THE INTERACTION OF FAST ELECTRON BEAMS WITH LONGITUDINAL PLASMA WAVES

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Equations are derived that yield the time variation of the plasma wave spectral density and of the fast electron distribution function for arbitrary velocity distributions. The dispersion relation for the stationary problem is solved, and the spatial extent of the exponential stage of the slowing down of the monochromatic beam is estimated.

1. There are many articles which deal with the generation of plasma oscillations by fast electron beams. It has been established that intense plasma oscillations can react on the beam and produce anomalous electron scattering, in which one observes a spatial localization of zones of intense plasma oscillation and zones of anomalous scattering.

In a recent paper Gabovich and Pasechnik[1] show that l, the distance traveled by an electron beam in a plasma before scattering, increases with a decrease in the beam current density, and that for a given l there exists a critical current density beyond which there occurs a considerable increase in fast-electron scattering. If the beam flux is more than critical, the average energy loss for an interaction with the plasma over a distance l amounts to 10 — 20%.

In the interpretation of their measurements, Gabovich and Pasechnik[1] refer to Vlasov[2] and Bohm and Gross,[3] who, in order to explain the anomalous scattering in the experiments of Merill and Webb,[4] assumed that an electron beam penetrating a plasma is velocity modulated by a fluctuating voltage ΔU. The beam modulation causes the formation of electron bunches and the appearance of intense plasma oscillations. Vlasov associates the voltage fluctuation with thermal surface oscillations of the plasma, the amplitude of which is on the order of the temperature T (in energy units). However, it can be shown that the amplitude of the surface oscillations is in fact many times less, $\approx T\sqrt{n_0\lambda_D^3}$ (where λ_D is the Debye length and n_0 is the electron density). Bohm and Gross attribute the appearance of the initial modulating voltage to the overall instability of the beam in the plasma and compare this voltage with the initial velocity spread in the beam. It is to be noted that the random fluctuations which are intensified by the action of the electron beam appear as longitudinal plasma waves. Their group velocity is directed along the beam, and for a semi-infinite plasma with no reflecting surfaces these waves cannot induce sufficient modulation at the penetration into the beam.* Besides, what is absent from the mechanism described by Vlasov and Bohm and Gross is the clear relationship between the scattering length l and the electron beam density n, a relationship observed by Gabovich and Pasechnik even when the frequency of the plasma oscillations was determined entirely by n_0 ($n_0 \gg n$).

To explain the anomalous phenomena in the interaction between a beam and plasma, one need not assume that there is a modulation voltage on the order of several volts at the plasma boundary; it is enough to assume that an electron beam in the plasma is unstable.[6,7] Because of this instability, the small disturbances (which always occur whenever the beam penetrates) grow exponentially with the distance from the boundary, and at a depth of about 1 cm their measured[1,4] amplitude becomes sufficient to effect anomalous beam scattering. We hasten to add that the above mechanism has no bearing on the interpretation of Looney and Brown's experiments,[8] in which the resonance properties of the system were sharply pronounced.

A theory of plasma wave excitation for two interpenetrating plasmas of the same density was developed in the hydrodynamic approximation by Kahn.[9] To explore the problem thoroughly, a kinetic equation must be used. This has been done by Klimontovich,[10] who has derived a kinetic equation describing the excitation and absorption processes for plasma waves. Similar equations, though of wider applicability, are derived in the present paper (Secs. 2 and 3). These are applicable even when the plasma oscillations are not in

*Gabor et al.[5] found a variable field at the plasma boundary with a large amplitude ($\approx$T) and with a frequency 4-5 times smaller than the Langmuir frequency. The nature of this field is not clear to us.

thermal equilibrium and the electron distribution function is non-Maxwellian.

