THE $\mu^+\mu^-$ AND e^+e^- PAIR PRODUCTION CROSS SECTIONS FOR NEUTRINOS SCATTERED BY NUCLEI

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The cross sections for muon and electron-positron pair production are computed for $E \sim 1$ BeV neutrinos scattered on nuclei. The values of these cross sections are estimated in the case of noncoherent scattering.

THE work done towards the production of neutrino beams in accelerators [1] gives grounds for hoping that experimental data will become available in the near future on lepton pair production in the scattering of neutrinos on nuclei in the neutrino energy region $E \sim 1$ BeV.

In the present paper we calculate, in this energy region, the cross sections of the processes

$$Z + \nu \rightarrow Z + \nu' + \mu^+ + \mu^-, \qquad (1)$$

$$Z + \nu \rightarrow Z + \nu' + e^+ + e^-. \qquad (2)$$

Processes (1) and (2) were investigated earlier [2,3] with an aim towards clarifying the character of the energy dependence of the cross sections at superhigh values of the neutrino energy ($E \gg 1$ BeV). In particular, Kozhushner and the author [3] calculated the cross section of process (1) and pointed out the errors in the results of many investigations [2] in which the influence of the form factor of the nucleus was not taken into account, and consequently an incorrect energy dependence was obtained for the cross section. Since, however, the formulas of [3] were obtained in the limit of superhigh values of the neutrino energy (on the order of several times 10 BeV), they cannot be used in the energy region of interest to us here. We shall show below, in Sec. 1, what changes must be made in the calculations of the cross section of the process (1) to accommodate lower values of the energy.

In Sec. 2 we estimate the cross section of process (2). Section 3 contains estimates of the cross section of processes (1) and (2) in the case when an excited nucleus is produced in the final state or in the case of nuclear breakup.

The cross section of processes (1) and (2), which occur in the Coulomb field of an infinitely heavy nucleus with zero spin, can be represented [3] in the form

$$\sigma_{1,2} = \frac{Z^2\alpha}{\pi}\int F^2(\Delta^2)\frac{dw^2}{w^2+\Delta^2}\frac{d\Delta^2}{\Delta^2}\left\{a(w^2,\Delta^2)\left[1-\frac{(w^2+\Delta^2)^2}{4E^2\Delta^2}\right] + b(w^2,\Delta^2)\frac{w^2}{2E^2}\right\}. \qquad (3)$$

In this formula $F(\Delta^2)$ is the electromagnetic form factor of the nucleus; $\Delta^2 = -q^2$, where q is the 4-momentum transferred to the nucleus; w is the energy in the center of mass of the particles $\nu' + \mu^+(e^+) + \mu^-(e^-)$. The functions $a(w^2, \Delta^2)$ and $b(w^2, \Delta^2)$ are determined from the relations

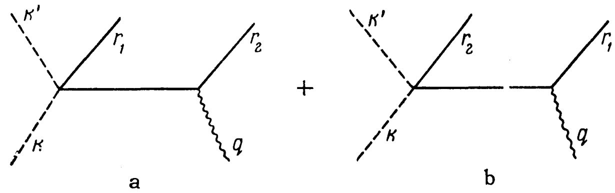
$$\Phi_{\alpha\beta} = a\left[(kq)\,\delta_{\alpha\beta} + \frac{k^2}{(kq)}q_\alpha q_\beta - q_\alpha k_\beta - q_\beta k_\alpha\right] + b\,[q^2\delta_{\alpha\beta} - q_\alpha q_\beta], \qquad (4)$$

$$\Phi_{\alpha\beta} = \frac{1}{(2\pi)^9}\sum_{\text{polar}} M_\alpha M_\beta^+ E_q E_k d^3r_1 d^3r_2 d^3k', \qquad (5)$$

where

$$M_\lambda = (2\pi)^4\frac{G}{\sqrt{2}}\frac{e}{\sqrt{2E_q}}\bar{u}(r_2)\left[\gamma_\lambda\frac{1}{\hat{r}_2-\hat{q}-m}\gamma_\tau(1+\gamma_5) + \gamma_\tau(1+\gamma_5)\frac{1}{-\hat{r}_1+\hat{q}-m}\gamma_\lambda\right]u(r_1)\bar{u}(k')\gamma_\tau(1+\gamma_5) \times u(k)\,\delta^4(k+q-r_1-r_2-k') \qquad (5')$$

and the summation is over the polarizations of all the spin particles, G is the weak-interaction constant, e is the electrical charge and u are spin-



Feynman diagram of processes (6) and (6′); k and k′ are the momenta of the incoming and scattered neutrinos, r_1 and r_2 are the momenta of the positron and electron respectively, and q is the photon momentum.

