-5} \text{ MeV},$$

$$W(\rho \to \pi + e^+ + e^-) = 10^{-3} \text{ MeV},$$

$$W(\rho \to \pi + \mu^+ + \mu^-) = 5.6 \cdot 10^{-5} \text{ MeV}. \tag{15}$$

For the percentage ratios to the main decays [see (7) and (8)] we have

$$\frac{W(\omega^0 \to \pi^0 + e^+ + e^-)}{W(\omega^0 \to 3\pi)} = 0.51\,\%,$$

$$\frac{W(\omega^0 \to \pi^0 + \mu^+ + \mu^-)}{W(\omega^0 \to 3\pi)} = 2.8 \cdot 10^{-2}\,\%,$$

$$\frac{W(\eta^0 \to \pi^0 + e^+ + e^-)}{W(\eta^0 \to \pi^0 + \gamma)} = 0.32\,\%,$$

$$\frac{W(\eta^0 \to \pi^0 + \mu^+ + \mu^-)}{W(\eta^0 \to \pi^0 + \gamma)} = 6.4 \cdot 10^{-3}\,\%,$$

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2)We have recently received a preprint of Nambu and Sakurai's "Rare Decays of $\omega(\eta)$ Mesons," in which the authors reach the same conclusion concerning the values of these ratios.

$$\frac{W(\rho\to\pi+e^{+}+e^{-})}{W(\rho\to 2\pi)}=10^{-3}\%,$$

$$\frac{W(\rho\to\pi+\mu^{+}+\mu^{-})}{W(\rho\to 2\pi)}=5.6\cdot 10^{-5}\%. \qquad (16)$$

The results (12)—(16) have been obtained under the assumption that β_V = const. In order to obtain information on the electromagnetic properties of e and μ, independently of the assumptions made concerning $\beta_V(q^2)$, we can, by choosing different values of q^2, compare the theoretical form with the experimental angular distributions of e and μ obtained for this q^2. Then $\beta_V(q^2)$ will be a certain proportionality coefficient, which is the same for the electron and muon. If we choose the angle between the momentum $\mathbf{k}$ of the outgoing pion and the difference in the lepton momenta $\mathbf{p}_1-\mathbf{p}_2$, we obtain

$$W(z,x)\,dzdx=\frac{\pi}{6}\alpha^2g^2\beta_V^2(z)\,m_V\left\{z+4\varepsilon^2+\frac{4x^2z\,(z-4\varepsilon^2)}{\gamma}\right\}$$
$$\times\frac{\sqrt{z-4\varepsilon^2}\,(1-\Delta^2+z)^2\,b^3}{z^2\gamma^{3/2}}\,dzdx,$$
$$b=\{[(1+\Delta)^2-z]\,[(1-\Delta)^2-z]\}^{1/2},$$
$$\gamma=(1-\Delta^2+z)^2-x^2b^2,$$
$$4\varepsilon^2\leqslant z=\frac{q^2}{m_V^2}\leqslant(1-\Delta)^2,\quad \varepsilon=\frac{m}{m_V},\quad \Delta=\frac{m_\pi}{m_V},$$
$$x=\cos\theta,\qquad -1\leqslant x\leqslant +1. \qquad (17)$$

Another possibility is to investigate the ratios of the distribution functions over the effective mass of the photon $W_e(z)/W_\mu(z)$ which does not contain $\beta_V(z)$. The z-spectrum (see Fig. 3) has the form

$$W(z)\,dz=\frac{\pi m_V}{6}\alpha^2g^2\beta_V^2(z)\sqrt{1-\frac{4\varepsilon^2}{z}}\left(1+\frac{2\varepsilon^2}{z}\right)[(1-\Delta)^2-z]^{3/2}$$
$$\times[(1+\Delta)^2-z]^{3/2}\frac{dz}{z}. \qquad (18)$$

We can thus investigate the electromagnetic properties of e and μ up to energies $\sim(m_V-m_\pi)$. Unfortunately, as can be seen from Fig. 3, the probability of events decreases rapidly with increasing q^2. At β_V = const the number of events with total c.m.s. muon energy $\geq 3m_\mu$ for the decays (10) amounts to about 40 per cent for the ω^0 and ρ mesons and about 13 per cent for the η^0 meson; for the decays (9), the number of events in the same energy range is about 2.5 per cent for ω^0 and ρ mesons and about 0.3 per cent for the η^0 mesons.

