ON THE THEORY OF THE SCATTERING OF SLOW NEUTRONS IN A FERMI LIQUID

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The scattering of slow neutrons in a Fermi liquid is investigated theoretically. The cross sections for direct scattering and scattering involving the excitation of zero and ordinary sound are determined.

1. The purpose of the present paper is the theoretical investigation of the scattering of slow neutrons in a Fermi liquid. It is known that, for sufficiently low temperatures, specific sound waves —the so-called zero sound (Landau,[1] Klimontovich and Silin[2])—may propagate in such a system. We shall show that these waves can be excited by irradiating the Fermi liquid with slow neutrons whose velocity exceeds the velocity of the zero sound.

The investigation of the scattering of slow neutrons in liquid He^3, which is a Fermi liquid, can therefore serve, in principle, to verify the existence of the zero sound.

Besides the scattering with excitation of the zero sound (and ordinary sound), direct scattering of the neutrons by the nuclei of the Fermi liquid is also possible. In these scattering processes the energy transfer and the scattering angle are not correlated, whereas in scattering processes connected with the excitation of collective degrees of freedom and having the character of Cerenkov radiation, the scattering angle is a unique function of the energy transfer.

2. The Hamiltonian for the interaction of a slow neutron with the nuclei of the Fermi liquid is given by a sum of Fermi pseudopotentials:

$$\mathcal{H} = -\frac{2\pi\hbar^2}{m'}\sum_i (a + b\mathbf{s}\mathbf{K}_i)\,\delta(\mathbf{r}-\mathbf{r}_i), \qquad (1)$$

where $\mathbf{r}$, $\mathbf{r}_i$, and $\mathbf{s}$, $\mathbf{K}_i$ are the radius vectors and the spins of the neutron and the i-th nucleus, m′ is the reduced mass of the neutron-nucleus system, and a and b are the coherent and incoherent scattering lengths for the scattering of a neutron from a free nucleus of the Fermi liquid (the quantities a and b are complex owing to the absorption of the neutrons by the nuclei). The effect of the Fermi liquid comes most clearly into play for small neutron momentum transfers $\Delta\mathbf{p}_n$. We shall therefore assume in the following that $|\Delta\mathbf{p}_n| \ll p_0$, where p_0 is the limiting momentum of the Fermi liquid. Under these conditions we can replace the summation over i in (1) by an integration over $\rho(\mathbf{r}_i, \alpha_i, t)\,d\mathbf{r}_i$, where ρ is the density of nuclei with spin projection α_i at the point $\mathbf{r}_i$ of the Fermi liquid:

$$\mathcal{H} = -(2\pi\hbar^2/m')\,(a + b\mathbf{s}\mathbf{K}(\mathbf{r}))\,\rho(\mathbf{r}, \alpha, t). \qquad (2)$$

The deviation of the density of the Fermi liquid from its equilibrium value ρ_0 is clearly equal to $\delta\rho = \int \delta n_p d\tau_p$, where $d\tau_p = (2\pi\hbar)^{-3}d\mathbf{p}$ ($\mathbf{p}$ is the momentum of a quasiparticle of the Fermi liquid) and δn_p is the deviation of the quasiparticle distribution function from its equilibrium value

$$n_p^0 = \left[\exp\left(\frac{\varepsilon_p - \zeta}{T}\right) + 1\right]^{-1}$$

(ε_p is the energy of the quasiparticle, ζ is the chemical potential). δn_p satisfies, according to Landau,[3] the equation

$$\left(\frac{\partial}{\partial t} + \frac{\partial \varepsilon_p}{\partial \mathbf{p}}\frac{\partial}{\partial \mathbf{r}}\right)\delta n_p(\mathbf{r}, t) - \frac{\partial n_p^0}{\partial \mathbf{p}}\,\mathrm{Sp}_{K'}\int f\frac{\partial}{\partial \mathbf{r}}\,\delta n_{p'}(\mathbf{r}, t)\,d\tau_{p'} = I\{\delta n_p\}. \qquad (3)$$

Here I is the collision integral, and $f = f(\mathbf{p}, \mathbf{p}') + \mathbf{K}\cdot\mathbf{K}'q(\mathbf{p}, \mathbf{p}')$ is a quantity which characterizes the interaction of the quasiparticles. At absolute zero $\delta n_p = (\nu + \mathbf{K}\cdot\boldsymbol{\mu})\,\delta(\varepsilon_p - \zeta)$, where ν and μ are certain functions of $\mathbf{n} = \mathbf{p}/p$ and $\mathbf{r}$, t. The monochromatic oscillations $\delta n_p\,(\sim e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)})$ satisfy the equations

$$(\eta_i - \cos\theta)\,\nu_{qi}(\mathbf{n}) = \cos\theta \int F(\widetilde{\chi})\,\nu_{qi}(\mathbf{n}')\,do'/4\pi,$$

$$(\eta_i' - \cos\theta)\,\mu_{qi}(\mathbf{n}) = \cos\theta \int G(\widetilde{\chi})\,\mu_{qi}(\mathbf{n}')\,do'/4\pi, \qquad (4)$$