Klimontovich developes a non-linear theory to explain why fast electron beams with a rather large velocity dispersion slow down in a plasma. However, the requirement that the fast electron beam be only slightly excited by the plasma waves induced by the beam renders Klimontovich's extension of the solution to steady waves incorrect. In Sec. 4 we derive equations that can be used to analyze the non-exponential growth in time of the plasma wave intensity, induced by beams with an arbitrary velocity distribution. It is shown that during the deceleration a strong scattering occurs in the beam, accompanied by the appearance of electrons with energies greater and smaller than the average initial energy. These equations, when applied to electron beams with a large velocity dispersion, reduce to the kinetic equations derived in Secs. 2 and 3. The accuracy of the equations is tested for a specific case in Sec. 5. The electric field in the equations that we have derived is characterized by the spectral density of plasma waves. Therefore, the phenomena of the velocity modulation of the entering beam and its subsequent bunching cannot be described by the proposed system of equations. In Sec. 7 we obtain the dispersion relation for the growth of the plasma wave intensity in the stationary linearized approximation and compare the theoretical and experimental values for the spatial extent of the linear stage of the deceleration.

In the following analysis it is assumed that the density of the fast electrons is much less than that of the plasma and that there are no external magnetic and electric fields.

2. Let us use the Focker-Planck equation to derive a kinetic equation for the beam electrons, in which we include the interaction with plasma waves, thus

$$\partial N/\partial t + u_i \partial N/\partial x_i + \partial I_i/\partial u_i = 0. \qquad (1)$$

Here I is the electron current in velocity space, which is found by computing the average decelerating force F_i and diffusion coefficient D_{ik}:

$$I_i = F_i N/m - \partial D_{ik} N/\partial u_k,$$

$$F_i = m\overline{\Delta u_i/\Delta t}, \qquad D_{ik} = \overline{\Delta u_i \Delta u_k/2\Delta t}.$$

The bar designates an average over the various possible variations in velocity Δu during time Δt.

N(u, r, t) is the electron distribution function and is obtained by averaging the exact distribution function over a volume whose linear dimension exceeds the plasma-oscillation wavelength that is characteristic for the problem. To describe the plasma oscillations we employ the plasma-wave spectral energy density $\epsilon(\mathbf{q}, \mathbf{r}, t)$, where q is the wave vector. The total plasma wave energy per unit volume, W, is given by:

$$W = \int \varepsilon\, d^3q.$$

The decelerating force F due to the collective interaction is composed of a force F_0, which is due to spontaneous plasma wave radiation ($\epsilon = 0$), and a force F_ϵ, which is proportional to the wave intensity. The force F_0 has been computed in different ways by various authors (see, for example, references 11 — 13) and is given by

$$F_0 = -(e^2\omega_0^2/u^2)\ln(q_{max}/q_{min}). \qquad (3)$$

The minimum wave number (or q_{min}) corresponds to waves radiated in the direction of motion of the particles ($\cos\theta = 1$) and is determined by the condition

$$\omega(q_{min})/q_{min}u = 1. \qquad (4)$$

Since plasma waves with $q > 1/\lambda_D$ are strongly damped, one can set q_{max} equal to $1/\lambda_D$.

To compute F_ϵ we need to find the average acceleration imparted to a particle by the electric field of the plasma wave, viz.,

$$\mathbf{F}_\varepsilon = m\frac{\overline{\Delta \mathbf{u}}}{\Delta t} = \frac{e}{\Delta t}\int_0^{\Delta t} \overline{\mathbf{E}(\mathbf{r}(t), t)\, dt},$$

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{u}_0 t + \frac{e}{m}\int_0^t \mathbf{E}(\mathbf{r}(t'), t')(t - t')\, dt'. \qquad (5)$$

The time interval Δt must satisfy the condition

$$1/\gamma \gg \Delta t \gg 1/\Delta q u, \qquad (6)$$

where γ is the logarithmic time derivative of the plasma wave amplitude, and Δq characterizes the width of the plasma wave spectrum.