From the foregoing estimates it follows that an experimental study of the conversion decays of vector mesons is quite feasible. Furthermore, the widths of ω^0 and η^0 can actually be much smaller than indicated in (7) and (8), and therefore

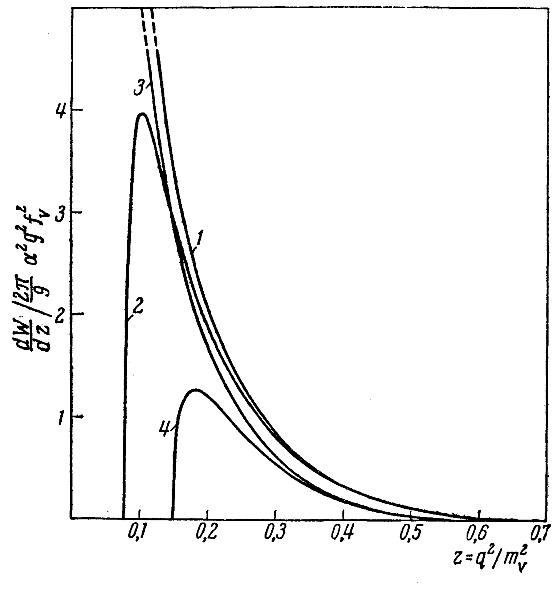

FIG. 3. Distribution with respect to q^2: Curves 1 and 2—for the ρ-meson decays (9) and (10), 3 and 4—for the η^0 meson decays (9) and (10). The maxima of the distributions 1 and 3 occur at $z = 5.5\,(m_e/m_V)^2$ and are equal to 0.23×10^6 in case 1 and 0.11×10^6 in case 3.

the percentage ratios of these decays to the main decays will turn out to be even larger.

3. Many papers (for example [8,9] dealt with the existence of two neutral pseudo-scalar mesons σ_1^0 and σ_2^0. The conversion decays of the σ^0 mesons

$$\sigma^0\to\gamma+e^{+}+e^{-}, \qquad (19)$$
$$\sigma^0\to\gamma+\mu^{+}+\mu^{-}, \qquad (20)$$

if such mesons were to be observed in experiment, could be used, in analogy with the decays (9) and (10), to investigate the electromagnetic properties of e and μ up to energies $\sim m_\sigma$. We note that unlike the π^0 meson, the decay of the σ^0 meson (20) is energetically allowed (see, for example, [10,11] concerning the $\pi^0\to\gamma+e^{+}+e^{-}$ decay).

The matrix element corresponding to (19) and (20) is

$$M=16\pi^2m_\sigma^{-1}q^{-2}F(q^2)\,\alpha^{3/2}\varepsilon_{\alpha\beta\gamma\delta}e_\alpha k_\beta q_\gamma\bar{u}\gamma_\delta v. \qquad (21)$$

Here e_α is the wave function of the photon, m_σ the mass of the σ^0 meson, and the vertex $\sigma^0\to2\gamma$ (Fig. 4) corresponds to the factor

$$(4\pi)^{3/2}m_\sigma^{-1}\alpha F(q^2)\,\varepsilon_{\alpha\beta\gamma\delta}e_\alpha k_\beta q_\gamma,$$

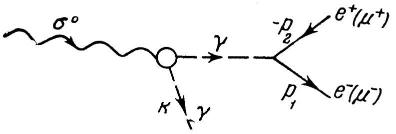

FIG. 4

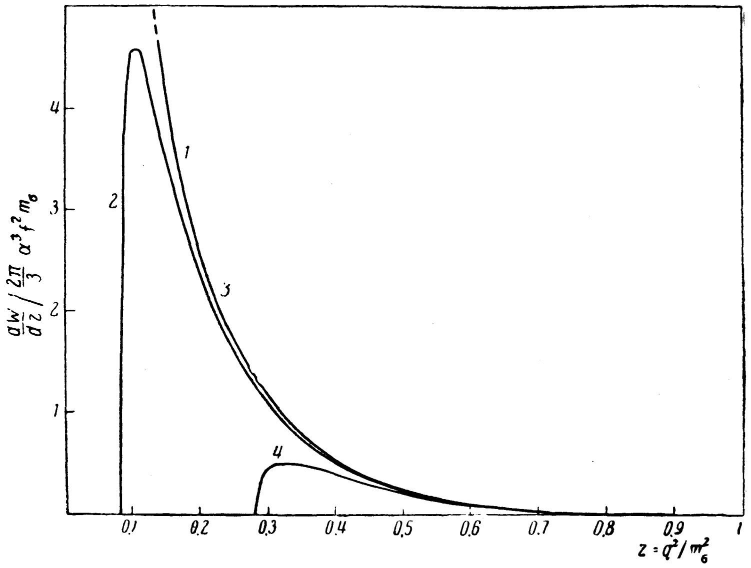

FIG. 5. Distribution with respect to q^2: Curves 1 and 2 are for the decays (19) and (20) of a σ^0 meson with $m_\sigma = 750$ MeV; 3 and 4 are for decays (17) and (18) of a σ^0 meson with $m_\sigma = 400$ MeV. The maxima of distributions 1 and 3 are located at $z \cong 5.5\,(m_e/m_\sigma)^2$ and are equal to 2.5×10^5 for 1 and 0.72×10^5 for 3.

