PLASMA TURBULENCE IN A MAGNETIC-MIRROR SYSTEM

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The convective turbulence of a rarefied plasma in a magnetic-mirror system is analyzed theoretically. The turbulence arises as a result of plasma instability. The results which are obtained are in satisfactory agreement with the experimental data of Ioffe, Tel'kovskiĭ, Sobolev, and Yushmanov on plasma lifetime in a system of this kind.[7]

1. INTRODUCTION

THE magnetic-mirror system[1] represents a possible means by which high-temperature plasma can be contained. In its simplest version, a magnetic-mirror system consists of a region of uniform magnetic field H_0 with mirrors at each end, i.e., regions of stronger magnetic field H_m. A schematic diagram of such a system is shown in Fig. 1.

If the magnetic field is strong the Larmor radius of the particles ρ is much smaller than any of the characteristic dimensions and the adiabatic invariant $\mu = Mw^2/2H$ (M is the mass of the particles, w is its transverse velocity) may be considered constant with a high degree of accuracy. For this reason a field configuration of this kind represents a good trapping system for particles for which $\sin\alpha = w/v > H/H_m$ where u is the longitudinal velocity while $v = (u^2 + w^2)^{1/2}$ is the total velocity.

However, the problem of containing a quasineutral plasma is quite difficult. In a magnetic-mirror system the magnetic field falls off in the radial direction and, because it is diamagnetic, the plasma is expelled from the central region toward the walls. As far as the motion of individual particles is concerned, this instability manifests itself in separation of the charges and the appearance of an electric field which causes particles of both signs to drift in the radial direction.

If the plasma pressure is much smaller than the pressure of the magnetic field, only the convective or interchange perturbations lead to instability;[2,3] these correspond to the interchange of neighboring lines of force without perturbation of the magnetic field. In these perturbations the electric field is irrotational and the field potential is constant along the lines of force. Hence, if there is good electrical contact with the end electrodes perturbations of this kind are impossible and a low-pressure plasma will be stable.

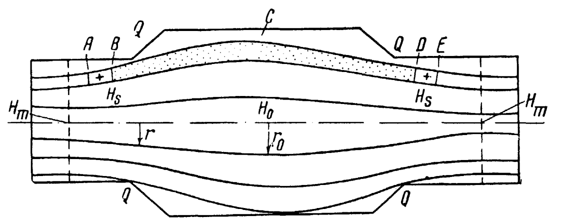

FIG. 1

The concrete details of a given experiment determine whether or not an electrical contact exists. For example, in the pyrotron, described by Post,[4] in which hot plasma is produced by adiabatic compression of a cold plasma which is injected from outside, contact is apparently realized by virtue of the rather dense cold plasma (this question is discussed in a paper by Post et al.[5]). Under other conditions no contact may be formed. Precisely this situation obtains in the system described by Ioffe et al.[6,7] The experimental finding that the plasma is lost from this system primarily in the direction transverse to the magnetic field verifies the fact that no electrical contact is realized.

2. QUALITATIVE ANALYSIS

In the experiment of Ioffe et al.[6] a hot plasma (ion energy, approximately 1 kev and density, approximately 10^9 cm^{-3}) is produced by accelerating ions in a pulsed radial electric field of the order of 1 kv/cm; the field is applied between the chamber and a cold pinch located along the axis of the system. Qualitatively, the way in which the system becomes filled with hot plasma may be described as follows. Following the application of the high voltage, the outer layers of the cold plasma are set into rotational motion. This rotational motion, however, is unstable. The centrifugal force causes the plasma to be "splashed" violently toward the periphery of the system; since this effect is characterized by high electric fields, the "splashing" results in an increase in the energy of the individual ions. As a result the system becomes more or less uniformly filled by plasma with high energy ions; it has been shown experimentally that this process is completed in a time of the order of $10 - 20\ \mu$sec.