Expanding (5) in a power series of the electric field vector E, we obtain for F_ϵ an expression which is proportional to ϵ, viz.,

$$\mathbf{F}_\varepsilon = \frac{4\pi^2 e^2 \omega_0^2}{m}\int \frac{\mathbf{q}}{\omega^2}\varepsilon\delta'(\mathbf{qu} - \omega)\, d^3q. \qquad (7)$$

To compute the diffusion coefficient D_{ik} we express the velocity increment in Eq. (2) in terms of the electric fields,

$$\overline{\Delta u_i \Delta u_k} = \frac{e^2}{m^2}\int_0^{\Delta t} dt \int_0^{\Delta t} \overline{dt' E_i(\mathbf{r}(t), t)\, E_k(\mathbf{r}(t'), t')}.$$

Effecting the above simple computation, we finally obtain

$$D_{ik}=\frac{4\pi e^2\omega_0^2}{m^2}\int\frac{q_iq_k}{q^2}\frac{\varepsilon}{\omega^2}\,\delta(\mathbf{qu}-\omega)\,d^3q. \tag{8}$$

Consequently the electron flux in velocity space is given by

$$I_i=-\frac{e^2\omega_0^2}{m}\left[\frac{u_iN}{u^3}\ln\frac{q_{max}}{q_{min}}+\frac{4\pi^2}{m}\frac{\partial N}{\partial u_k}\int\frac{q_iq_k}{q^2\omega^2}\varepsilon\delta(\mathbf{qu}-\omega)\,d^3q)\right]. \tag{9}$$

In deriving Eq. (9) we have neglected terms proportional to ϵ^2 in the series expansion of (5). This is permissible if the inequality

$$\int\varepsilon d^3q\ll(\Delta q/q)^4\,n_0mu^2 \tag{10}$$

is satisfied. Condition (10) applies even when the plasma wave spectrum differs appreciably from a thermal spectrum. In the particular case when the wave spectrum differs only slightly from equilibrium, Eq. (9) reduces to Klimontovich's.[10]

3. Let us set up an equation to define the plasma wave spectrum, i.e., an equation for the function $\epsilon(\mathbf{q},\mathbf{r},t)$. The variation of ϵ in time will be determined by the transfer of plasma wave quanta (plasmons) with the group velocity $\partial\omega/\partial\mathbf{q}$, by the spontaneous radiation of plasmons (a term independent of ϵ), and by their absorption or induced emission which is proportional to $\epsilon(\mathbf{q},\mathbf{r},t)$:

$$\frac{\partial\varepsilon}{\partial t}+\frac{\partial\omega}{\partial q_i}\frac{\partial\varepsilon}{\partial x_i}=A+B\varepsilon.$$

Here B is twice the growth rate of the plasma wave amplitude.

The quantity B is to be found from a dispersion relation that depends on the electron distribution function N(u). If the function N(u) can be expanded in a series in the complex plane near the point $\mathbf{q}\cdot\mathbf{u}=\omega$ and if moreover the value of the function at the pole $\mathbf{q}\cdot\mathbf{u}-\omega-i\gamma$ can be found from just the first term of the expansion, then the following expression is obtained for B,

$$B=2\gamma=\frac{4\pi\omega_0^2e^2}{m\omega q^2}\int q_i\frac{\partial N}{\partial u_i}\delta(\mathbf{qu}-\omega)\,d^3u. \tag{11}$$

In the particular case of a Maxwell distribution, the Landau[14] damping formula is obtained. A corresponding expression for γ was previously derived by Bohm and Gross.[6]

The next pure imaginary term in the expansion of N(u) can be neglected if the inequality

$$|u^6n_0^{-2}N'(u)\,N'''(u)|\ll 1 \tag{12}$$

is satisfied. For a beam of fast electrons of density n, moving at velocity u through a plasma with an electron density n_0 and possessing a velocity dispersion Δu, this inequality assumes the following form

$$(u/\Delta u)^3(n/n_0)\ll 1. \tag{13}$$

On the other hand, condition (6), when applied to a beam of fast electrons, has the following form

$$\Delta q/q\gg(n/n_0)\,(u/\Delta u)^2, \tag{14}$$

where, as in Eq. (10), Δq and q characterize the plasma wave spectrum. If the plasma waves are generated by the fast electrons themselves, then $\Delta q/q\approx\Delta u/u$, and conditions (10), (13), and (14) become equivalent.

