WAVE RESONANCE OF LIGHT AND GRAVITIONAL WAVES

M. E. GERTSENSHTEĬN

Submitted to JETP editor July 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 113-114 (July, 1961)

The energy of gravitational waves excited during the propagation of light in a constant magnetic or electric field is estimated.

ACCORDING to general relativity, light and gravitational waves propagate at the same speed, and their rays coincides with no geodetics. Therefore, if the waves of gravitational and light waves are linearly related, wave resonance, a well known phenomenon in radio physics, sets in and makes possible an appreciable transfer of energy even at low coupling. In the present note we estimate the extent of excitation of gravitational waves by light.

The equations of a weak gravitational field in the presence of an electromagnetic field (see [1]) are

$$\Box\psi^{ik} = -16\pi\gamma c^{-4}\tau^{ik}, \qquad \tau_k^k = 0, \qquad \tau^{ik}_{,k} = 0,$$

$$\tau_k^i = \frac{1}{4\pi}\left[F^{il}F_{kl} - \frac{1}{4}\delta_k^i\,(F^{lm}F_{ml})\right], \qquad \psi_i^k = h_i^k - \frac{1}{2}h\delta_i^k, \tag{1}$$

where τ^{ik} is the energy-momentum tensor of the electromagnetic field, F^{ik} is the electromagnetic field tensor, γ is the gravitational constant, and h_{ik} is the perturbation of the metric tensor.

Let us apply Eq. (1) to the propagation of light (field F^{ik}) in the presence of a strong "magnetizing" field $F^{(0)ik}$, constant in space and in time. The energy-momentum tensor will be the sum of three terms: the square of the constant term, the square of the field of the light wave, and an interference term describing the wave resonance. Leaving out the nonresonant terms we obtain

$$\Box\psi_k^i = -\frac{8\gamma}{c^4}\left[F^{(0)il}\,F_{kl} - \frac{1}{4}\delta_k^i\,(F^{(0)lm}F_{lm})\right]. \tag{2}$$

We align the x axis with the wave vector and normalize the wave amplitudes to unit energy density:

$$F_{kl} = b(x)\,f_{kl}\,e^{ikx}, \qquad f^{0l}f_{0l} = 1, \qquad k = \omega/c,$$

$$\psi^{ik} = a(x)\sqrt{16\pi\gamma/c^4k^2}\,\zeta^{ik}e^{ikx}, \qquad \zeta_{ik}\zeta^{ik} = 1, \qquad \zeta_i^i = 0; \tag{3}$$

The amplitudes f_{kl} and ζ^{ik} are dimensionless. We then have in the approximation of slowly varying amplitudes [2]:

$$i\,da(x)/dx = \sqrt{\gamma/\pi c^4}\,F^{(0)il}f_{kl}\,\zeta_i^k\,b(x). \tag{4}$$

The solution of (4) has the form

$$a(x) = i\sqrt{\gamma/\pi c^4}\,f_{kl}\,\zeta_i^k\int_0^x F^{(0)il}(s)\,b(s)\,ds + a(0), \tag{5}$$

where the integration is along the ray. If a(0) = 0, the external field is constant and the absorption or scattering of light along the ray is small in the region under consideration, i.e., b(s) = const, and then

$$|a(x)/b(0)|^2 = (\gamma/\pi c^2)\,F^{(0)2}T^2, \tag{6}$$

where T is the time of travel of the ray in a constant field. In the derivation of (6) we assumed the convolutions of the dimensionless amplitudes to be equal to unity. If the field $F^{(0)}$ is turbulent and random, we can estimate the energy of the gravitational wave by assuming that $F^{(0)}$ is constant over a section of length R_0 (where R_0 is the correlation radius of the field $F^{(0)}$), after which it changes abruptly and at random. The amplitude of the light b(x) is practically constant along the ray. We then have for the amplitude of the gravitational wave

$$a(x) = \sum a_n; \qquad a_n = i\sqrt{\gamma/\pi c^4}\,f_{kl}\,\zeta_i^k\int_{x_{n-1}}^{x_n}F^{(0)il}(s)\,b(s)\,ds.$$

The gravitational waves excited on each section will be incoherent, and we must add their energies and not their amplitudes. As a result we obtain

$$|a(x)/b|^2 = (\gamma/\pi c^3)\,F^{(0)2}R_0T. \tag{7}$$

For interstellar fields, putting $F^{(0)} = 10^{-5}$ gauss, $R_0 = 10$ light years, and $T = 10^7$ years, we obtain $|a/b|^2 \sim 10^{-17}$. The frequency of the excited gravitational wave is determined by the frequency of the light.

There are strong magnetic fields also inside the stars, and consequently generation of gravitational waves is also possible. Formula (5) is applicable in this case, too, and the "constant" field $F^{(0)}$ changes relatively slowly, whereas the amplitude b(s) varies rapidly as a result of light scattering. Thus, the correlation radius a(x) is determined for this case essentially by the free path of the radiation, which is the reciprocal of the opacity.[3] Since the star is transparent to the exciting gravitational radiation, we can show that formula (7) yields the ratio of the gravitational radiation of the star to the optical one. The intensity of the gravitational radiation is small and of no importance to the energy balance. The frequency of the gravitational radiation is determined essentially by the electromagnetic radiation inside the star, where γ quanta predominate.

Thus, the spectrum of the gravitational radiation of stars has maxima both at very low frequencies (the frequencies of planetary orbits), as well as in the range of γ-quantum frequencies; both portions of the spectrum have comparable energy.

From general relativity follows also the possibility of the inverse conversion of gravitational waves into light waves, but this problem is hardly of interest.

[1] L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), 3d Ed., Fizmatgiz, 1960.

[2] M. E. Gertsenshteĭn and N. F. Funtova, Radiotekhnika i elektronika (Radio Engineering and Electron Physics) **4**, 805 (1959).

[3] D. A. Frank-Kamenetskii, Fizicheskie yavleniya vnutry zvezd (Physical Phenomena Inside the Stars), Fizmatgiz, 1959.

Translated by J. G. Adashko
26