where θ is the angle between $\mathbf{n}$ and $\mathbf{q}$, $\widetilde{\chi}$ is the angle between $\mathbf{p}$ and $\mathbf{p}'$, do′ is the angular part of the volume element of $\mathbf{p}'$, $\eta_i = s_i/v_0$, $\eta_i' = s_i'/v_0$, $s_i = \omega_i/q$, $s_i' = \omega_i'/q$ are the propagation velocities of the oscillations of the quantities ν and μ (the index i denotes the kind of oscillation), v_0 is the Fermi velocity, and the quantities F and G, which are proportional to $f(\mathbf{p}, \mathbf{p}')$ and $g(\mathbf{p}, \mathbf{p}')$, determine the change of the energy of the quasiparticle, which is related to the change of the distribution function by

$$\delta\varepsilon_p = \int\left(F\nu' + \tfrac{1}{4}G\boldsymbol{\mu}'\mathbf{K}\right)do'/4\pi,$$
$$\nu' = \nu(\mathbf{n}', \mathbf{r}, t), \qquad \boldsymbol{\mu}' = \boldsymbol{\mu}(\mathbf{n}', \mathbf{r}, t). \tag{5}$$

For sufficiently low temperatures, when there is very little absorption of zero sound, the scattering of the neutron and the excitation of the zero sound can be interpreted as the emission of a quantum of zero sound by the neutron. This process is described by a Hamiltonian which differs from (2) in that ρ is replaced by the change of the density of the Fermi liquid caused by the zero sound oscillations.

In order to calculate the emission probability for a quantum of zero sound, we must quantize the zero sound. For this purpose we must first determine the energy of the oscillations of the zero sound,

$$E = \int d\mathbf{r}\,\mathrm{Sp}\int_{p=p_0}^{p_0+\delta p_0}\varepsilon_p\,d\tau_p + \frac{1}{2}\int d\mathbf{r}\,\mathrm{Sp}\int\delta\varepsilon_p\,\delta n_p\,d\tau_p, \tag{6}$$

where $\varepsilon_p = v_0(p - p_0)$ and $\delta p_0 = (\nu + \mathbf{K}\cdot\boldsymbol{\mu})/v_0$ is the change of the Fermi momentum connected with the oscillations of the Fermi surface [the second term in (6) takes account of the interaction between the quasiparticles]. Using (5), we obtain

$$E = \frac{p_0^2}{(2\pi\hbar)^3 v_0}\int d\mathbf{r}\left\{\int\left(\nu^2 + \frac{1}{4}\boldsymbol{\mu}^2\right)do + \iint\left(F\nu\nu' + \frac{1}{16}G\boldsymbol{\mu}\boldsymbol{\mu}'\right)\frac{do\,do'}{4\pi}\right\}. \tag{7}$$

This expression should, of course, have the form

$$E = \sum_{\mathbf{q}i}\left(N_{qi} + \frac{1}{2}\right)\hbar\omega_i + \sum_{\mathbf{q},\,i,\,\lambda}\left(N_{qi\lambda} + \frac{1}{2}\right)\hbar\omega_i', \tag{8}$$

where N_{qi} is the number of quanta of zero sound of the type ν_i and frequency ω_i, and $N_{qi\lambda}$ is the number of quanta of zero sound of the type μ_i, frequency ω_i', and polarization $\lambda = 1, 2, 3$. Expression (8) follows from (7), if we expand ν and μ in terms of plane waves:

$$\nu = \frac{(2\pi\hbar)^{3/2}}{p_0}\sum_{\mathbf{q},\,i}\left(\frac{1}{2}v_0\hbar\omega_i\right)^{1/2}\{\nu_{qi}(\mathbf{n})\,c_{qi}e^{i(\mathbf{qr}-\omega_i t)} + \nu_{qi}^*(\mathbf{n})\,c_{qi}^+e^{-i(\mathbf{qr}-\omega_i t)}\},$$
$$\boldsymbol{\mu} = \frac{(2\pi\hbar)^{3/2}}{p_0}\sum_{\mathbf{q},\,i,\,\lambda}(2v_0\hbar\omega_i')^{1/2}\{\boldsymbol{\mu}_{qi\lambda}(\mathbf{n})\,c_{qi\lambda}e^{i(\mathbf{qr}-\omega_i' t)} + \boldsymbol{\mu}_{qi\lambda}^*(\mathbf{n})\,c_{qi\lambda}^+e^{-i(\mathbf{qr}-\omega_i' t)}\} \tag{9}$$

interpret the quantities c_{qi}, c_{qi}^+, $c_{qi\lambda}$, $c_{qi\lambda}^+$ as boson absorption and creation operators for the corresponding zero sound quanta, and subject the functions $\nu_{qi}(\mathbf{n})$ and $\boldsymbol{\mu}_{qi\lambda}(\mathbf{n})$ to the normalization conditions*

$$\eta_i\int|\nu_{qi}(\mathbf{n})|^2\,do/\cos\theta = 1.$$
$$\eta_i'\int|\boldsymbol{\mu}_{qi\lambda}(\mathbf{n})|^2\,do/\cos\theta = 1. \tag{10}$$

Substituting (9) in (2), we find for the energy of the interaction of a neutron with the zero sound oscillations

$$\mathscr{H}^{(0)} = -\frac{m^*}{m'}\sqrt{\frac{\hbar v_0}{\pi}}\left\{a\sum_{\mathbf{q}i}\sqrt{\hbar\omega_i}\,(A_i c_{qi}^+e^{-i(\mathbf{qr}-\omega_i t)} + \mathbf{h.c.}) + 2\,[(a + bs\mathbf{K})\,\mathbf{K}]_c\sum_{\mathbf{q}i\lambda}\sqrt{\hbar\omega_i}\,(\mathbf{B}_{i\lambda}c_{qi\lambda}^+e^{-i(\mathbf{qr}-\omega_i' t)} + \mathbf{h.c.}\right\}, \tag{11}$$

where

$$A_i = \int\nu_{qi}(\mathbf{n})\,do, \qquad \mathbf{B}_{i\lambda} = \int\boldsymbol{\mu}_{qi\lambda}(\mathbf{n})\,do, \tag{12}$$

and the bracket $[\,Y\,]_c$ denotes the part of Y which is coherent in the spins of the nuclei and does not contain the nuclear spin operators $\mathbf{K}$ linearly; $m^* = p_0/v_0$.

