Angular correlations in inelastic scattering of high-energy nucleons

G. L. Vysotskii

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

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Angular correlations for the inelastic scattering of high-energy nucleons by nuclei with zero spin and zero isotopic spin are examined. The calculation is carried out in the impulse approximation at small angles. It is shown that the correlation function and its dependence on the nucleon scattering angle are mainly determined by the parity and isotopic spin of the excited level.

A great amount of data has accumulated in recent years on the polarization of high-energy protons in the inelastic scattering by nuclei with excitation of the low-lying levels. Some of the features of these experimental results have been successfully explained by Kerman, McManus, and Thaler with the help of the impulse approximation. The experiments show that in a number of cases the angular distribution of the polarization of the inelastic group of protons is the same or almost the same as for the elastic group. This fact has been regarded as a confirmation of the assumption of the rotational nature of the excited levels, since the Born approximation calculations with a nonspherical optical model potential yield the same polarization for the elastic and inelastic scattering.

In the impulse approximation, which is valid for high energies, the amplitude for the inelastic scattering of the nucleons by nuclei is expressed in terms of the amplitude for nucleon-nucleon scattering and the reduced nuclear matrix elements, which play the role of phenomenological parameters.

In the impulse approximation, the observed similarity between the polarization of the groups of elastically and inelastically scattered nucleons for even-even nuclei connected with the excitation of a level with \( \pi = (-)^J \) (\( J \) is the spin of the level) is explained in a natural way by the smallness of the ratio of the reduced matrix element with spin flip over the matrix element without spin flip, independently of the nature of the excited level. One can also explain the small polarization connected with the excitation of levels of even-even nuclei with the parity \( \pi = (-)^{J+1} \) and a number of other features. It is, therefore, of interest to consider the predictions of the theory of inelastic scattering in the impulse approximation with regard to the p-\( \gamma \) correlations. The investigation of the p-\( \gamma \) correlations in the inelastic scattering of high-energy nucleons is of interest not only as a test of the theory of inelastic scattering, but also from the point of view of using this process for the determination of the spectroscopic properties of the levels. This refers, in particular, to the isotopic spin, which does not play an important role in low energy scattering experiments.

In the first nonvanishing approximation, the inelastic scattering amplitude \( T \) is related to the nucleon-nucleon scattering amplitude \( M \) in the following way (we consider small angle scattering):

\[
T = -\frac{\hbar^2}{2\pi m N} \bar{M}, \quad M = \langle f | M e^{-i\bar{Q}R} | i \rangle,
\]

(1)

\[
M = A + B (\sigma, n) (\sigma, n) + C (\sigma, n + \sigma, n) + E (\sigma, q) (\sigma, q) + F (\sigma, p) (\sigma, p).
\]

(2)*

Here \( N \) is the number of particles in the nucleus, \( f \) and \( i \) are the labels of the excited and ground states of the nucleus, \( \sigma \) is the wave vector of the incident and scattered nucleons. We have omitted the index for the total isotopic spin \( t = 0 \) in the system of the two nucleons in the quantities \( A \), \( B \), \( C \), etc. in (2). In place of the quantities \( A_0 \) and \( A_1\), \( B_0 \) and \( B_1 \), etc. it is convenient to introduce the following linear combinations: \( A (0) = \frac{1}{4} (3A_1 + A_0) \), \( A (1) = \frac{1}{4} (A_1 - A_0) \) and analogously for \( B \), \( C \), etc. The quantities \( A \), \( B \), etc., also depend on \( Q^2 \). The lower indices 0 and 1 of the spin operators \( \sigma \) denote the nucleons.

\*\( \langle \sigma, p \rangle = \sigma \cdot p; \langle qn \rangle = q \times n \).
We shall use a coordinate system with the z axis along q and the x and y axes along the vectors \( \mathbf{a} \) and \( \mathbf{p} \), respectively. It is convenient to separate out the components of \( \mathbf{M} \) which transform according to an irreducible representation of the rotation group:

\[
M = M^{(0)} + \sum_p (-)^p M^{(1)} \sigma^{(1)}_{\mathbf{p}m,\mathbf{r}m},
\]

\[
\sigma^{(1)}_{\mathbf{p}m,\mathbf{r}m} = \mp (\alpha_{1p} \pm i \alpha_{2p}) / \sqrt{2}.
\]

In our system of coordinates, we have

\[
M^{(0)} = A + C \sigma_0 \mathbf{n}, \quad M^{(1)} = -(C + B \sigma_0 \mathbf{n} + i F \sigma_2 \mathbf{p}) / \sqrt{2},
\]

\[
M^{(2)} = E \sigma_0 \mathbf{q}, \quad M^{(3)} = (C + B \sigma_0 \mathbf{n} - i F \sigma_2 \mathbf{p}) / \sqrt{2}.
\]

We then obtain the following expression for \( \mathbf{M} \) in terms of the reduced matrix elements:

\[
M = \sum_{\alpha=0,1} \sum_{m} \sum_{l} \left( \frac{(\mu_0/\mu_0 M_0) || M ||}{(2L+1)\gamma} \right) N_{\alpha l} \left( \frac{1}{2L+1} \right) Q_{\alpha l}.
\]