Cross section of processes (1) and (2) as a function of the neutrino energy

E, Bev	σ_1, cm²		σ_2, cm²	
	Lead	Iron	Lead	Iron
20	$9\cdot10^{-41}$	$1.8\cdot10^{-41}$	$7\cdot10^{-40}$	10^{-40}
2	$7\cdot10^{-43}$	$2\cdot10^{-43}$	$4\cdot10^{-41}$	$5.6\cdot10^{-42}$
1	$7.4\cdot10^{-44}$	$2.6\cdot10^{-44}$	$1.6\cdot10^{-41}$	$2\cdot10^{-42}$
0.5	$3\cdot10^{-45}$	$1.3\cdot10^{-45}$	$6\cdot10^{-42}$	$7.2\cdot10^{-43}$
0.4	$6.6\cdot10^{-46}$	$2.8\cdot10^{-46}$	$4\cdot10^{-42}$	$5\cdot10^{-43}$
0.3	$3.4\cdot10^{-47}$	$1.6\cdot10^{-47}$	$2.6\cdot10^{-42}$	$2.8\cdot10^{-43}$

ors. The remaining symbols are explained in the caption to the figure.

It follows from (4), (5), and (5′) that when $q^2 = 0$ the quantity

$$\frac{1}{2(kq)}\,\mathrm{Sp}\,\Phi_{\alpha\beta} = a$$

is the cross section of the photoprocesses

$$\gamma + \nu \to \nu' + \mu^+ + \mu^-, \qquad (6)$$

$$\gamma + \nu \to \nu' + e^+ + e^-. \qquad (6')$$

Using these formulas, let us consider the reactions of interest to us.

1. THE CROSS SECTION OF PROCESS (1).

This cross section was calculated earlier [3] for the case $E \gg 1$ BeV under the following assumptions:

1) The form factor of the nucleus can be chosen to be

$$F(\Delta) = \begin{cases}1, & \Delta \leqslant 1/R = \Delta_0,\\ 0, & \Delta > \Delta_0,\end{cases} \qquad (7)$$

where R is the nuclear radius, with value $A^{1/3}/m_\pi$, A is the mass number of the nucleus, and m_π is the mass of the pion.

2) The values of $a(w^2, \Delta^2)$ differ little, in the considered region of Δ, from $a(w^2, 0)$—the cross sections of the photoprocess (6), the latter being taken in the region $w \ll 2m_\pi$.

The ratio of the minimum momentum transfer $\Delta_{min} = 2m_\mu^2/E$ to the maximum transfer, defined by formula (7), is $\Delta_{min}/\Delta_0 \approx A^{1/3}/7E$, and can be made larger than unity for heavy nuclei at energies $E < 1$ BeV, which is absurd. Therefore, if we consider the energy region $E \sim 1$ BeV, we must choose the form factor in the form of a smooth function. We shall make use of a form factor

$$F(\Delta^2) = (1 + \Delta^2/6\Delta_0^2)^{-1}, \qquad (8)$$

which corresponds to the "Yukawa-2" model of the charge distribution in the nucleus [4].

The region of values of momentum transfer that are allowed by the kinematics is determined by the relation

$$\Delta^2_{\substack{max\\min}} = 2E^2 - w^2 \pm 2E^2\sqrt{1 - w^2/E^2}, \qquad (9)$$

and is quite broad. However, because of the action of the form factor, large values of Δ^2 are little likely and it turns out that the effective values of Δ^2 in the functions $a(w^2, \Delta^2)$ and $b(w^2, \Delta^2)$ in formula (3) are $\Delta^2 \sim 6\Delta_0^2 = 6m_\pi^2/A^{2/3}$. It will be shown in Sec. 2 that for values of Δ^2 satisfying the condition $\Delta^2 \ll m_\mu^2$, the values of $a(w^2, \Delta^2)$ and $b(w^2, \Delta^2)$ differ little from their values for $\Delta^2 = 0$. In the case of heavy nuclei $6m_\pi^2/A^{2/3} \ll m_\mu^2$, and consequently one can substitute in (3) the values of a and b for $\Delta^2 = 0$. Then, integrating (3) with respect to Δ^2, we get