Using the expression for $\mathscr{H}^{(0)}$, we can easily determine the cross sections for the scattering of a neutron (from a single nucleus) with emission of the different types of zero sound quanta:

$$d\sigma_{\uparrow\uparrow}^{(\nu_i)} = \frac{3}{2}(m^*/m')^2\,|A_i|^2\,|a|^2(s_i v_0/v_n^2)(\hbar^3 q^2\,dq/p_0^3), \quad d\sigma_{\uparrow\downarrow}^{(\nu_i)} = 0,$$
$$d\sigma_{\uparrow\uparrow}^{(\mu_i)} = \frac{3}{2}\left(\frac{m^*}{m'}\right)^2|B_{iz}|^2\frac{|b|^2}{16}\frac{s_i' v_0}{v_n^2}\frac{\hbar^3 q^2\,dq}{p_0^3},$$
$$d\sigma_{\uparrow\downarrow}^{(\mu_i)} = \frac{3}{2}\left(\frac{m^*}{m'}\right)^2|B_i^+|^2\frac{|b|^2}{16}\frac{s_i' v_0}{v_n^2}\frac{\hbar^3 q^2\,dq}{p_0^3}, \tag{13}$$

where v_n is the velocity of the neutron, $B_i^+ = B_{ix} + iB_{iy}$ (the z axis is directed along $\mathbf{s}$), the arrows indicate whether the directions of the neutron spin before and after the scattering are parallel or antiparallel with respect to each other, and the indices ν_i and μ_i denote the type of zero sound quantum emitted.

It is very probable [1] that the simultaneous propagation of ν_i and μ_i waves in the Fermi liquid is not possible (at least, if $F(\tilde{\chi}) = \text{const}$). In the following we shall therefore only consider the excitation of ν_i waves. In this case the conservation laws

$$\mathbf{p}_n = \mathbf{p}_n' + \hbar\mathbf{q}, \qquad E_n = E_n' + \hbar q s_i, \tag{14}$$

*To derive these conditions, we must use the relations

$$\iint F\nu_{qi}\nu_{qi}'\,do\,do'/4\pi = \int(\eta_i/\cos\theta - 1)\,|\nu_{qi}|^2 do,$$
$$\iint G\boldsymbol{\mu}_{qi\lambda}\boldsymbol{\mu}_{qi\lambda}'\,do\,do'/4\pi = \int(\eta_i'/\cos\theta - 1)\,|\boldsymbol{\mu}_{qi\lambda}|^2\,do.$$

which follow from (4).

must hold, where $\mathbf{p}_n$, E_n and $\mathbf{p}'_n$, E'_n are the momentum and energy of the neutron before and after the scattering. It follows from (14) that the angle ϑ between $\mathbf{p}_n$ and $\mathbf{q}$, as well as the scattering angle χ, are uniquely determined by the energy transfer:

$$\cos\vartheta = \frac{s}{v_n} + \frac{\hbar q}{2p_n}, \qquad \cos\chi = \frac{2p_n^2 - 2ms\hbar q - (\hbar q)^2}{2p_n\sqrt{p_n^2 - 2ms\hbar q}}, \quad (15)$$

where $s = s_i$ and m is the mass of the neutron. The first relation shows that v_n must be larger than s.

For small scattering angles we have

$$\cos\vartheta = s/v_n, \qquad \chi^2 = (\hbar q/p_n)^2(1 - s^2/v_n^2), \qquad \hbar q \ll p_n. \quad (15')$$

In this case the zero-sound quanta are emitted under a rigorously defined angle with respect to the direction of motion of the neutron (as in the Cerenkov radiation). The largest possible momentum of the quantum is equal to $\hbar q_{max} = 2(p_n - ms)$ and the smallest momentum of the neutron after the scattering is equal to $p'_{n\,min} = |p_n - 2ms|$ (the neutrons with the minimal energy will move along the initial direction of the beam; all these relations hold for $\hbar q_{max} \ll p_0$).

It is easy to see that the scattering angle χ reaches a maximum for $\hbar q = (2/3ms)(p_n^2 - m^2s^2)$. This limiting angle is equal to (with $p_n \approx ms$)

$$\chi_0 = \arccos\left[\frac{(1+2\xi_0^2)\sqrt{4-\xi_0^2}}{3\sqrt{3}\xi_0}\right] \approx 2\left(\frac{\xi_0^2-1}{3}\right)^{3/2}, \quad \xi_0 = \frac{p_n}{ms} \quad (16)$$

Near the maximal scattering angle χ_0 the scattering cross section behaves like

$$d\sigma^{(\nu_i)} \sim q^2\,dq \sim \sin\chi\,d\chi/\sqrt{\cos\chi - \cos\chi_0}, \qquad \chi \approx \chi_0, \quad (17)$$

i.e., it contains an integrable singularity. However, this result, as well as the proof of the existence of χ_0 itself, was obtained under the assumption that the absorption of the zero sound can be neglected (see Sec. 4).