Here the quantities \( \mathbf{M} (\alpha) \) are expressed in terms of \( A (\alpha) \), \( B (\alpha) \), etc., and

\[
N_{\alpha l} = \langle JT \mid \rho \mathbf{Y} \mid J'T' \rangle, \quad N_{l1} = \langle TT' \rangle \langle JT \mid \rho \mathbf{Y}_r \tau \mid J'T' \rangle,
\]

\[
Q_{\alpha l} = \langle JT \mid \rho \mathbf{T}_k (\theta) \mid J'T' \rangle, \quad Q_{l1} = \langle TT' \rangle \langle JT \mid \rho \mathbf{T}_k (\theta) \tau \mid J'T' \rangle;
\]

\[
e^{-iQ\mathbf{R}} = \sum_{\mathbf{a}} \rho \mathbf{Y}_0 \cdot e^{-iQ\mathbf{R}} \sigma^{(1)}_{\mathbf{a}} = \sum_{\mathbf{a}} (1/2) \rho \mathbf{T}_k (\theta) \rho \mathbf{Y}_{\mathbf{k}0} (\theta),
\]

\[
f (TT') = \pm (T'T' 0) || TT' || (2T + 1)^{-1/2}.
\]

The problem therefore reduces to the calculation of the spin tensors \( \rho \mathbf{F}_k \), for which we have the expression

\[
\rho \mathbf{F}_k = \sum_{M_{JM}} (-)^{J_0 - M_0} \langle JM | M' \rangle | F_0 \rangle \text{ Sp} \mathbf{M}_{JM} \mathbf{M}_{JM}'\mathbf{M}_{JM}^\dagger,
\]

where

\[
I_0 = \sum_{M_{JM}} \text{ Sp} \mathbf{M}_{JM} \mathbf{M}_{JM}^\dagger
\]

agrees with the cross section for the process except for a factor.

Let us consider the angular correlations for a nucleus with vanishing spin and isotopic spin, \( J_0 = T_0 = 0 \). In this case the angular correlations are essentially determined by the parity and isotopic spin of the excited level. The selection rules permit the values \( T = 0, 1 \) for the isotopic spin of the level. For \( T = 0 \) only the terms with \( k = 0 \) are different from zero, and for \( T = 1 \) the only nonvanishing terms are those with \( \alpha = 1 \). Furthermore, if the parity of the level is "normal," \( \pi = (-)^J \), we have the selection rule \( l = J \) for the reduced matrix elements without spin flip and \( l = k = J \) for the matrix elements with spin flip. On the other hand, if the parity of the level is "anomalous," \( \pi = (-)^{J+1} \), the matrix elements without spin flip vanish, and the matrix elements with spin flip are subject to the selection rule \( k = J, l = J - 1 \). As a result we obtain the following expressions for the spin tensors defining the polarization of nuclei with an excited level with parity \( \pi = (-)^J \):

\[
\rho_{F0} = (-)^J \langle J 0 J 0 \mid F 0 \rangle | A |^2 + | C |^2 - \frac{1}{2} (| B |^2 + | C |^2)
\]

\[
+ | F | \lambda (J|J - 1\rangle \langle J 0 | J 0 \rangle \langle 0 J 0 \mid F 0 \rangle | K^{-1},
\]

\[
\lambda = | Q_{JJ} |^2 / | N_{JJ} |^2,
\]

\[
K = | A |^2 + | C |^2 + \frac{1}{2} (| B |^2 + | C |^2 + | F |^2) \lambda; \quad \Phi = \arctan (AC + C' B, \quad \Phi = \arctan Q_{JJ} / N_{JJ}; \quad \rho_{F2} = (-)^{J+1} \langle J 0 J 1 \mid F 2 \rangle (11 J 0 \mid J 1 \rangle \langle J 1 \mid | A |^2
\]

\[
+ | C |^2 - | F | | K^{-1} \lambda K^{-1},
\]

\( F \) takes the value 2 for dipole transitions (electric and magnetic) and the values 2, 4 for electric quadrupole transitions. The value \( \rho_{q0} = (2J + 1)^{1/2} \) follows from the normalization of the density matrix. The quantities \( \rho_{F2} \) and \( \rho_{F3} \), which are measured directly in experiment, are real and depend, just like the polarization of the elastically scattered nucleons, on the single parameter \( \lambda \), the ratio of the reduced matrix element with spin flip over the
\[ P_{F2} = (\frac{-1}{2}) (J | I | F 2) (| B |^2 + | C |^2) (1 - \mu) k^{-1}; \]
\[ K' = (| B |^2 + | C |^2 + | F |^2) (1 - \mu) + 2 | E |^2 \mu, \]
\[ \mu = \left( \sum_{i=1}^{10} Q_{ji} Q_{ji} \right)^{-1} \sum_{i=1}^{10} (10 | 10 | 0 | 0 | 10 | 0) \left( \frac{2i+1}{2i+1} \right) Q_{ji} Q_{ji}. \]
In the case of transitions with anomalous change of parity, the spin tensors also depend only on the single parameter \( \mu \). \( P_{F1} \) is in this case identically equal to zero, and the term with \( \sin \varphi \) in the correlation function is absent.

The formulas above, of course, also applicable to those cases where the levels are de-excited by the emission of an \( \alpha \) particle or a \( \beta \) particle instead of a \( \gamma \) ray. Only formula (6) will have to be modified.

Our discussion has shown that the measurement of the correlation function in the inelastic scattering of high-energy nucleons yields information on the parity of the excited states. That is, if we observe a term of the form \( P_{21} (\cos \varphi) \sin \varphi \) in the correlation function (this implies that \( P_{F1} \neq 0 \)), we know that the parity changes in the normal way \( \{ \pi = (-) \} \). If instead we observe a term of the form \( P_{22} (\cos \varphi) \cos 2\varphi \), the parity changes in an anomalous manner \( \{ \pi = (-) \} \). Moreover, the investigation of the dependence of the coefficients of the correlation function on the scattering angle of the proton permits us in certain appropriate cases \( \lambda \) sufficiently large) to determine the isotopic spin of the level.

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