The intensity of the spontaneous radiation is found by analyzing the motion of a charged particle in the plasma.[12]

Thus, for the kinetic equation for $\epsilon(\mathbf{q},\mathbf{r},t)$, we obtain

$$\frac{\partial\varepsilon}{\partial t}+\frac{\partial\omega}{\partial q_i}\frac{\partial\varepsilon}{\partial x_i}=\frac{e^2\omega_0^2}{2\pi q^2}\int N\delta(\mathbf{qu}-\omega)\,d^3u+\frac{4\pi^2e^2\omega_0^2}{m\omega q^2}\varepsilon\int q_i\frac{\partial N}{\partial u_i}\delta(\mathbf{qu}-\omega)\,d^3u. \tag{15}$$

Equations (1), (9), and (15) can be derived from a quantum representation of the plasma waves if the probabilities of emission and absorption by the electron of a quantum with frequency ω and wave vector $\mathbf{q}$ are assumed to be, respectively,[15]

$$W^+=\frac{\omega_0^2}{2\pi\omega q^2}(N_\omega+1),\qquad W^-=\frac{\omega_0^2}{2\pi\omega q^2}N_\omega,$$

where N_ω is the density in phase space of quanta with frequency ω.

It should be noted further that the description of plasmon transfer by a group velocity concept is justified provided that

$$\frac{\partial\varepsilon}{\partial t}\gg\frac{\partial\omega}{\partial q_i}\frac{\partial\varepsilon}{\partial x_i}. \tag{16}$$

If this inequality is not satisfied then the region of applicability of Eq. (15) is determined by a more stringent requirement on the velocity dispersion of the electron beam, viz.,

$$\Delta u/u\gg(n/n_0)^{1/3}(mu^2/T)^{1/6}. \tag{17}$$

Condition (17) defines simultaneously the boundaries of the quantum representation of plasma waves.

4. Equations (1), (9), and (15) are applicable when the electron distribution is a sufficiently smooth function of the velocity. Now let us derive equations for the time variation of the plasma wave spectral density and for the distribution function of fast electrons with an arbitrary velocity spread. We begin with the dispersion relation[14]

$$1=\frac{4\pi e^2i}{mq^2}\int_C q_i\frac{\partial N}{\partial u_i}\frac{d^3u}{i(\mathbf{qu}-\omega)+\gamma}. \tag{18}$$

We are interested in the case in which the electron distribution function can be represented as the sum of a Maxwellian distribution, $N_0(u)$, and a fast-electron distribution, $N(u, r, t)$, in which the average velocity is assumed to be considerably larger than the thermal velocity of the plasma. Since the distribution function for the fast electrons is different from zero only within a certain limited region of velocity space, this region is best separated in the dispersion relations where the integration is over u. Neglecting terms proportional to $(n/n_0)^{2/3}$ and $(a/\omega)^4$ where $a^2 = q^2T/2m$, we find from (18) that

$$1 - \frac{\omega_0^2}{\omega^2} - \frac{6a^2\omega_0^2}{\omega^4} = -\frac{\omega_0^2}{q^2 n_0}\int d^3u \frac{\omega - \mathbf{qu}}{\gamma^2 + (\omega - \mathbf{qu})^2} q_i \frac{\partial N}{\partial u_i}, \tag{19}$$

$$\gamma\left(1 + \frac{12a^2}{\omega^2}\right) = \frac{\omega^2}{2q^2 n_0}\int d^3u \frac{|\gamma|}{\gamma^2 + (\omega - \mathbf{qu})^2} q_i \frac{\partial N}{\partial u_i}. \tag{20}$$

The integration in (19) and (20) is to be performed over the region $\omega - \mathbf{q}\cdot\mathbf{u} \ll \omega_0$. These equations are applicable if ω and ω_0 differ only slightly, which is not the case when $q \ll \omega/u$, although such small values of q are essentially absent from the plasma-wave spectrum when the waves are generated by fast electrons.

In the case of a monochromatic electron beam ($(n/n_0)(u/\Delta u)^3 \gg 1$, $n \ll n_0$) with velocity u_0, Akhiezer and Faĭnberg[7] have found that

$$\gamma = \gamma_a - (\omega - \mathbf{qu}_0)^2/12\gamma_a,$$
$$\gamma_a = \sqrt{3}\cdot 2^{-4/3}(n/n_0)^{1/3}\omega_0^{2/3}(\tilde\omega - q\partial\tilde\omega/\partial q)^{1/3}. \tag{21}$$

Here $\tilde\omega$ represents the oscillation frequency for a Maxwellian distribution ($n = 0$):

$$\tilde\omega(q) = \omega_0 + \tfrac{3}{2}Tq^2/m\omega_0. \tag{22}$$

When the electron beam is not monochromatic ($(n/n_0)(u/\Delta u)^3 \ll 1$), then γ is determined by Eq. (11).