$$\sigma_1 = \frac{Z^2\alpha}{\pi}\int_{4m_\mu^2}^{E^2}\frac{dw^2}{w^2}\Big\{a(w^2,0)$$

$$\times\left[f(w^2)\ln\frac{f(w^2)+\sqrt{1-w^2/E^2}}{f(w^2)-\sqrt{1-w^2/E^2}} - 2\sqrt{1-w^2/E^2}\right]$$

$$+ b(w^2,0)\frac{w^2}{2E^2}\left[\ln\frac{f(w^2)+\sqrt{1-w^2/E^2}}{f(w^2)-\sqrt{1-w^2/E^2}}\right.$$

$$\left.-\frac{\sqrt{1-w^2/E^2}}{1-w^2/2E^2+w^4/24E^2\Delta_0^2+3\Delta_0^2/2E^2}\right]\Big\},$$

$$f(w^2) = 1 - w^2/2E^2 + w^4/12E^2\Delta_0^2. \qquad (10)$$

The cross section of the reaction (6), $a(w^2, 0)$, calculated for the case when ultrarelativistic muons are produced is [1)]

$$a(w^2)_{rel} = \frac{2}{9\pi^2}\alpha G^2 w^2\left(\ln\frac{w}{m_\mu} - \frac{65}{48}\right). \qquad (11)$$

We cannot use this formula, however, because for a neutrino with energy $E \sim 1$ BeV the effective values of w in formula (10) lie near the threshold of the reaction (6). In this case, namely when

[1)]This formula differs from the result of Kozhushner and the author[3] by the numerical factor in front of the second term, because in the present calculations we have taken into account terms proportional to m² in the formula for the probability for the process (6).

$w - 2m_\mu \ll w$, the formula for $a(w^2)$ has the form

$$a(w^2) = 2^{13/2}\alpha\, G^2 (w - 2m_\mu)^{7/2} / 7!!\,\pi^2 w^{3/2}. \tag{12}$$

From a comparison of formulas (11) and (12) we see that $a(w^2) \sim w^2$ over a wide range of the argument w. Since the order of magnitude of the function $b(w^2)$ and its increase growth with energy are not larger than those of $a(w^2)$, (see Sec. 2), the term proportional to $b(w^2)$ in formula (10) has an order of smallness $\sqrt{6}\,\Delta_0/E$ and can be neglected. As a result we obtain

$$\sigma_1 = \frac{2^{13/2} Z^2 \alpha^2 G^2}{7!!\,\pi^3} \int\limits_{4m_\mu^2}^{E^2} \frac{dw^2}{w^2} \frac{(w - 2m_\mu)^{7/2}}{w^{3/2}} \left\{ \left(1 - \frac{w^2}{2E^2} + \frac{w^4}{12E^2\Delta_0^2}\right) \right.$$
$$\times \ln \frac{1 - w^2/2E^2 + w^4/12E^2\Delta_0^2 + \sqrt{1 - w^2/E^2}}{1 - w^2/2E^2 + w^4/12E^2\Delta_0^2 - \sqrt{1 - w^2/E^2}}$$
$$\left. - 2\sqrt{1 - w^2/E^2} \right\}. \tag{13}$$

The table lists the cross sections of process (1) for two nuclei, lead and iron, at different values of neutrino energy. For lead and at E = 1 BeV we obtain, in particular,

$$\sigma_1 = 0.7 \cdot 10^{-43}\ \text{cm}^2.$$

2. CROSS SECTION OF PROCESS (2)

This case differs from the foregoing in the following:

1) If we turn again to formula (7), we see that the ratio Δ_{min}/Δ_0 in the case of production of an e^+e^- pair is a quantity much smaller than unity and we can choose (to simplify the calculations) $F(\Delta)$ in the form (7).

2) Owing to the smallness of m_e compared with m_μ, the condition $\Delta^2_{eff} \gg m^2$, which would enable us to substitute the values of a and b for $\Delta^2 = 0$ in the case of muon pair production, is not satisfied.