where $F(q^2)$ is an unknown function of q^2. Using (21) we obtain[3)]

$$W(z, x)\,dzdx = \frac{\pi}{2}\alpha^3F^2(z)\,m_\sigma\left\{z + 4\varepsilon^2 + \frac{4x^2z(z-4\varepsilon^2)}{(1+z)^2-(1-z)^2x^2}\right\} \times \frac{(1-z)(1+z)^2\sqrt{z-4\varepsilon^2}}{z^2[(1+z)^2-(1-z)^2x^2]^{3/2}}\,dzdx, \quad (22)$$

$$W(z)\,dz = \frac{2\pi}{3}\alpha^3F^2(z)\,m_\sigma\frac{(1-z)^3(z+2\varepsilon^2)}{z^2}\sqrt{1-\frac{4\varepsilon^2}{z^2}}\,dz, \quad (23)$$

where

$$4\varepsilon^2 \leqslant z = q^2/m_\sigma^2 \leqslant 1, \qquad \varepsilon = m/m_\sigma, \qquad -1 \leqslant x \leqslant 1.$$

Figure 5 shows the distributions (23) for the electrons and muons at $m_\sigma = 750$ and 400 MeV. The distributions $W(x)$ for e and μ are obtained by numerical integration of the distribution (22) with respect to z under the assumption $F(q^2)$ = const. They are indicated in the table. For $F(q^2)$ = const we obtain from (23) the total probability of the decays (19) and (20)

$$W = \frac{2\pi}{3}\alpha^3F^2m_\sigma\left\{(1-18\varepsilon^4+8\varepsilon^6)\ln\frac{1-2\varepsilon^2+\sqrt{1-4\varepsilon^2}}{2\varepsilon^2} - \left(\frac{7}{2}-13\varepsilon^2-4\varepsilon^4\right)\sqrt{1-4\varepsilon^2}\right\}. \quad (24)$$

	m_σ, MeV	$x=0$	$x=\pm 1/2$	$x=\pm 0.9$	$x=\pm 1$
$W_e(x)$	750	0.1	0.172	0.83	$1.1\cdot 10^4$
	400	0.1	0.144	0.75	$3.2\cdot 10^3$
$W_\mu(x)$	750	0.037	0.051	0.112	0.51
	400	0.00756	0.00843	0.01098	0.0121

3)Formula (23) for the $\pi^0 \to \gamma + e^+ + e^-$ decay was first obtained by Dalitz.[10]

For the electron, formula (24) simplifies if we make use of the fact that $m_e \ll m_\sigma$:

$$W(\sigma^0 \to \gamma + e^+ + e^-) = \frac{2\pi}{3}\alpha^3F^2m_\sigma\left(\ln\frac{m_\sigma^2}{m_e^2} - \frac{7}{2}\right). \quad (25)$$

If the mass of the σ^0 meson is close to $2m_\mu$, then (24) assumes the form

$$W(\sigma^0 \to \gamma + \mu^+ + \mu^-) = \frac{32\pi}{315}\alpha^3F^2m_\sigma\delta^{7/2}, \text{ where}$$

$$\delta = \frac{m_\sigma^2 - 4m_\mu^2}{m_\sigma^2} \ll 1. \quad (26)$$

When $m_\sigma = 750$ MeV we have

$$W(\sigma^0 \to \gamma + e^+ + e^-)/W(\sigma^0 \to \gamma + \mu^+ + \mu^-) = 15;$$

when $m_\sigma = 400$ MeV, this ratio is equal to 110. Since $W(\sigma^0 \to 2\gamma) = \pi^2\alpha^2F^2m_\sigma$ (if the fact that the photons are identical is taken into account), we obtain in the limit of (26)

$$\frac{W(\sigma^0 \to \gamma + \mu^+ + \mu^-)}{W(\sigma^0 \to 2\gamma)} = \frac{32\alpha}{315\pi}\delta^{-7/2}.$$

If $m_\sigma = 400$ MeV, then

$$\frac{W(\sigma^0 \to \gamma + e^+ + e^-)}{W(\sigma^0 \to 2\gamma)} = \frac{1}{66}, \qquad \frac{W(\sigma^0 \to \gamma + \mu^+ + \mu^-)}{W(\sigma^0 \to 2\gamma)} = \frac{1}{7250}.$$

When $m_\sigma = 750$ MeV

$$\frac{W(\sigma^0 \to \gamma + e^+ + e^-)}{W(\sigma^0 \to 2\gamma)} = \frac{1}{58}, \qquad \frac{W(\sigma^0 \to \gamma + \mu^+ + \mu^-)}{W(\sigma^0 \to 2\gamma)} = \frac{1}{870}$$

(for the π^0 meson, $W(\pi^0 \to \gamma + e^+ + e^-)/W(\pi^0 \to 2\gamma) = 1.80$[11]).

If it is assumed that the decay $\sigma^0 \to 2\gamma$ can be the main one, the situation for the study of the decays (19) and (20) turns out to be quite favorable (one decay for each 100 or 1000 events).

In conclusion the authors are deeply grateful to L. B. Okun' for suggesting the topic and for continuing interest in the work, and to I. Yu. Kobzarev for useful remarks.

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