## *CRITICAL VELOCITIES IN SUPERFLUID HELIUM*

V. P. PESHKOV

Institute for Physics Problems, Academy of Sciences, U.S.S.R.

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MORE than 20 years ago Daunt and Mendelssohn,[1] in films, and Allen and Misener,[2] in fine capillaries, noted that the flow velocity of helium II is independent of the pressure head. The clearest picture of the critical velocity was derived by Kapitza,[3] in his papers on the investigation of the thermo-mechanical effect. Critical velocities are manifested in the fact that beyond a certain value the flow velocity of the superfluid component of helium, in films and in fine capillaries and slits, does not increase with further increase in head. In large capillaries (larger than $10^{-3}$ cm), beginning at a certain critical velocity, there appear forces of interaction between the normal and superfluid components. Landau, in his first paper on superfluid helium,[4] formulated the conditions for destruction of superfluid motion as a consequence of the appearance of excitations in the superfluid helium. The ratio of the energy of an excitation to its momentum must be smaller than the flow velocity of the superfluid component relative to the walls: $\epsilon/p < v_s$. Proceeding from these premises, one may conclude that a velocity greater than the velocity of sound is required for formation of phonons, and one greater than 70 m/sec for formation of rotons.

Using dimensional and certain other concepts, a number of authors have proposed for the critical velocity the relations $v_s = \hbar/md$ and $v_s = \hbar/m\sqrt{ad}$, where m is the mass of the helium atom, d the diameter of the capillary or width of the slit, and a a quantity of the same order as the interatomic distance. Feynman,[5] assuming that superfluidity is destroyed as a result of the formation of vortices at the exit of the superfluid current from the capillary, derived the formula $v_s = (\hbar/md)\cdot\ln(d/a)$. These formulas give the correct order of magnitude for the critical velocity and its dependence upon the dimensions, for a certain range of capillary diameters, but for capillary dimensions of less than $10^{-3}$ cm they are not borne out, either qualitatively or quantitatively. Suggestions indicating that the reason for the destruction of superfluidity lies in the generation of vortices, however, appear to be correct. As is well-known,[5,6] the energy of a vortex ring of radius R is, approximately

$$\varepsilon \approx (\rho_s R h^2/2m^2)\ln(R/a), \qquad (1)$$

while its momentum $p \approx \rho_s \pi R^2 h/m$, where $\rho_s$ is the superfluid component density, and $h = 2\pi\hbar$. Thus, in addition to phonons and rotons, there exists still another form of excitations — vortex rings. Assuming that a vortex ring of minimal size is a roton of energy $\Delta = k\cdot 8.9° = 1.23 \times 10^{-15}$ erg, and momentum $p_0 = 2.1\times 10^{-19}$ g cm/sec, we obtain $a \approx 10^{-8}$ cm, and $R = 2.6\times 10^{-8}$ cm, while the vortex ring spectrum will have the form

$$\varepsilon = \Delta\sqrt{\frac{p}{p_0}}\left(1+\frac{1}{2}\ln\frac{p}{p_0}\right). \qquad (2)$$

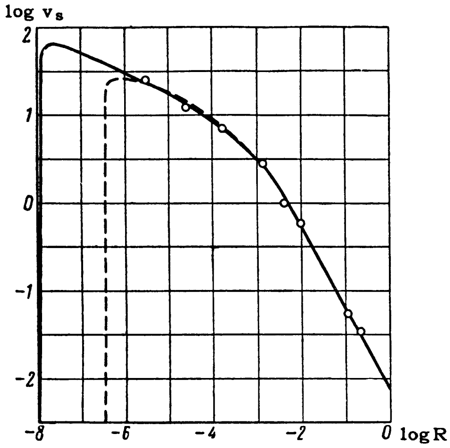
Inasmuch as the condition for generation of the excitations is $v_s > \epsilon/p$, it is evident from the form of the spectrum that the development of vortices with the largest p — i.e., the largest radius — is the most profitable. Moreover, there should exist an analogy between the development of turbulence in an ordinary liquid and the generation of vortices in superfluid helium; i.e., the energy for formation of the vortices may not be acquired instantaneously, but only after a relaxation time $\tau$. One may therefore presume that superfluidity is destroyed through formation of a vortex of maximal radius R with the energy (1). This energy is acquired at the expense of a fraction ($\alpha$) of the kinetic energy of the superfluid current in the capillary over a segment of its length on the order of $R + v_s\tau$; i.e.,

$$\alpha\frac{\rho_s v_s^2}{2}\pi R^2(R+v_s\tau) = \rho_s\frac{Rh^2}{2m^2}\ln\frac{R}{a}$$