The latter leads to the need for calculating the functions $a(w^2, \Delta^2)$ and $b(w^2, \Delta^2)$, and consequently also the tensor $\Phi_{\alpha\beta}$ for $\Delta^2 \neq 0$. The determination of the tensor $\Phi_{\alpha\beta}$ entails, however, cumbersome calculations. We therefore attempt to find $a(w^2, \Delta^2)$ and $b(w^2, \Delta^2)$ by another means, namely comparison of the traces of (4) and (5).

For estimates we can confine ourselves to an examination of the contribution of one term in (5′), say the first one. We denote the part of the tensor $\Phi_{\alpha\beta}$ corresponding to the contribution of this term by $\Phi^{I}_{\alpha\beta}$. Then

$$\text{Sp}\,\Phi^{I}_{\alpha\beta} = \frac{\alpha G^2 w^2 (w^2 - \Delta^2)}{2\pi^2 (w^2 + \Delta^2)} \int\limits_{m_e}^{w/2} (w - 2E_1)\, dE_1$$
$$\times \left\{ \ln \frac{|\mathbf{r}_1| + E_1 + \Delta^2 (w - 2E_1)/(w^2 + \Delta^2)}{-|\mathbf{r}_1| + E_1 + \Delta^2 (w - 2E_1)/(w^2 + \Delta^2)} \right.$$
$$- \frac{\Delta^2 w (w^2 - \Delta^2 - 2E_1 w)}{(w^2 + \Delta^2)(w^2 - \Delta^2)} \left[\frac{1}{-|\mathbf{r}_1| + E_1 + \Delta^2 (w - 2E_1)/(w^2 + \Delta^2)} \right.$$
$$\left.\left. - \frac{1}{|\mathbf{r}_1| + E_1 + \Delta^2 (w - 2E_1)/(w^2 + \Delta^2)} \right] \right\}. \tag{14}$$

The argument of the logarithm in (14) can be rewritten

$$\left[2E_1 + \frac{\Delta^2 (w - 2E_1)}{w^2 + \Delta^2}\right]^2 \left\{ m^2 + \left[2E_1 + \frac{\Delta^2 (w - 2E_1)}{w^2 + \Delta^2}\right] \frac{\Delta^2 (w - 2E_1)}{w^2 + \Delta^2} \right\}^{-1}$$
$$\approx \begin{cases} 4E_1^2/m^2, & \Delta^2 \ll m^2, \quad (15) \\ (2E_1 w + \Delta^2)\, w/(w - 2E_1)\,\Delta^2, & \Delta^2 \gg m^2. \quad (15') \end{cases}$$

The relation (15) is realized in the reaction (6). For reaction (6′) it is necessary to use (15′), in view of the smallness of the electron mass. We then obtain

$$\text{Sp}\,\Phi^{I}_{\alpha\beta} = \frac{\alpha G^2}{8\pi^2} \left\{ (w^2 + \Delta^2) \left[w^2 \ln \frac{w^2 + \Delta^2}{\Delta^2} - \frac{3}{2} \frac{w^4}{w^2 + \Delta^2} \right] \right.$$
$$\left. + \Delta^2 \left[(w^2 - \Delta^2) \ln \frac{w^2 + \Delta^2}{\Delta^2} + \frac{w^2 \Delta^2}{w^2 + \Delta^2} \right] \right\}, \qquad \Delta^2 > m^2. \tag{16}$$

From formula (4) we have

$$\text{Sp}\,\Phi_{\alpha\beta} = (w^2 + \Delta^2)\, a(w^2, \Delta^2) - 3\Delta^2 b(w^2, \Delta^2). \tag{17}$$

When an attempt is made to obtain the functions $a(w^2, \Delta^2)$ and $b(w^2, \Delta^2)$ from a comparison of (16) and (17) we encounter two kinds of ambiguities. The first consists in the fact that in formula (16) itself we can separate $a(w^2, \Delta^2)$ only accurate to terms of order $\Delta^2/(w^2 + \Delta^2)$. The second ambiguity is connected with the operation of taking the trace, during the course of which, generally speaking, some terms which previously were included in $a(w^2, \Delta^2)$ may cancel out terms contained in the function $b(w^2, \Delta^2)$, and consequently may not appear in formula (16) at all. It is easy to see from (4), however, that such terms are also of order $\Delta^2/(w^2 + \Delta^2)$ compared with the principal terms.