The derivation of Eqs. (24) and (25) which follow is based on the assumption that the electron distribution function changes more slowly than the plasma wave amplitude, i.e., that the system is "quasi-stationary." If $(n/n_0)(u/\Delta u)^3 \gg 1$, then one can show that the "quasi-stationary" approximation holds when

$$\int \varepsilon d^3q \ll (n/n_0)^{2/3}(\Delta u/u)^2 n_0 m u^2. \tag{23}$$

Despite the fact that the "quasi-stationary" conditions are essentially fulfilled only for electron beams that are sufficiently spread out in velocity, this approximation is useful in describing the entire slowing down process. In this case the equation for the plasma wave intensity is given by

$$\frac{\partial\varepsilon}{\partial t} + \frac{\partial\omega}{\partial q_i}\frac{\partial\varepsilon}{\partial x_i} = 2\gamma\varepsilon. \tag{24}$$

As before, the description of the plasma wave transfer by means of the group velocity concept is applicable if

$$\frac{\partial\omega}{\partial q}\frac{\partial\varepsilon}{\partial x} \ll \frac{\partial\varepsilon}{\partial t}.$$

This inequality is not satisfied, for example, in the case of the stationary problem whose solution is examined below. Equation (24) does not take into account the radiation of plasma waves by individual, randomly moving electrons, but this radiation is important only for finding the initial values of ϵ in the exponential stage of growth of the oscillation and during the approach to equilibrium.

The corresponding equation for the electron distribution function for a given spectral density $\epsilon(\mathbf{q}, \mathbf{r}, t)$ will be of the form

$$\partial N/\partial t + u_i \partial N/\partial x_i + \partial I_i/\partial u_i = 0, \tag{25}$$

where

$$I = \int d^3q \frac{A_{ik}}{\gamma^2 + (\omega - \mathbf{qu})^2}\frac{\partial N}{\partial u_k}. \tag{26}$$

A_{ik} is determined uniquely by stipulating that Eqs. (18), (24), (25), and (26) satisfy the conservation of energy and also that in the limit, when $(n/n_0)(u/\Delta u)^3 \ll 1$, Eq. (26) reduces to the corresponding term in Eq. (9). In this case we have

$$A_{ik} = -\frac{8\pi e^2}{m^2}\frac{q_i q_k \varepsilon(\mathbf{q})|\gamma|}{q^2[1 + \omega_0^2/\omega^2 + 18a^2\omega_0^2/\omega^4]}. \tag{27}$$

Unlike the linearized kinetic equations, Eqs. (24), (25), and (26) take into account the reaction of the plasma wave field on the fast electrons. On the other hand, the effects due to the nonlinearity and non-sinusoidality of the wave are not included in our equations. These effects can be described by terms proportional to ϵ^2 and are seemingly essential when the plasma waves die out.

5. Equation (25) describes the diffusion of the fast electrons in velocity space during their interaction with the plasma waves. In case the beam is almost monochromatic one can obtain, assuming $(\omega - \mathbf{q}\cdot\mathbf{u})^2 \ll \gamma^2$, the following expression for I_i,

$$I_i = -\frac{4\pi e^2}{m^2\gamma}\frac{\partial N}{\partial u_k}\int \varepsilon \frac{q_i q_k}{q^2} d^3q. \tag{28}$$

Let us assume that, in the main, waves are generated with a wave vector directed along the z axis, in a direction parallel to the velocity of the fast electrons, i.e., that $q_z \approx q$ and $q_r \ll q_z$. This occurs, for example, in the case of a fast electron beam with a small cross section. Neglecting, moreover, the density variation in a homogeneous electron beam due to scattering, we can write the equation for diffusion in the u_z direction as

$$\frac{\partial N}{\partial t} = \frac{4\pi e^2}{m^2\gamma}\int \varepsilon d^3q \frac{\partial^2 N}{\partial u_z^2}. \quad (29)$$

