ON THE POLARIZATION OF RECOMBINATION RADIATION

B. A. LYSOV, L. P. BELOVA, and L. I. KOROVINA

Moscow State University

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The polarization of radiation following the capture of a relativistic electron into the K shell is considered. Partial elliptical polarization is shown to occur in this case. The expression for the intensity of the unpolarized part of the radiation is given. The electron-spin contribution is discussed. The calculations are performed to the lowest order in αZ.

1. The photoelectric effect and its inverse processes have recently, after a long lapse, again begun to attract attention. Special attention has been given to the study of the polarization phenomena which occur in these processes. Polarization phenomena in the photoelectric effect and also in the inverse process of single-photon positron annihilation have first been studied in detail by McVoy.[1] However, the results obtained were found to contradict the formula of Sauter-Sommerfeld, since McVoy, who used Born's method, limited himself to the zero Born approximation.

Recently, in a series of articles by Fano, McVoy, and Albers,[2] the earlier results of McVoy have been corrected.* The authors have abandoned Born's method, although there is no fundamental reason for doing so, and have used the Sauter approximation.

In the present paper, we shall discuss in detail the so-far untreated case of polarization effects in the inverse process of electron capture by an ionized atom. Such a process is determined by the first-order matrix element

$$S^{(1)}_{i\to f}=-\frac{e}{\hbar c}L^{-3/2}\sum_{\varkappa}\sqrt{\frac{2\pi c\hbar}{\varkappa}}\int\psi_f^+(\boldsymbol{\alpha}\mathbf{a}^+)e^{-i(\varkappa x)}\psi_i d^4x. \quad (1)$$

Henceforth, we shall limit ourselves to the electron capture into the K shell by a nucleus with charge Ze and, assuming that $\alpha Z \ll 1$ (where $\alpha = e^2/\hbar c$), we shall carry out all calculations to the lowest order in αZ.

It is interesting to note that the matrix element (1), as has been shown by Fano, McVoy, and Albers,[2] can, in calculations to the lowest order in αZ, also be used to describe the short-wave bremsstrahlung, where almost all the initial kinetic electron energy is carried away by the radiated photon. In connection with the above, the results referring to the polarization properties of recombination radiation are equally applicable to bremsstrahlung near the upper limit of its spectrum.

2. We shall find the wavefunction of the initial state using Born's method. It has been shown by Gavrila,[3] who considered the relativistic photoeffect from the K shell, that, in order to obtain the correct result, it is necessary to take into account the first-order Born approximation even for the lowest order in αZ. In this approximation, we have

$$\psi_i=\left\{1-\frac{k_0'}{2\pi^2k_0}\int\frac{[K+\boldsymbol{\alpha}(\mathbf{k}-\boldsymbol{\varkappa})+\rho_3k_0]\,e^{-i\varkappa' r}}{\varkappa'^2[k^2-(\mathbf{k}-\boldsymbol{\varkappa}')^2+i\eta]}d^3x\right\}\psi, \quad (2)$$

where the wavefunction

$$\psi=L^{-3/2}\sum_{s=\pm1}C_sb(s)\,e^{-icKt+i\mathbf{kr}} \quad (3)$$

describes the free electron with energy $\hbar cK$ with momentum $\hbar\mathbf{k}$. $b(s)$ is the spinor amplitude, and the coefficient C_s characterizes the initial electron polarization.[4,5] The real infinitesimal quantity η is chosen as positive so that, for larger r, the function ψ_i represents asymptotically the sum of a plane and of a diverging spherical wave.[6]

In the approximation under consideration, the finite wavefunction of the K state can be written as follows, neglecting the electron binding energy, which is small as compared with its rest energy:

$$\psi_f=\pi^{-1/2}k_0'^{3/2}\left\{1+\tfrac{1}{2}\,i\alpha Z\,(\boldsymbol{\alpha}\mathbf{r}/r)\ \exp(-ick_0t-k_0'r)\right\}, \quad (4)$$

where $\hbar ck_0$ is the electron rest energy, $k_0' = \alpha Zk_0$, and the row matrix b_0^+ is (1,0,0,0) and (0,-1,0,0) for the K state with magnetic quantum number $j_z = 1/2$ and $j_z = -1/2$ respectively.