After the high-voltage pulse ends, plasma rotation ceases and a quieter phase, characteristic of the motion of a diamagnetic plasma in a magnetic field that decays in the radial direction, is observed. The characteristic time for this motion is considerably greater than the ion time-of-flight between the mirrors $t_s \sim 5\ \mu$sec. Hence, an equilibrium distribution of ions and electrons can be set up along the lines of force.

We consider an individual tube containing plasma, ABCDE. Since the electrons remain cold (the electron temperature T_e is of the order of 10 ev) we can write $T_e = 0$; the, from the electron equilibrium condition it follows that φ, the electrical potential along each tube of force containing plasma, is a constant. The potential can not be negative; if it were, the excess electrons would be rapidly repelled from a given tube towards the ends, which are at zero potential. Thus, an individual tube of force can have a potential different from zero only if there is a charge due to positive ions at its ends, the regions AB and DE, where the electron density vanishes.

This charge, however, cannot be arbitrarily large because if the ion charge increases the electric field repels ions toward the ends and then they can be contained only by virtue of the increase in magnetic field from the value H_s at the boundary of the tube to the value H_m at the mirror. It follows that the potential φ satisfies the following condition:

$$0 < \varphi < (T/e)(H_m/H_s - 1), \tag{1}$$

where T is the ion temperature, i.e., $^2\!/_3$ of the ion mean energy.

This relation allows us to understand why there may be no electrical contact with the ends under the conditions being considered. Because there are large fluctuations in the electric field during the ion acceleration stage, in a plasma produced in this way there are no ions which are reflected very close to the mirrors. In other words, the boundary of the hot plasma is located at some distance from the surface of maximum magnetic field, that is to say, $H_s \neq H_m$. Hence, φ can vary over some finite limit (1) and this means that there is no contact with the end walls.

A consequence is that there is a convective instability in this system: a tube with more dense plasma is expelled towards the walls as a result of the production of the azimuthal electric field, causing a drift $v_e = cH^{-2}(\mathbf{E} \times \mathbf{H})$ and when the tube comes into contact with the wall, ions are emitted and absorbed by the wall; these ions are at a distance of the order of the mean Larmor radius ρ from the wall. Because of the loss of ions, the plasma potential close to the walls drops to zero and the excess electrons escape to the ends along the lines of force. As a result, a layer of thickness ρ next to the walls will be at zero potential and the azimuthal component of the electric field (consequently, the normal component of the velocity) V_e vanishes in the entire wall layer.

It follows that an individual plasma tube cannot be lost to the wall immediately. The tube is slowed down* near the wall and as soon as its density becomes lower than that of the surrounding plasma (because of wall losses) it is forced back into the system. Thus, each individual tube must come in contact with the walls and is forced back into the chamber several times before becoming completely detached from the plasma. The motion as a whole is similar to thermal convection in an ordinary incompressible fluid: the role of the fluid is played by the tubes of force of the magnetic field while the plasma pressure acts as the temperature. For this reason we can make use of the semi-quantitative methods of analysis of ordinary convection.

3. BASIC EQUATIONS

For the conditions considered here, collisions between particles are unimportant so that an exact description of the ion motion is given by the kinetic equation without the collision term:

$$\frac{\partial f}{\partial t} + (\mathbf{v}\nabla) f + \frac{e}{M}\{\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{H}]\}\frac{\partial f}{\partial \mathbf{v}} = 0. \tag{2}$$

For convective flow of a plasma characterized by a plasma frequency which is much lower than the cyclotron frequency $\Omega_H = eH/Mc$ but a characteristic length considerably greater than the Larmor radius ρ, the important terms in Eq. (2) are the last two terms. Hence, in the zeroth approximation

$$\{\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{H}]\}\frac{\partial f}{\partial \mathbf{v}} = 0, \tag{3}$$

whence we have $f = F(\mathbf{v} - \mathbf{v}_0)$, where $\mathbf{v}_0 = H^{-2}c(\mathbf{E} \times \mathbf{H}) = -H^{-2}c(\nabla\varphi_0 \times \mathbf{H})$ while the function F has

*The fact that a metal wall along the walls of force reduces the normal component of the electric drift to zero, causing retardation of the plasma, was pointed out by L. A. Artsimovich.

an axis of symmetry with respect to the direction of the magnetic field.