For small scattering angles the cross section $d\sigma^{(\nu_i)}$ has the form

$$d\sigma^{(\nu_i)} = \frac{3}{2}\left(\frac{m}{m'}\right)^2 |A_i|^2 |a|^2 \frac{s_i p_n}{v_0 p_0}\left(1 - \frac{s_i^2}{v_n^2}\right)^{-3/2} \chi^2\,d\chi. \quad (18)$$

If $\hbar q_{max} \ll p_0$, we can determine the total cross section for the emission of a quantum of zero sound:

$$\sigma^{(\nu_i)} = 4\left(\frac{m^*}{m'}\right)_i^2 |A_i^2|\,a\,|^2 \frac{s_i v_0}{v_n^2}\left(\frac{p_n - ms_i}{p_0}\right)^3. \quad (19)$$

The quantity A_i depends on the form of the function $F(\tilde{\chi})$. If we assume that $F(\tilde{\chi}) = \text{const}$, we have, according to (10) and (12),

$$A = \sqrt{2\pi(\eta_0^2-1)}\left(\eta_0 \ln\frac{\eta_0+1}{\eta_0-1} - 2\right) \times \left[2 - (\eta_0^2-1)\left(\eta_0 \ln\frac{\eta_0+1}{\eta_0-1} - 2\right)\right]^{-1/2}, \qquad \eta_0 = \frac{s}{v_0}. \quad (20)$$

We note that A vanishes for $F \to 0$.

3. We now turn to the calculation of the cross section for the direct scattering of the neutron by the nuclei of the Fermi liquid and to the consideration of the absorption of the zero sound in the scattering accompanied by the excitation of zero sound. For simplicity we shall assume that the energy of the quasiparticle does not depend on the orientation of its spin. The cross section for the coherent scattering of the neutron without change of orientation of its spin, $d\sigma_c$, is, according to (2), determined by the Fourier component of the deviation of the density of the Fermi liquid $\delta\rho(\mathbf{r}, t)$ from its equilibrium value ρ_0, i.e., by the quantity $\int \delta\rho(\mathbf{r}, t)\, e^{-i(\mathbf{q}\cdot\mathbf{r} - \omega t)}\,d\mathbf{r}\,dt$, where $\hbar\mathbf{q} = \mathbf{p}_n - \mathbf{p}'_n$ and $\hbar\omega = E_n - E'_n$ are the changes of the momentum and energy of the neutron:

$$d\sigma_c = 2\pi(|a|^2/v_n\rho_0)(2\pi\hbar/m')^2\,\Phi(q, \omega)\,d\mathbf{p}'_n/(2\pi\hbar)^3, \quad (21)$$

where Φ is the correlation function of the density of the nuclei,

$$\Phi(q, \omega) = \frac{1}{2\pi}\int d\mathbf{r}_1\, e^{-i\mathbf{q}(\mathbf{r}_1 - \mathbf{r}_2)} \int dt_1\, e^{i\omega(t_1 - t_2)} \times \langle \delta\rho(\mathbf{r}_1, t_1)\,\delta\rho(\mathbf{r}_2, t_2)\rangle, \quad (22)$$

and the brackets $\langle \ldots \rangle$ indicate the (quantum mechanical and thermodynamical) average.* According to the general theory of fluctuations, one can calculate the quantity Φ (for $T, \hbar\omega \ll \zeta$) first in the temperature region $T \gg \hbar\omega$, and then multiply the result by the factor $\hbar\omega(N_\omega + 1)/T$, where N_ω is Planck's distribution function:†

$$\Phi(q, \omega) = \frac{\hbar\omega}{T}(N_\omega + 1)\frac{1}{2\pi}\int d\mathbf{r}_1\, e^{-i\mathbf{q}(\mathbf{r}_1 - \mathbf{r}_2)} \int dt_1\, e^{i\omega(t_1 - t_2)} \times \overline{\delta\rho(\mathbf{r}_1, t_1)\,\delta\rho(\mathbf{r}_2, t_2)} \quad (23)$$

(the bar denotes the thermodynamical average).

In order to calculate the correlation function $\Phi(q, \omega)$ with account of the collisions we use the method of Abrikosov and Khalatnikov.[5] Introducing the stray force $y(\mathbf{p}, \mathbf{r}, t)$ in the kinetic equation (3) and choosing the collision integral $I\{\nu\}$ in the simplest form satisfying the requirements of conservation of the number of particles, momentum, and energy,

*The general relation between the neutron scattering cross section and the correlation function of the density of nuclei was established by Van Hove.[4]

†This method was used by Abrikosov and Khalatnikov[5] in their calculation of the correlation function with neglect of the collisions.