By means of (29) it is possible to determine the induced velocity dispersion, viz.,

$$\overline{\Delta u_z^2} = (4\pi e^2/m^2\gamma^2)W. \quad (30)$$

Equation (30) is derived from Eq. (25), whose applicability to a monochromatic beam has not been, strictly speaking, proved. Therefore, in order to determine the accuracy of Eq. (25), a numerical calculation was made of the variation in the velocity of single electrons in a beam initially uniform in space and monochromatic in velocity.*

In dimensionless variables the equation for the motion of an electron in the field of an exponentially increasing plasma wave is

$$d^2x/dt^2 = \alpha e^{\gamma t}\cos(x - t), \quad \alpha = 10^{-4}, \quad \gamma = 0{,}1.$$

The initial electron coordinate x_0 ($0 \le x_0 \le \pi$) and initial velocity, u_0 ($|u_0 - 1| < 10^{-3}$) were selected randomly. The value of $\alpha^{-2}(u-1)^2e^{-2\gamma t}$ at various times, which is shown below, was obtained by averaging over 135 different trajectories:

$t =$	0	10	20	30	40	50
$\alpha^{-2}(u-1)^2e^{-2\gamma t} =$	32	22.4	36.5	44	46.3	43.5

These results are in good agreement with Eq. (30), which gives

$$\alpha^{-2}(u-1)^2e^{-2\gamma t} = 1/2\gamma^2 = 50.$$

6. Let us now consider an idealized problem dealing with the growth of the instability in a uniform monochromatic beam of electrons moving in an infinite plasma of constant density. Let us imagine that the interaction between the fast electrons and plasma is switched on instantly at t = 0. For t < 0, only a thermal background of oscillations exists in the plasma these being due to the random movement of Maxwell electrons ($\epsilon = T/(2\pi)^3$. Once the interaction is turned on, the plasma wave intensity which increases exponentially is given by

$$\varepsilon(\mathbf{q}, t) = (2\pi)^{-3}\left[\tfrac{1}{9}\, 2^{2/3}(n/n_0)^{1/3}mu_0^2 + \tfrac{4}{9}T\right]e^{2\gamma t}. \quad (31)$$

The pre-exponential factor is obtained by solving the kinetic equation with the inclusion of the random motion of the electrons.

Integrating (31) over **q** we obtain the total intensity of the excited plasma waves. This result is applicable so long as it is possible to neglect the variation in γ due to the fact that the monochromaticity of the fast electron beam is disturbed. During this exponential growth stage a narrow region of wave vectors stand out for which q_z is close to ω/u_0, so that when ϵ is integrated over

*L. P. Strotseva and M. P. Bronnikova performed the computation for this problem.

q_z with γ given by Eq. (21), the dependence of γ_a on q_z becomes insignificant. The integration over q_r is to be restricted at some upper limit, $q_{r\,max}$, which in the case of a beam is determined by its diameter. Hence we find that

$$W = \int \varepsilon d^3q \sim \frac{1}{(2\pi)^3}\left[\frac{2^{2/3}}{9}\left(\frac{n}{n_0}\right)^{1/3}mu_0^2 + \frac{4}{9}T\right]\sqrt{\frac{6\gamma_a\pi^3}{t}}\,\frac{1}{u_0}q_{r\,max}^2 e^{2\gamma_a t}. \quad (32)$$

From geometric considerations we have that $q_{r\,max} \approx q_z d/u_0 t$, where d is the diameter of the beam. If the diameter of the beam is significantly larger than the path length to the scattering zone, then the integral over q_r converges when Landau damping and the dependence of γ_a on q are taken into account.