If we take account of the remaining terms, then in addition to the electric drift there are supplementary drift velocities which are different for different particles. As an approximation, however, we may assume that all ions move accross the magnetic field with the same velocity $\mathbf{v}_0$; φ_0, however, is no longer the potential of the electric field, but some effective potential which takes account of all forces acting on the ions. This approximation allows us to go from the kinetic description to the hydrodynamic description. If we take $\varphi_0 = \text{const}$ along the lines of force a further simplification is possible and the motion of the individual tubes of force becomes two-dimensional.

In Eq. (2) we can write $f = F(\mathbf{v} - \mathbf{v}_0)$, multiply by $\mathbf{w} = \mathbf{v} - \mathbf{u}\cdot\mathbf{H}/H$, integrate over velocity, and average over the lines of force, thereby obtaining the equation for the transverse motion. We shall assume that the mirror ratio H_m/H_0 is small and that the lines of force of the magnetic field are only slightly distorted. Then we can then neglect the curvature of the lines in the inertia terms, thereby obtaining the following equation for the transverse motion:

$$Mn\left\{\frac{\partial \mathbf{v}_0}{\partial t} + (\mathbf{v}_0\nabla)\,\mathbf{v}_0\right\} + \nabla p + en\nabla\varphi - \frac{ne}{c}[\mathbf{v}_0\times\mathbf{H}] = Mn\mathbf{g}. \quad (4)$$

Here,

$$n = \frac{H_0}{L}\iint f d\mathbf{v}\frac{dl}{H}$$

is some mean density over the tube of force, L is the mean length of the tubes of force,

$$p = \frac{H_0}{L}\int\frac{p_\perp}{H}dl$$

is the mean transverse pressure, the vector **g** is along the radius and is given by

$$g = \frac{1}{MnL}\int(p_\perp + p_H)\frac{H_0^2 r_0}{H^2 Rr}dl \approx \frac{1}{MnL}\int(p_\perp + p_H)\frac{H_0^{3/2}}{RH^{3/2}}dl,$$

H_0 is the field at the center of the system, $r = r(z)$ is the distance from the line of force to the axis of the system, $r_0 = r(z = 0)$ is the distance from the line of force to the axis in the central plane, dl is an element of length of the line of force, R is its radius of curvature, and

$$p_\parallel = \int Mu^2 f dv, \qquad p_\perp = \int\frac{Mw^2}{2}fd\mathbf{v}$$

are respectively the longitudinal and transverse pressures.

The average field **H** which appears in Eq. (4) may be approximated by a constant so that Eq. (4) coincides exactly with the hydrodynamic equation for the two-dimensional motion of ions in a uniform magnetic field in the presence of a gravitational force Mg. This force, which takes account of the curvature of the lines of force, expresses precisely the average effect of the expulsion of the diamagnetic plasma from the field. We may note that the expression for g given above can be obtained from energy considerations, as has been done by Rosenbluth and Longmire.[2]

The ions can reach an equilibrium state in the longitudinal direction so that the ion distribution function depends explicitly only on v^2 and $\mu = Mw^2/2H$. If the particle density is high enough the plasma is quasi-neutral and the ion density is regions AB and DE (cf. Fig. 1), where the electron density vanishes, is very small. However, this region is entered only by particles characterized by $\sin\alpha = w/v < H/H_s$; consequently, the ion velocity distribution contains no velocities within the cone defined by $\sin\alpha < H/H_s$. We approximate the ion velocity distribution by a Maxwellian function (no velocities in this cone) and assume that the temperature T is a constant. For this distribution

$$n = N\sqrt{1 - H/H_s}, \qquad p_\parallel + 2p_\perp = 3nT,$$
$$p_\parallel = nT(1 - H/H_s).$$

In principle, we can calculate g for these relations and find the connection between n and p for a given magnetic field, in which case the problem is reduced to an analysis of the transverse motion alone.