$$I\{\nu\} = -\tau^{-1}\Big\{\nu(\mathbf{n}) - \nu_0 - \sum_{m=-1}^{1} \nu_1^m P_1^m(\cos\theta)\, e^{im\varphi}\Big\}, \tag{24}$$

where ν_l^m are the coefficients in the expansion of the function $\nu(\mathbf{n})$ in terms of spherical harmonics, we obtain the following expression for the average value of the product of stray forces:

$$\overline{y(\mathbf{p}, \mathbf{r}, t)\, y(\mathbf{p}', \mathbf{r}', t')} = \frac{2T}{\tau}\Big(\frac{d\varepsilon_p}{d\tau_p}\Big)_\zeta \delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \times \delta(\varepsilon-\zeta)\,\delta(\varepsilon'-\zeta) \sum_{l=2}^{\infty} \frac{2l+1}{1+F_l/(2l+1)} P_l(\cos\widetilde{\chi}),$$

$$(d\tau_p/d\varepsilon_p)_\zeta = m^* p_0/\pi^2\hbar^3, \tag{25}$$

where F_l are the spherical harmonics of the function $F(\widetilde{\chi})$. Solving the kinetic equation (3) with the stray force y, we obtain a relation between the Fourier components of the functions $\delta\rho$ and y ($\sim e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$). Considering only the first two harmonics in the expansion of F in terms of spherical harmonics, we find

$$\delta\rho = -i\Big(\frac{d\tau_p}{d\varepsilon_p}\Big)_\zeta \frac{1}{qv_0 D}\int y(\mathbf{n})\, h(\mathbf{n})\, \frac{do}{4\pi}, \tag{26}$$

where

$$D = 1 + (A_1F_0 + i\eta\xi A_0) - g(A_2F_0 + i\eta\xi A_1),$$

$$h(\mathbf{n}) = \frac{1 - g\cos\theta}{\cos\theta - \eta(1+i\xi)},$$

$$g = \frac{3i\xi\eta + A_2F_1}{1 + 3i\xi\eta A_2 + F_1A_3}, \qquad A_n = \frac{1}{2}\int_{-1}^{1} \frac{x^n\, dx}{x - \eta(1+i\xi)}$$

($\eta = \omega/qv_0$, $\xi = 1/\omega\tau$). Using (24) and (25), we obtain finally the following expression for Φ:

$$\Phi(q, \omega) = \frac{2\hbar\omega}{\tau(qv_0)^2}\Big(\frac{d\tau_p}{d\varepsilon_p}\Big)_\zeta (N_\omega + 1)\frac{1}{|D|^2}\Big\{\int |h(\mathbf{n})|^2 \frac{do}{4\pi} - \Big|\int h(\mathbf{n})\frac{do}{4\pi}\Big|^2 - 3\Big|\int h(\mathbf{n})\cos\theta \frac{do}{4\pi}\Big|^2\Big\}. \tag{27}$$

If the condition $|\omega|\tau \gg 1$ ($\tau \sim T^{-2}$) is satisfied, the function $\Phi(q, \omega)$ has poles for values of ω which are close to the real axis, $\omega = \eta_0 v_0 q - i\gamma_0$ ($\eta_0 v_0$ is the velocity of the zero sound and γ_0 is its damping coefficient; an expression for γ_0 was found in reference 6). Separating out the poles of the function $\Phi(q, \omega)$, we write the latter in the form

$$\Phi(q, \omega) = \Phi_0(q, \omega) + \Phi_d(q, \omega), \tag{28}$$

where the function Φ_0 contains the poles of Φ, while Φ_d is free of singularities. Assuming for simplicity that F = const, we have

$$\Phi_0(q, \omega) = \frac{\hbar\omega}{\pi}\Big(\frac{d\tau_p}{d\varepsilon_p}\Big)_\zeta (N_\omega + 1)\frac{\eta_0^2 - 1}{F_0(1 - \eta_0^2 + F_0)} \times \Big\{\frac{\gamma_0}{(\omega - \eta_0 v_0 q)^2 + \gamma_0^2} + \frac{\gamma_0}{(\omega + \eta_0 v_0 q)^2 + \gamma_0^2}\Big\}. \tag{29}$$

The quantity Φ_d is a smooth function of ω. We can therefore neglect the collisions of the quasiparticles in the computation of Φ_d.

If F = const, we have*

$$\Phi_d(q, \omega) = \frac{\hbar\omega}{2qv_0}\Big(\frac{d\tau_p}{d\varepsilon_p}\Big)_\zeta (N_\omega + 1)\, R_0\Big(\frac{\omega}{qv_0}\Big)\theta\Big(1 - \frac{|\omega|}{qv_0}\Big),$$

$$R_0\Big(\frac{\omega}{qv_0}\Big) = \Big\{(1 + F_0 w)^2 + \Big(\frac{\pi}{2}\frac{\omega}{qv_0}F_0\Big)^2\Big\}^{-1},$$

$$w = 1 - \frac{\omega}{2qv_0}\ln\Big|\frac{qv_0 + \omega}{qv_0 - \omega}\Big|, \qquad \theta(x) = \begin{cases} 0, & x<0 \\ 1, & x>0 \end{cases}. \tag{30}$$

The separation of the poles out of the correlation function corresponds to the separation of the coherent neutron scattering cross section into two parts:

$$d\sigma_c = d\sigma^{(0)} + d\sigma_c^{(d)},$$

$$d\sigma^{(0)} = \frac{2\pi|a|^2}{v_n p_0}\Big(\frac{2\pi\hbar}{m'}\Big)^2 \Phi_0(q, \omega)\frac{d\mathbf{p}_n'}{(2\pi\hbar)^3},$$

$$d\sigma_c^{(d)} = \frac{2\pi|a|^2}{v_n p_0}\Big(\frac{2\pi\hbar}{m'}\Big)^2 \Phi_d(q\omega)\frac{d\mathbf{p}_n'}{(2\pi\hbar)^3}. \tag{31}$$