The average velocity loss by the beam, Δu, which follows from the conservation of total energy, is found from the relation

$$W = nmu_0\Delta u.$$

On the other hand, according to (30) the rms spread in velocities is proportional to the square root of W. Furthermore, bearing in mind that $\gamma \approx \omega_0(n/n_0)^{1/3}$, we have

$$\sqrt{\overline{(\Delta u)^2}} \sim (n_0/n)^{1/3}(W/mn_0)^{1/2}.$$

The mean velocity loss is comparable with the rms spread for $W \approx nmu_0^2(n/n_0)^{1/3}$, at which time γ already differs considerably from its original value and Eq. (32) becomes inapplicable. Assuming in (32) that $W = nmu_0^2(n/n_0)^{1/3}$, we can find the time τ, during which the oscillation energy increases exponentially,

$$\tau = L/2\gamma_a, \quad (33)$$

where L is the logarithm determined from Eq. (32).

7. In the case of the stationary linearized problem the solution for the growth of the longitudinal oscillations due to a beam of fast electrons in a semi-infinite plasma with specified boundary plasma oscillations can be determined from the dispersion relations derived by A. A. Luchina.[16] As applied to a monochromatic electron beam entering a plasma, the dispersion relation is given by

$$\frac{1}{\omega_0^2} - \frac{1}{\omega^2} = \frac{n}{n_0}\frac{1}{(\omega - ku)^2} + \frac{3Tk^2}{m\omega^4}, \quad (34)$$

where $k = q + i\mu$ is the complex wave vector whose imaginary part μ characterizes the spatial growth in the plasma wave intensity.

If the plasma temperature is zero, T = 0, which corresponds to the hydrodynamic description of two mutually penetrating beams, then $\mu_{max} = \infty$, i.e. there is no stationary solution. The value of μ_{max} is finite if the temperature of the plasma is different from zero. Provided that $T \ll mu^2$, the solution of Eq. (34) becomes

$$\mu_{max} = \frac{\omega_0}{u}\left(\frac{nmu^2}{6n_0T}\right)^{1/3}\frac{\sqrt{\mathrm{ch}^2t - 1/4}}{\mathrm{ch}^2 t}, \quad \mathrm{sh}\, 2t = \left(\frac{4nmu^2}{3n_0T}\right)^{1/2} \quad (35)^*$$

The spatial extent of the exponential growth of the plasma wave intensity, which corresponds to l, the distance to the first scattering zone in the electron beam, is found, as in (33), from the equation

$$l = L/2\mu. \quad (36)$$

Equations (35) and (36), which determine the dependence of the length l, on the plasma density and the beam density, are qualitatively confirmed by the experimental results. Unfortunately it is impossible to make a thorough quantitative comparison because we do not know accurately enough some of the quantities which occur in Eqs. (35) and (36). In the first place, Eq. (32) only sets the upper limit to the value for the logarithm ($L \approx 20$); in fact the true value of L could turn out to be somewhat smaller if the excitations on the plasma boundary were strengthened by reflection of plasma waves in the experimental apparatus. In the second place, our estimates disregard the finite energy spread in the electron beam, which would diminish the value of μ. These data are not given in the paper by Gabovich and Pasechnik.[1] Finally, the published results[1,4] do not contain the actual plasma temperature.

If, nevertheless, we neglect the initial velocity spread in the beam and assume T = 2 ev, then on the basis of the experimental data given by Merill and Webb,[4] the logarithm, L, for the two different measurements made by these authors proves to be 15 and 19 respectively, which can be considered as a completely satisfactory confirmation of Eq. (36). The value of L in the experiments of Gabovich and Pasechnik falls within the range of from 12 to 70 for the different beam and plasma parameters employed. This disagrees with what would be theoretically expected for this quantity ($15 < L < 30$). Evidently additional measurements are needed to definitely clarify the problem of the divergence between the theoretical and empirical values for the length, l, which differ in some cases by as much as a factor of two or three.

The concepts developed above also provide for an understanding of such experimentally observed phenomena accompanying anomalous scattering as the change in the monochromaticity of the beam, which causes the appearance of electrons with an energy greater than the initial, and the small value of the average energy lost by the beam in the first scattering zone. The appearance of a second scattering zone, which was especially sharply defined in the experiments performed by Merill and Webb,[4] should, apparently, be attributed to the modulation of the beam velocity and a subsequent bunching of the beam after passage through the first scattering zone.

*sh = sinh, ch = cosh.

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22