From Eqs. (1) to (4) we find the following expression for the differential cross section for electron capture into the K shell of the nucleus with the charge Ze:

$$d\sigma=\frac{32\,e^2k_0'^5\varkappa K\,d\Omega}{\hbar ck\,(\mathbf{k}-\boldsymbol{\varkappa})^8}\sum_{s,s'}R_{s'}^+R_s, \quad (5)$$

*Correct results have also been obtained by Banerjee.[14]

$$R_s = C_s b(s)\{A(\varkappa a^+) - iB(\sigma[a^+(k-\varkappa)]) + C(a^+(k-\varkappa))\};$$

$$A = 1 - \frac{K(k-\varkappa)^2}{2k_0^2\varkappa}, \quad B = \frac{1}{2k_0}\left(1 - \frac{(k-\varkappa)^2}{2k_0\varkappa}\right), \quad C = \frac{1}{2k_0}\left(1 + \frac{(k-\varkappa)^2}{2k_0\varkappa}\right), \tag{6}$$

where $\hbar c\kappa$ and $\hbar\kappa$ are the energy and momentum of the radiated photon. (See also reference 7, where a detailed derivation of an analogous formula for the photoeffect is given.)

3. As is well known,[8,9] the polarization properties of radiation may be described by calculating the intensity of the radiation in two linear W_λ ($\lambda = 2, 3$) and two circular W_l ($l = \pm 1$) polarization states. In calculating W_λ, the quantized photon amplitude should be resolved into two mutually perpendicular components

$$a = \sum_\lambda g_\lambda \beta_\lambda, \quad [g_\lambda \overset{+}{g}_{\lambda'}]_- = \delta_{\lambda\lambda'}, \tag{7}$$

where β_2 and β_3 are arbitrary unit vectors satisfying the orthogonality condition

$$\beta_\lambda \varkappa = 0. \tag{8}$$

A different resolution of the photon amplitudes is necessary for the calculation of W_l:

$$a = \sum_l g_l \beta_l, \quad [g_l g_{l'}]_- = \delta_{ll'}, \tag{9}$$

where β_l are related to β_λ by the equation

$$\beta_l = 2^{-1/2}(\beta_2 + il\beta_3).$$

It should be noted that, in classical optics, the polarization of a radiation is characterized by the amplitudes of the oscillations of electrical vectors in two mutually perpendicular directions and by the phase difference δ between these vibrations. Both methods of describing the polarization properties of radiation are equivalent, and this equivalence is established by the formula relating the quantities W_λ and W_l with the quantity δ. Such a formula for the case of a totally polarized radiation has been derived by Sokolov and Ternov[8] [see also reference 9, Eq. (28.41)].

In the case where the radiation is only partially polarized, we should use a somewhat more general formula

$$\sin\delta = \tfrac{1}{2}(W_{-1} - W_1)[(W_2 - W_0/2)(W_3 - W_0/2)]^{-1/2} \tag{10}$$

where W_0 is the intensity of the unpolarized component of the radiation. The quantity W_0 may be found from the following considerations. In Eq. (10), only the quantities W_2 and W_3 depend on the actual resolution of the photon amplitude, i.e., on the choice of the vectors β_2 and β_3. We shall now choose these vectors in such a way that the quantities W_2 and W_3 reach their extremum values. In such a case, the polarization ellipse of the polarized component of the radiation will be brought to the main axes and, consequently, for such a choice of β_2 and β_3 we have $|\sin\delta| = 1$.

Taking this into account, we find from Eq. (10)

$$W_0/W = 1 - (P_\lambda^2 + P_l^2)^{1/2}, \tag{11}$$

where P_λ and P_l are the degrees of linear and circular polarization of the radiation respectively:

$$P_\lambda = (W_3 - W_2)/W, \quad P_l = (W_1 - W_{-1})/W, \tag{12}$$

and W is the total radiation intensity. Moreover, in view of the above, in order to calculate P_λ the photon amplitude is divided so that P_λ will reach a maximum. It should be noted that an equation analogous to (11) is well known* in classical electrodynamics [see reference 10, Eq. (50.13)]. We shall now directly apply the results given above to the study of the polarization properties of recombination radiation.