For F = const we find, according to (29) and (30),

$$d\sigma^{(0)} = \frac{3}{\pi}\Big(\frac{m^*}{m'}\Big)^2 |a|^2 \omega(N_\omega + 1)\frac{v_0}{v_n}\frac{\eta_0^2 - 1}{F_0(1 - \eta_0^2 + F_0)} \times \Big\{\frac{\gamma_0}{(\omega - \eta_0 v_0 q)^2 + \gamma_0^2} + \frac{\gamma_0}{(\omega + \eta_0 v_0 q)^2 + \gamma_0^2}\Big\}\frac{d\mathbf{p}_n'}{p_0^3}, \tag{32}$$

$$d\sigma_c^{(d)} = \frac{3}{2}\Big(\frac{m^*}{m'}\Big)^2 |a|^2 \frac{\omega}{qv_n}(N_\omega + 1) R_0\Big(\frac{\omega}{qv_0}\Big)\theta\Big(1 - \frac{|\omega|}{qv_0}\Big)\frac{d\mathbf{p}_n'}{p_0^3}. \tag{33}$$

The quantity $d\sigma^{(0)}$ is the cross section for the scattering of the neutron with excitation (or absorption) of zero sound, and $d\sigma_c^{(d)}$ is the cross section for the direct scattering of the neutron without change of orientation of its spin.

If $\gamma_0 \to 0$, we find†

$$\Phi_0(q, \omega) = \frac{\hbar\omega}{qv_0}\Big(\frac{d\tau_p}{d\varepsilon_p}\Big)_\zeta (N_\omega + 1)\frac{\eta_0^2 - 1}{F_0(1 - \eta_0^2 + F_0)}\Big\{\delta\Big(\frac{\omega}{qv_0} - \eta_0\Big) + \delta\Big(\frac{\omega}{qv_0} + \eta_0\Big)\Big\}. \tag{29'}$$

Substitution of this expression in $d\sigma^{(0)}$ leads immediately to formulas (13) and (20) for the cross section for the scattering of the neutron with excitation of zero sound for F = const.

We note that the validity of expression (29) is limited by the inequalities $\tau^{-1} \ll |\omega| \ll T/\hbar$. If $T \lesssim \hbar\omega$, we can use only expression (29′) for Φ_0, since expression (29) does not take account of the quantum mechanical effect in the absorption of the

*The expression for Φ_d with $\gamma = 0$ and F = const can be taken directly from formula (23) of the paper of Abrikosov and Khalatnikov.[5]

†Expression (29′) can be taken directly from formula (23) of[5]. Replacing in this formula

$$\delta(\Delta\omega \pm \eta_0 v_0 q) \text{ by } \pi^{-1}\gamma\,[(\Delta\omega \pm \eta_0 v_0 q)^2 + \gamma^2]^{-1},$$

we take account of the effect of the absorption of zero sound on the scattering of light in a Fermi liquid.

zero sound, as pointed out by Landau.[1] According to the results of[1], however, we may expect that formula (29) remains valid also for $T \lesssim \hbar\omega$, if we replace γ_0 by $\gamma = \gamma_0\{1 + (\hbar\omega/2\pi T)^2\}$.

4. We have seen above that for $\gamma \to 0$ and $\eta_0 v_0 < v_n < 2\eta_0 v_0$ the scattering angle of the neutron in the scattering with excitation of zero sound cannot exceed a certain limiting value χ_0, and the cross section for this process becomes infinite for $\chi \sim \chi_0$. If the absorption of the zero sound is taken into account, the scattering cross section for $\chi \sim \chi_0$ will, according to (32), have the form

$$d\sigma^{(0)} = G(J + O(\xi)) \sin\chi\, d\chi, \qquad J = \int \frac{\xi\, dt}{(\eta - \eta_0)^2 + \xi^2},$$

where

$$t = p_n/m\eta_0 v_0, \quad \xi = \xi(\chi) = \gamma/qv_0, \quad \eta = \eta(\chi, t) = \omega/qv_0$$

and G is some function of χ. The limiting angle χ_0 is evidently determined by the conditions

$$\eta(\chi_0, t_0) = \eta_0, \qquad \partial\eta(\chi_0, t)/\partial t\,|_{t_0} = 0.$$

For $\chi > \chi_0$ the difference $\eta(\chi, t) - \eta_0$, regarded as a function of t, does not become zero and the integral J is of the order of ξ: $J = O(\xi)$. For $\chi < \chi_0$ and $\xi \ll 1$ the integral J is equal to

$$J = \pi \int \delta(\eta - \eta_0)\, dt + O(\xi) = \pi\, |\partial\eta/\partial t\,|_{t_0}^{-1} + O(\xi) \sim 1,$$

and $d\sigma^{(0)}$ is given by (13) with an accuracy up to terms of order ξ.