To Eq. (4) we must add the electric field equation

$$\Delta\varphi = -4\pi e(n - n_e) \quad (5)$$

and the ion and electron continuity equations:

$$\partial n/\partial t + \mathrm{div}(\mathbf{v}_0 n) = 0, \quad (6)$$
$$\partial n_e/\partial t + \mathbf{v}_e\nabla n_e = 0. \quad (7)$$

Here we have neglected end leakage because it has been shown experimentally[7] that leakage is responsible for less than 20% of the total particle loss from the system; we assume that the electrons experience an electric drift $\mathbf{v}_e = cH^{-2}(\mathbf{H}\times\nabla\varphi)$ only. Eqs. (4) — (7) completely describe the behavior of plasma in the system.

4. STABILITY

We first find the density at which the plasma becomes unstable. We assume that

$$4\pi e^2 n/M\Omega_H^2 = 4\pi nMc^2/H^2 \ll 1.$$

Under these conditions the inertia terms in Eq. (4) can be neglected and we obtain the following expression

$$\mathbf{v}_0 = -\frac{1}{\Omega_H}[\mathbf{h}\times\mathbf{g}] + \frac{1}{M\Omega_H n}[\mathbf{h}\times\nabla p] + \frac{c\,[\mathbf{h}\times\nabla\varphi]}{H}, \quad (8)$$

where **h** is a unit vector along **H** (i.e., the z axis).

We take $p = nT$ and assume that T is a constant. Furthermore, for the curvature of the lines of force we can write $g = Tr/MaR_0$ where a is the radius of the chamber and R_0 is some mean radius of curvature for the lines of force close to the walls. Substituting Eq. (8) in Eq. (6) we have

$$\frac{\partial n}{\partial t} - \omega_0 \frac{\partial n}{\partial \vartheta} + \frac{c}{H}\mathbf{h}[\nabla\varphi, \nabla n] = 0, \quad (9)$$

where $\omega_0 = T/MaR_0\Omega_H$ is the angular drift velocity of the ions under the effect of the force Mg.

Suppose that in the equilibrium state $n = n_e = n_0$ and $\varphi_0 = 0$. Assuming that the perturbations of density and potential n', n_e' and φ' vary according to $\exp\{-i\omega t + im\vartheta\}$, from Eqs. (5), (7), and (9) we can obtain a single equation for φ:

$$\omega(\omega + m\omega_0)\Delta\varphi' + \frac{4\pi e^2 m^2 \omega_0}{M\Omega_H}\frac{1}{r}\frac{dn_0}{dr}\varphi' = 0. \quad (10)$$

In the simplest case, where the density n_0 is given by the parabolic relation $n_0 = N(1 - r^2/a^2)$, the solution of Eq. (10) is of the form $\varphi^1 = J_m(\alpha_{mn} r/a)$ where α_{mn} is the n-th root of the Bessel function J_m. Then the oscillation frequency is given by

$$\omega = -\tfrac{1}{2}m\omega_0 \pm m\sqrt{\tfrac{1}{4}\omega_0^2 - 2\Omega_0^2\omega_0/\Omega_H\alpha_{mn}^2},$$

where $\Omega_0^2 = 4\pi e^2 N/M$. From these considerations it is apparent that an instability arises only when the density becomes high enough to make Ω_0^2 greater than $\frac{1}{8}\alpha_{mn}^2\omega_0\Omega_H$. Under these conditions the instability arises first for perturbations with the greatest wavelength, i.e. for $m = 1$ and $\alpha_{mn} = \alpha_{11} = 2.83$. The stability condition can be written approximately in the form

$$r_d^2 > aR_0, \quad (11)$$

where $r_D^2 = T/M\Omega_0^2$ is the square of the Debye radius.