4. We consider first the radiation accompanying the K capture of an unpolarized electron by a nucleus with charge Ze. For this case, we should, in Eqs. (5) and (6), set

$$\overset{+}{C}_1 C_1 = \overset{+}{C}_{-1} C_{-1} = 1/2, \quad \overset{+}{C}_1 C_{-1} = \overset{+}{C}_{-1} C_1 = 0. \tag{13}$$

We furthermore choose the vector β_2 as perpendicular to the plane of the vectors **k** and $\boldsymbol{\kappa}$, and place the vector β_3 in this plane. It can easily be seen that such a resolution of the photon amplitude results in the extremum of W_λ. Omitting the rather trivial calculations, we present the final expressions for the degree of polarization

$$P_l = 0,$$
$$P_\lambda = [2 - \gamma(\gamma+1)(1-\beta\cos\theta)] / [2 + \gamma(\gamma^2-1) \times (1-\beta\cos\theta)]. \tag{14}$$

where $\gamma = \kappa/k_0$, $\beta = k/K$, and θ is the angle between the vectors **k** and $\boldsymbol{\kappa}$.

Thus, the recombination of the unpolarized electron is, according to Eqs. (11)—(13), accompanied by radiation which is partially linearly polarized. In the nonrelativistic limit, where $\gamma \ll 1$, the polarization of the radiation becomes total. In the ultrarelativistic case, the radiation is fully depolarized ($W_0 = W$). The variation of the degree of linear polarization with the photon energy for

*The relation between Eq. (11) and Eq. (50.13) of reference 10 can be established directly by taking into account that $|J_{yz}| = 1/2\,|W_1 - W_{-1}|$ if, for the direction of x, y, we take the direction of the principal axes of the polarization ellipse.

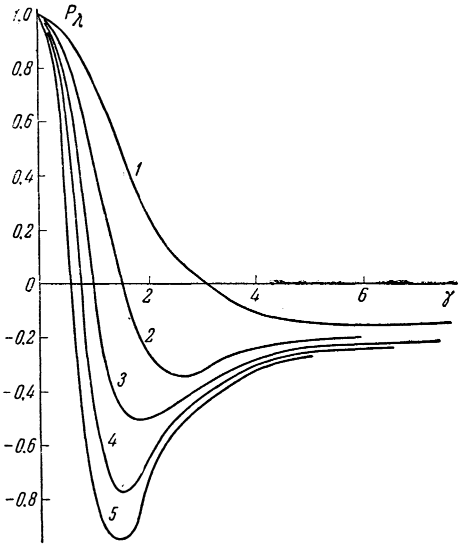

FIG. 1. Variations of the degree of linear polarization with energy. Curves 1–5 correspond to angles θ = 30, 60, 90, 120, and 150° respectively.

the intermediate cases is shown in Fig. 1 for certain values of the angle θ.

5. We shall now study the capture of longitudinally polarized electrons. In such a case, $C_1 = 1$ and $C_{-1} = 0$, or $C_1 = 0$ and $C_{-1} = 1$. Choosing the vectors β_λ as in the previous case, we find that P_λ, as before, is given by Eq. (14), and the degree of circular polarization is equal to

$$P_l = \frac{s\gamma}{\sqrt{\gamma(\gamma+2)}} \frac{2+\gamma(\gamma+1)(\gamma^2-2)(1-\beta\cos\theta)}{2+\gamma(\gamma^2-1)(1-\beta\cos\theta)}, \quad (15)$$

where $s = \pm 1$ for the electron polarized along the direction of motion or along the opposite direction respectively. Equation (15), with an accuracy limited by the substitution $s \to l$ (where l characterizes the circular photon polarization), coincides with the formula for the longitudinal polarization of electrons in the photoeffect from the K shell. The corresponding curves are given in reference 11.* In a nonrelativistic approximation ($\gamma \ll 1$), P_l vanishes, so that the radiation is, as before, fully linearly polarized in the plane of the vectors **k** and $\boldsymbol{\kappa}$. In the case of high energies, the radiation becomes fully circularly polarized, and its helicity coincides with that of the incident electron. In the intermediate cases, $P_\lambda^2 + P_l^2 \neq 1$, and, according to Eq. (11), the recombination radiation is partially elliptically polarized.