or

$$v_s^2 R(R+v_s\tau) = \frac{h^2}{\alpha\pi m^2}\ln\frac{R}{a}. \qquad (3)$$

For $\alpha = 0.122$, $\tau = 4\times 10^{-4}$ sec, and $a = 10^{-8}$ cm, (3) yields for $v_s(R)$ a dependence that agrees to within the limits of experimental error with the experimental data available (cf. reference 6), at T = 1.3°K, from $R = 3\times 10^{-6}$ cm to R = 0.2 cm. For films, R is taken to be the total thickness d, and for slits, the quantity d/2. If one takes $a = 3\times 10^{-7}$ cm (the value obtained by Hall[7] from experiments on the oscillation of vortices), then for $\alpha = 0.116$ and $\tau = 2\times 10^{-4}$ sec satisfactory agreement is also obtained. In the figure, the circles represent the experimental data, the solid curve is computed from equation (3) with $a = 10^{-8}$ cm, $\tau = 4\times 10^{-4}$ sec, and $\alpha = 0.122$, and the




dashed curve, with $a = 3 \times 10^{-7}$ cm, $\tau = 2 \times 10^{-4}$ sec, and $\alpha = 0.116$.

The appearance of a trans-critical regime in the capillary under conditions of slight supercriticality is represented in the following form: At certain points in the capillary, due to nonuniformity of its walls, there will exist more suitable conditions for the formation of a ring vortex. When the vortex has moved along the capillary, another vortex arises at the same point, but with two vortices present near one another one vortex begins to move into the other. After a third vortex has formed, it too begins to take part in the collective vortex motion. Thus, the vortices forming at individual points due to slight supercriticalities begin gradually to fill the capillary, through an extremely complex collective motion. In the presence of a thermal current along the capillary the vortices existing therein increase the thermal resistance; the gradual filling of the capillary with vortices is therefore accompanied by a real, experimentally observable, slow (e.g., 0.2 cm/sec) advance of a front of increased thermal gradient. This pattern has been observed by Mendelssohn and Steele.[8] The vortices formed are more stable when their momenta are in the direction of motion of the normal component than when they are oppositely directed; when trans-critical regimes are present in capillaries, therefore, a velocity distribution is established which is similar to that proposed by Gorter and Mellink,[9] wherein the difference $v_s - v_n$ remains constant over the entire cross-section of the capillary.

[1] J. G. Daunt and K. Mendelssohn, Proc. Roy. Soc. **A170**, 423, 439 (1939).

[2] J. F. Allen and A. D. Misener, Proc. Roy. Soc. **A172**, 467 (1939).

[3] P. L. Kapitza, JETP **11**, 581 (1941).

[4] L. D. Landau, JETP **11**, 592 (1941).

[5] R. P. Feynman, Progr. in Low Temp. Phys. **1**, Amsterdam, 1955, p. 17.

[6] K. R. Atkins, Liquid Helium, Cambridge, 1959.

[7] H. E. Hall, Proc. Roy. Soc. **A245**, 546 (1958).

[8] K. Mendelssohn and W. A. Steele, Proc. Phys. Soc. **73**, 144 (1959).

[9] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).

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## *ORIGINS OF THE NERNST EFFECT IN FERROMAGNETIC METALS*

E. I. KONDORSKIĬ

Moscow State University

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THIS article treats a relativistic effect connected with the movement of current carriers that possess a nonvanishing mean magnetization. An estimate of the transverse electric field intensity connected with this effect shows that it needs to be taken into account in the theory of the Nernst effect in ferromagnetic metals. It is shown that from the sign of the Nernst field one can determine how the magnetization of the current carriers is directed with respect to the resultant spontaneous magnetization of the metal.

In the existing theories, the fractional values of the magnetic moments of ferromagnetic metals are explained[1] on the basis of the assumption that the current carriers possess a magnetization directed parallel or antiparallel to the magnetization of the metal.

Let us consider the simplest possible model, in which the current carriers are free electrons with mean magnetization $I_e = a_e I_z$, where $a_e$ is a positive or negative coefficient and $I_z$ is the magnetization. Under the influence of an electric field $E_x$, there is excited an electric current of density $j_x = env_x = -enuE_x$, where u is the mobility. It is known that if a system possessing magnetic moment **I** moves with velocity **v**, a stationary observer records in it an electric polarization **P** and an electric field $\mathbf{E} = -4\pi\mathbf{P} = -(4\pi/c)\mathbf{v}\times\mathbf{I}$. In the case being considered, this field is directed