The stability condition assumes a similar form for other density distributions $n_0(r)$. In particular, if n_0 experiences a steep drop at a distance δ from the walls (which are assumed to be metal), the stability condition becomes $r_D^2 > R_0\delta$. Since δ cannot be smaller than ρ, for a plasma density high enough so that $r_D^2 < R_0\rho$ there will be an instability for any distribution n_0 which vanishes at the walls. This is precisely the situation which obtains in the experiment[6] where $r_D \sim \rho \sim 1$ cm, while R_0 is approximately 10^2 cm.

5. TURBULENT CONVECTION

The condition $r_D^2 < R_0\rho$ together with the plasma neutrality condition, represent the criteria for convective motion on all scales of length to the minimum of approximately ρ. The equations of motion can be simplified for this particular case. For this purpose we subtract Eq. (7) from Eq. (6) and express the difference $n - n_e$ in terms of φ by means of Eq. (5). We thus obtain

$$\frac{\partial}{\partial t}\Delta\varphi + \mathbf{v}_e\nabla(\Delta\varphi) - 4\pi e\,\mathrm{div}\,(n(\mathbf{v}_0 - \mathbf{v}_e)) = 0, \quad (12)$$

where $\mathbf{v}_e = cH^{-1}(\mathbf{h}\times\nabla\varphi)$. For two-dimensional flow, however, $\nabla\varphi = (H/c)\,\mathrm{curl}\,\mathbf{v}_e$ so that Eq. (12) can be integrated once, yielding

$$\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e\nabla)\mathbf{v}_e - \frac{4\pi ec}{H^2} n[\mathbf{v}_0 - \mathbf{v}_e]\times\mathbf{H} + \nabla f = 0, \quad (13)$$

where f is an arbitrary function of r and ϑ.

As an approximation, in the first term of Eq. (4) we replace $\mathbf{v}_0$ by $\mathbf{v}_e$ and n by some mean density N, multiply Eq. (4) by $4\pi c^2/H^2$, and subtract from Eq. (13). We then obtain the approximate equation

$$\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e\nabla)\mathbf{v}_e + \nabla p^* = g^* n, \quad (14)$$

where $\mathbf{v}_e \approx \mathbf{v}_0$ is the macroscopic plasma velocity, p^* is an arbitrary function of r and ϑ, and $g^* = 4\pi e^2 g/M(\Omega_H^2 + \Omega_0^2)$, $\Omega_0^2 = 4\pi e^2 N/M$. If we take account of the neutrality condition $n = n_e$ and the condition

$$\mathrm{div}\,\mathbf{v}_e = 0 \quad (15)$$

Eqs. (14) and (7) coincide exactly with the equations for an incompressible inhomogeneous fluid in a gravitational field of force. Hence there is a very close analogy with the familiar convection of an incompressible fluid.

Since Eq. (14) does not contain a viscosity term, the convection described by this equation is of a turbulent nature. Thus, the problem of plasma lifetime in the system becomes essentially the determination of the coefficient of turbulent diffusion. We shall assume that the chamber walls are exactly along the lines of force. Let x be the distance from the wall while q is the diffusion flux of plasma to the wall. In the region next to the walls the flux q may be assumed constant. Eqs. (14) and (15) contain only one dimensional parameter g^* which appears in the form of a product with n. It then follows from similitude considerations that the turbulent state of the plasma must be determined only by the parameter g^*q. But, from the quantities g^*q and x the dimensionality of the diffusion coefficient can be constructed in only one way, namely,

$$D = A(g^*q)^{1/3}x^{4/3} = A[4\pi e^2 Tq/R_0 M(\Omega_H^2 + \Omega_0^2)]^{1/3}x^{4/3}, \quad (16)$$

where A is a numerical factor of order unity. Knowing D, from the relation $q = D - dn_0/dr = D\,dn_0/dx$ we find the distribution close to the walls n_0:

$$n_0 = N - 3q / A (g^* q)^{1/3} x^{1/3}, \qquad (17)$$

where $N = \text{const}$ is the density inside the chamber, i.e., at $x \to \infty$.