Thus, $a(w^2, \Delta^2)$ can be determined only accurate to terms of order $\Delta^2/(w^2 + \Delta^2)$. With respect to the function $b(w^2, \Delta^2)$ it follows that the order of its magnitude and its increase with energy is not higher than that of $a(w^2, \Delta^2)$. If we recognize that the effective values $[\Delta^2/(w^2 + \Delta^2)]_{eff}$ and $(w^2/E^2)_{eff}$ are of the order of $m_\pi/EA^{1/3}$ and neglect terms that have the same order of smallness, we obtain

$$d\sigma_2 \approx \frac{Z^2\alpha^2G^2}{4\pi^3}\frac{d\Delta^2}{\Delta^2}\frac{dw^2}{w^2+\Delta^2}\left[1-\frac{(w^2+\Delta^2)^2}{4E^2\Delta^2}\right]$$

$$\times\left[(w^2+\Delta^2)\ln\frac{w^2+\Delta^2}{\Delta^2}-\frac{3w^4}{2(w^2+\Delta^2)}\right]. \quad (18)$$

Integrating (18) we get

$$\sigma_2 \approx \frac{2Z^2\alpha^2G^2}{3\pi^3}E\Delta_0\left[\ln\frac{2E}{\Delta_0}-\frac{11}{6}\right]. \quad (19)$$

The numerical values of σ_2 for lead and iron nuclei are listed in the table. In particular, at E = 1 BeV and Z = 82 we get

$$\sigma_2 \approx 2\cdot 10^{-41}\ \mathrm{cm}^2.$$

3. CROSS SECTIONS OF THE PROCESSES (1) AND (2) IN THE CASE OF NONCOHERENT SCATTERING

We have so far considered processes in which the nucleus interacts with the virtual protons as a whole, which corresponds to a momentum transfer $q \sim 1/R$. At large momentum transfers, the internal structure of the nucleus begins to play a significant role and consequently the interactions between the virtual photons and individual nucleons of the nucleus can cause the latter to go over into an excited state or to break up, producing a star. The cross section of processes (1) and (2) can in this case be estimated in the following fashion:

$$\sigma_{1,2\,\mathrm{noncoh}} \approx Z\sigma_{1,2p}, \quad (20)$$

where σ_{1p} and σ_{2p} are respectively the cross sections of processes (1) and (2) which take place on the proton.

By way of an estimate of the cross section σ_{1p} we can use formula (13), in which we put Z = 1 and $\Delta_0 = m_\pi/0.6$. For a neutrino energy E = 1 BeV we obtain $\sigma_{1p} \approx 1.6\times 10^{-45}\ \mathrm{cm}^2$. This estimate is quite crude, since formula (13) is applicable generally speaking only to heavy nuclei. A more correct analysis will lead apparently to a reduction in the obtained value of σ_{1p}. If we assume $\sigma_{1p} = 1.6\times 10^{-45}$, we obtain for lead

$$\sigma_{1\,\mathrm{noncoh}} = 1.3\cdot 10^{-43}\ \mathrm{cm}^2.$$

The noncoherent cross section of the process (2), estimated by formulas (19) and (20), is

$$\sigma_{2\,\mathrm{noncoh}} = 10^{-43}\ \mathrm{cm}^2.$$

We see that in the production of $\mu^+\mu^-$ and e^+e^- pairs the noncoherent cross sections are differently related to the corresponding coherent cross sections. Namely $\sigma_{1\mathrm{noncoh}}/\sigma_1 \sim 1$, whereas $\sigma_{2\mathrm{noncoh}}/\sigma_2 \sim 1/Z$. The strong increase in the ratio $\sigma_{1\mathrm{noncoh}}/\sigma_1$, compared with $\sigma_{2\mathrm{noncoh}}/\sigma_2$, in the energy region under consideration is due to the following circumstance: the creation of a large mass ($\mu^+\mu^-$ pair) is accompanied by large momentum transfers, which, however, are suppressed by the action of the form factor. The suppression is much less in the case of a proton than in the case of a nucleus, and this leads to a strong increase in the noncoherent cross section. In the case of production of a small mass (e^+e^- pair), the difference between the effects of the nuclear and proton form factors is not so significant.

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