Finally, if $\chi \approx \chi_0$, we use the expansion

$$\eta_0 - \eta = \alpha(\chi - \chi_0) + \beta(t - t_0)^2,$$

where

$$\alpha = -(\partial\eta/\partial\chi)\,|_{\chi_0, t_0}, \qquad \beta = -\tfrac{1}{2}(\partial^2\eta/\partial t^2)\,|_{\chi_0, t_0},$$

and write J in the form

$$J = \int_{-\infty}^{\infty} \frac{\xi\, dz}{[\alpha(\chi - \chi_0) + \beta z^2]^2 + \xi^2}, \qquad z = t - t_0.$$

For $\chi = \chi_0$ we then obtain $J \sim (\beta\xi)^{-1/2} \gg 1$.

We see that the absorption of the zero sound leads to a "smearing out" of the limiting scattering angle.

5. The cross section for the direct coherent scattering of the neutron by the nuclei of the Fermi liquid (without change of orientation of the neutron spin), given by formula (33), differs from the cross section for the coherent scattering of the neutron without change of orientation of its spin in a Fermi gas of the same density by the factor R_0. For $|\omega|/qv_0 \ll 1$ this factor is independent of the transferred energy and momentum and is equal to $R_0 \cong (1 + F_0)^{-2}$. $R_0 \to 0$ for $|\omega|/qv_0 \to 1$. Therefore the cross section for direct coherent scattering into angles close to

$$\chi_d = \arc\cos\left\{\frac{1}{2p_n p_n'}[p_n^2 + p_n'^2 - (2mv_0)^{-2}(p_n^2 - p_n'^2)^2]\right\}$$

is appreciably smaller in a Fermi liquid than in an ideal gas.

If we keep only the first two harmonics in the expansion of F, we find for the direct scattering cross section

$$d\sigma_c^{(d)} = \frac{3}{2}\left(\frac{m^*}{m'}\right)^2 |a|^2 \frac{\omega}{qv_n}(N_\omega + 1)\, R_1\left(\frac{\omega}{qv_0}\right)\theta\left(1 - \frac{|\omega|}{qv_0}\right)\frac{dp_n'}{p_n^3}, \tag{33'}$$

where the factor R_1, which distinguishes this cross section from the corresponding cross section in the ideal Fermi gas, is equal to

$$R_1\left(\frac{\omega}{qv_0}\right) = \left(1 + \frac{F_1}{3}\right)^2 L^{-2}\left\{(1 + F_0 Q_1)^2 + \left(\frac{\pi}{2}\eta F_0 Q_2\right)^2\right\}^{-1},$$

$$Q_1 = w - \frac{\eta^2 F_1}{Q}\left\{L\left(w^2 - \frac{\pi^2}{4}\eta^2\right) + \frac{\pi^2}{2} w \eta^4 F_1\right\},$$

$$Q_2 = 1 + \frac{\eta^2 F_1}{Q}\left\{\eta F_1\left(w^2 - \frac{\pi^2}{4}\eta^2\right) - 2wL\right\},$$

$$L = 1 + F_1\left(\tfrac{1}{3} + \eta^2 w\right), \qquad Q = L^2 + \frac{\pi^2}{4}\eta^6 F_1^2.$$

6. Let us now consider the incoherent scattering of the neutron in a Fermi liquid. This scattering, which is due to the quantity b in the pseudopotential (1), is of the same type as the scattering of a neutron in an ideal Fermi gas of nuclei whose mass is equal to the effective mass m^* of the quasiparticles of the Fermi liquid. The total cross section for the incoherent scattering (with and without change of orientation of the neutron spin) per nucleus is equal to

$$d\sigma' = \frac{9}{32}\left(\frac{m^*}{m'}\right)^2 |b|^2 \frac{\omega}{qv_n}(N_\omega + 1)\,\theta\left(1 - \frac{|\omega|}{qv_0}\right) p_0^{-3} dp_n'. \tag{34}$$

We note that this expression does not contain (as $d\sigma_c^{(d)}$ does) the factor R which depends on the correlation between the nuclei.

It follows from (33) and (34) that there is no unique relation between the transferred energy and momentum for the direct scattering, whereas such a relation does exist (for $\gamma \to 0$) in the scattering with excitation of the zero sound [owing to the presence of the δ functions in formula (29')]. This circumstance allows us, in principle, to distinguish between the two types of scattering.

Let us finally discuss the angular distribution of the neutrons for the direct scattering in the region of small angles, $\chi \ll 1$. If $v_n \sim v_0$, the cross section will be proportional to $\chi d\chi$. For large neutron velocities the cross section will behave like $\chi^2 d\chi$. For an estimate of the order of magnitude of the cross section, we set $R \sim 1$, and obtain for $v_n \gg v_0$ and $\chi \ll 1$

$$d\sigma^{(d)} \sim \frac{3}{8}(m/m')^2\sigma_0\,(p_n/p_0)\,\chi^2\,d\chi,$$
$$\sigma_0 = 4\pi\left(|a|^2 + \frac{3}{16}|b|^2\right). \quad (34')$$

This expression depends on χ in the same way as $d\sigma^{(\nu i)}$ does [formula (18)]. The ratio of the cross sections is, in order of magnitude, equal to

$$d\sigma^{(\nu)}/d\sigma^{(d)} \sim (s/v_0)\,(1 - s^2/v_n^2)^{-3/2} \quad (35)$$

and can be larger than unity if $v_n \sim s$; in general, however, $d\sigma^{(0)} \sim d\sigma^{(d)}$.