The distribution given by Eq. (17) holds only when $x > \rho = \sqrt{T/M\Omega_H^2}$. At $x = \rho$ we must impose a boundary condition. Let ξ be the fraction of the plasma lost in contact of the tube with the wall and let n_s be the mean density (17) at $x = \rho$. The density fluctuation n' due to the fact that the plasma tubes move back and forth at the wall is a quantity of order $n' \sim \frac{1}{2} \xi n_s$. In order-of-magnitude terms, the flux at the wall q is $n'v'$ where $v' \sim D/x$ is the velocity fluctuation. Hence, as an approximation we can write the following boundary condition:

$$q = \frac{1}{2} \xi n_s (D/x)_{x=\rho}.$$

Now, expressing n_s by means Eqs. (17) we can find the relation between q and N and the plasma lifetime τ_0:

$$\tau_0 \approx \frac{\pi a^2 N}{2\pi a q} = Ca \left(\frac{\Omega_H^2 + \Omega_0^2}{\Omega_0^2} \frac{R_0 M}{\rho T} \right)^{1/2}, \qquad C = \frac{1}{2} \left(\frac{2+3\xi}{A\xi} \right)^{3/2}. \qquad (18)$$

The expression for τ_0 contains one unknown parameter ξ. The quantity τ_0 is rather sensitive to ξ, but it is an extremely complicated problem to calculate ξ because this parameter is directly related to the "roughness" of the wall; experimentally, the walls never coincide completely with the lines of force. For this reason, the tube running along the wall does not lose ions over its entire length, but only at separated points of contact. In the remaining portions the tube potential (consequently, its velocity) is maintained for some period of time. Hence, to find ξ we must solve the complete kinetic equation, taking account of the longitudinal motion. In view of the complexity of this probelm and the lack of adequate experimental data on wall roughness we do not treat this problem and shall assume that ξ is a numerical constant of order unity.

In the derivation of Eq. (18) we have not taken any account of end effects. According to Eq. (1), however, end effects impose certain limitations on the potential φ. As a result, the velocity $v'_\lambda \sim C\varphi'_\lambda/\lambda H$ of a fluctuation of order λ cannot be larger than $(cT/\lambda H)(H_m/H_s - 1)$. Hence, the turbulent diffusion coefficient $D \sim \langle \lambda' v \rangle$ cannot be larger than

$$D_m = B \frac{cT}{H} \left(\frac{H_m}{H_s} - 1 \right), \qquad (19)$$

where B is a numerical factor of order unity.

An estimate shows that (16) is actually much smaller than (19); however, if there is a diaphragm this relation may not hold. When plasma retardation at the wall is not important and we modify the first approach by introducing velocity limitations due to end effects, the plasma flow loses its diffusion nature because the tubes do not close inside the chamber. For this reason Eq. (19) does not give the true diffusion coefficient; it can, however, be used for making estimates.

	V, kv	45	30	20	10
H, kg	T, kev	1.2	1.0	0.9	0.4
5	experiment	0.15	0.2	0.4	1.0
	theory	0.35	0.4	0.45	0.7
6	experiment	0.16	0.4		
	theory	0.45	0.5		
8	experiment	0.7	1.5		
	theory	1.0	1.1		

6. COMPARISON WITH THE EXPERIMENTAL DATA

In the table we compare the dependence of τ_0 (in milliseconds) on T and H as computed from Eq. (18) with the experimentally measured relation[6] for a mirror ratio $H_m/H_0 = 1.5$. The calculation is carried out as follows: the radius of curvature $R_0 \approx 3 \times 10^2$ cm is computed from the experimentally measured magnetic field under the assumption of a Maxwellian ion-velocity distribution with the cone excluded (depending on the cone angle α, R_0 can vary by 20%.) The ion temperature is $\frac{2}{3}$ of the mean ion energy, which is estimated experimentally from the rate of removal due to charge exchange. According to reference 6 the density N is 10^9 for fields of 5 and 6 kilogauss and 5×10^8 for a field of 8 kilogauss. The constant C is taken as 20, corresponding approximately to $A = \xi \approx \frac{1}{2}$. The chamber radius $a = 22$ cm and V is the acceleration potential.