The variation of the unpolarized-component intensity of the radiation with photon energy is shown in Fig. 2. As can be seen from the figure, the depolarization is greatest in the energy range $\gamma \sim 1$.

*It should be kept in mind that the final formula (35) for the degree of the longitudinal polarization given by Fano et al.[11] contains an error; the graphs are correct.

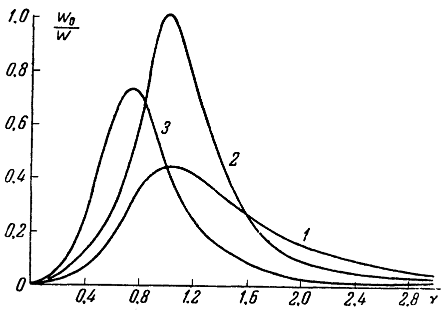

FIG. 2. Variation on the intensity of the unpolarized component of the radiation with energy. Curves 1–3 correspond to angles θ = 60, 90, and 120° respectively.

Since the condition that P_λ be maximum is the same for any γ and θ, the orientation of the polarization ellipse is independent of γ and θ. (One of its axes always lies in the plane of the vectors **k** and $\boldsymbol{\kappa}$).

6. We consider finally the case of transverse polarization of the incident electron. Without loss of generality, we can assume that the electron initially moves along the z axis. Furthermore, let the spin vector make an angle φ with the **k**, $\boldsymbol{\kappa}$ plane. In this case, the coefficient C_s should be chosen in the following manner:[12]

$$C_1 = 1/\sqrt{2}, \quad C_{-1} = e^{i\varphi}/\sqrt{2}. \quad (16)$$

In the calculation, we have to use the formula

$$b^+(s)\, \alpha'_\nu b_0 b_0^+ \alpha_\mu b\,(-s) = \tfrac{1}{8}\,\mathrm{Sp}\, \alpha'_\nu (1+\rho_3)\, \alpha_\mu (1 + \rho_1 sk/K + \rho_3 k_0/K)(1+s\sigma_3)\,\rho_1\sigma_3, \quad (17)$$

where α'_ν and α_μ are any of the sixteen Dirac matrices.[9,13]

Omitting the calculations, the final result is

$$P_l = 2\gamma\,[2+\gamma(\gamma^2-1)(1-\beta\cos\theta)]^{-1}\sin\theta\cos\varphi. \quad (18)$$

P_λ is again given by Eq. (14). It follows from Eqs. (11), (14), and (18) that, for $\gamma \ll 1$ and for $\gamma \gg 1$, the polarization is the same as for the case of an unpolarized electron. For intermediate energies, we have partial elliptical polarization, the degree of which considerably depends on the angle φ.

7. The above discussion of the polarization properties of the radiation accompanying the electron capture into the K shell of a nucleus with charge Ze shows that, for the case of low energies, the radiation is fully linearly polarized in the plane of the vectors **k** and $\boldsymbol{\kappa}$, irrespective of the initial polarization of the electron. With increasing energy, the linear polarization becomes elliptical on one hand, and, on the other, an unpolarized component appears, whose magnitude depends considerably on the initial polarization of the electron.

In conclusion, it is necessary to note that the results obtained are in need of a further increase of accuracy for angles θ close to zero. This is due to the fact that the quantities W_λ and W_l calculated by us to the lowest order in αZ contain a factor $\sin^2\theta$. Consequently, if by taking higher powers of αZ into account we obtain a cross section for the recombination process that does not vanish for $\theta = 0$, then this will considerably change the picture of the polarization properties of radiation for $\theta \sim 0$. Until now, this problem has not been solved. Thus, Gavrila,[3] who calculated the cross section for the photoeffect taking the second Born approximation into account, found that, for $\theta = 0$, the cross section does not vanish. On the other hand, Banerjee,[14] using the Sommerfeld approximation, found that it does vanish. Only recently, it has been possible to carry out an exact integration in the matrix element for the relativistic photoeffect from the K shell for $\theta = 0$.[15] A nonvanishing expression was obtained for the cross section.

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