7. We saw earlier that the cross section for the scattering of neutrons with excitation of zero sound oscillations is determined by the poles of the function $\Phi(q, \omega)$ for $|\omega|\tau \gg 1$. If $|\omega|\tau \sim 1$, then Φ does not have poles for values of ω close to the real axis. For $|\omega|\tau \sim 1$ we cannot, therefore, separate out of the scattering cross section a term corresponding to the excitation of sharply defined collective oscillations of the Fermi liquid. On the other hand, if $|\omega|\tau \ll 1$, the function Φ again has poles for values of ω close to the real axis. These poles ($\omega = \eta_s v_0 q - i\gamma_s$) determine the dispersion law and the damping coefficient of the zero sound oscillations (expressions for the velocity $\eta_s v_0$ and the damping coefficient γ_s of the sound have been found in the work of Landau [1] and Khalatnikov and Abrikosov [6]).

Denoting the quantity Φ for $|\omega|\tau \ll 1$ by Φ_s, we have

$$\Phi_s(q, \omega) = \frac{\hbar\omega}{6\pi\eta_s^2}\left(\frac{d\tau_p}{d\varepsilon_p}\right)_\zeta (N_\omega + 1)\left\{\frac{\gamma_s}{(\omega - \eta_s v_0 q)^2 + \gamma_s^2} + \frac{\gamma_s}{(\omega + \eta_s v_0 q)^2 + \gamma_s^2}\right\}. \quad (36)$$

Substituting (36) in (21), we obtain the cross section for the scattering of the neutron with excitation ($\omega > 0$) and absorption ($\omega < 0$) of zero-sound oscillations:

$$d\sigma^{(s)} = \frac{1}{2\pi}\left(\frac{m^*}{m'}\right)^2 |a|^2 \frac{v_0}{v_n}\,\omega\,(N_\omega + 1)\frac{1}{\eta_s^2} \times \left\{\frac{\gamma_s}{(\omega - \eta_s v_0 q)^2 + \gamma_s^2} + \frac{\gamma_s}{(\omega + \eta_s v_0 q)^2 + \gamma_s^2}\right\}\frac{d\mathbf{p}_n'}{p_0^3}. \quad (37)$$

We note that the cross section for the scattering of neutrons with small energy transfer ($|\omega| \ll \tau^{-1}$) is of the order $|\omega|\tau$; for $|\omega|\tau \ll 1$ we can therefore neglect the probability for direct scattering as compared to the probability for the scattering with excitation of ordinary sound.

If $\gamma_s = 0$, the cross section for excitation of ordinary sound* contains the function $\delta(\omega - \eta_s v_0 q)$ [see formula (37)]. Replacing $(2\pi\hbar)^{-3}d\mathbf{p}_n'$ by $(2\pi)^{-3}d\mathbf{q}$ and integrating over the angular part of the vector $\mathbf{q}$, we find for the cross section for the excitation of ordinary sound with a wave vector in the interval $(q, q + dq)$

$$d\sigma^{(s)} = \pi\,(m^*/m')^2\,|a|^2\,(N_\omega + 1)\,(v_0/v_n)^2\,\hbar^3 q^2\,dq/\eta_s p_0^3. \quad (38)$$

Excitation of ordinary sound is possible only if $v_n > \eta_s v_0$.

Comparing (37) and (32), we see that we can obtain the part of $d\sigma^{(s)}$ which depends on ω and $\mathbf{q}$ by making the replacements $\eta_0 \rightarrow \eta_s$, $\gamma_0 \rightarrow \gamma_s$ in the corresponding part of $d\sigma^{(0)}$. The relations for the scattering of neutrons with excitation of zero sound are therefore also valid for the scattering with excitation of ordinary sound. In particular, if $\gamma_s = 0$ and $\eta_s v_0 < v_n < 2\eta_s v_0$, the neutrons can not be scattered into an angle which is larger than some limiting angle χ_s. This angle is given by formula (16), if we replace ξ_0 by $\xi_s = v_n/\eta_s v_0$. As in the case of the zero sound, the damping of the ordinary sound leads to a "smearing out" of the limiting scattering angle. Here the scattering cross section for the neutron within the angular interval $d\chi$ (integrated over the absolute value of the vector $\mathbf{p}_n'$) near the angle χ_s is proportional to $\gamma_s^{-1/2}$.

8. Unfortunately, the only known Fermi liquid — He^3—has the property that the absorption of slow neutrons is very strong. For energies of the order of 1° K the capture cross section for neutrons in He^3 is $\sigma_c \sim 10^5 \times 10^{-24}$ cm^2, whereas the scattering cross section is only $\sigma_0 \sim 10^{-24}$ cm^2. For every scattering event there will thus be a very large number of neutron capture events leading to a strong absorption of the neutron wave. Since the mean free path of the neutron with respect to capture is equal to $l_c \sim (\rho_0\sigma_c)^{-1} \sim 10^{-3}$ cm, the neutrons will be scattered primarily in the surface layer of the He^3, the effective thickness of which is $\sim 10^{-3}$ cm. Moreover, one must consider the fact that the liquid He^3 will be heated up as a consequence of the nuclear reactions caused by the capture of the neutrons (this energy amounts to 5×10^5 ev per captured neutron, which means that 10^{15} He^3 nuclei will be heated up by 1°K for every scattering event). All these complications make the investigation of the scattering of slow neutrons in liquid He^3 very difficult. One may hope, however, that it will be possible to carry out such experiments as time goes on.

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*An expression for $d\sigma^{(s)}$ without account of the attenuation of sound has been found independently and by a different method by M. Kaganov.

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