The fact that the lifetime which we have computed is of the order of magnitude of the experimental lifetime for a completely reasonable choice of the constants A and ξ is an indication of the usefulness of the considerations developed above. However, the experimental dependence of τ_0 on H and T is much sharper than the theoretical dependence. This result is probably due to the fact that the chamber in the experimental apparatus has a complicated shape with protrusions (Q in Fig. 1) which act as diaphragms or that ξ itself is not a constant but increases with diminishing τ_0, that is to say, with increased intensity of the convective oscillations.

The finding that convection is responsible for plasma loss from this system is also supported by probe measurements.[7] These measurements show

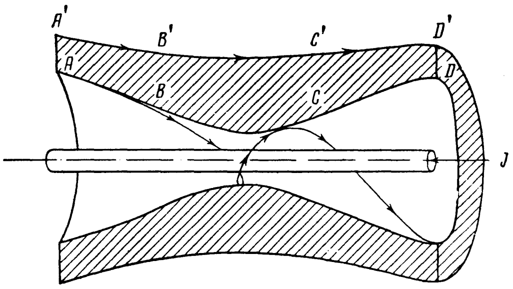

FIG. 2

that the currents to the wall probes are well correlated along the lines of force, that is to say, the plasma actually escapes to the wall in complete tubes. Furthermore, the probe currents exhibit oscillations with a period of approximately 5 — 10 μsec. If we assume that in the vicinity of the walls the ions rotate in azimuth with a velocity $v_0 = a\omega_0 = T/MR_0 \sim 10^5$ (and that the electric drift velocity is of the same order of magnitude) while the minimum correlation length $\rho \sim 1$ cm, the oscillation period is approximately $\rho/v_0 \sim 10$ μsec, in agreement with experimental results.

The dependence of lifetime on density N can be determined from the experimental dependence of τ_0 on time.[6] For example, using the experimental $\tau = \tau(\mathrm{pt})$ curve for $p = 2 \times 10^{-6}$ mm Hg we can estimate the time for τ_0 to increase by a factor of e as 0.3 — 0.4 msec. But the mean lifetime for this interval (taking account of charge exchange) is approximately 0.15 msec. Thus, τ_0 increases in time in accordance with a function of the form $\tau_0 \sim N^{-1/2}$, which follows from Eq. (18) if $\Omega_0^2\Omega_H^2 < 1$, as is the case if $N < 10^9$.

7. CONCLUSION

In spite of the numerous simplifications which have been introduced, our theoretical analysis is in satisfactory agreement with the experimental results and both lead to the conclusion that plasma instability arises in a system of this kind as a consequence of convection. In order to avoid instability it is necessary to have a magnetic field configuration which increases in all directions as seen from the region occupied by plasma. One configuration of this kind, which has only two mirrors, is shown in Fig. 2. This field consists of a meridian field, with lines of force of which lie in the planes of r and z, and an azimuthal field which is produced by a current I along the axis. If the force line A′B′C′D′ of the meridian field has a negative curvature, as shown in Fig. 2, the field increases with increasing radial distance from this line. The field increases in the outward direction from the cross-hatched region at sections AB and CD in exactly the same way.

If we now choose a current I which makes the azimuthal field small at the line A′B′C′D′ but converts the force line ABCD into a rather sharp spiral in BC, than all the force lines at the boundary of the cross-hatched torroidal region exhibit negative curvature, that is to say, the field increases in all directions. Actually, the presence of a current I causes some elongation of the lines of force close to the axis of the system so that the longitudinal pressure of the plasma will tend to push the plasma tube toward the axis. But if the longitudinal pressure is smaller than the transverse pressure, effects due to elongation of the lines of force will be small and the plasma will be stable in the cross-hatched region, since a rotational diamagnetic force acts upon it.

In conclusion, the author wishes to express his gratitude to M. S. Ioffe and V. G. Tel'kovskiĭ, who maintained constant contact with this work.

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