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PHOTOPRODUCTION OF π MESONS ON NUCLEONS IN PERIPHERAL COLLISIONS

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The production of π mesons in peripheral collisions between γ rays and nucleons is investigated. The photoproduction amplitudes are calculated using one-meson and two-meson approximations. The dispersion relations as well as the unitarity condition for the photoproduction amplitude and its analytical properties were used for this purpose. The deduced relations together with the corresponding experimental data make it possible to calculate the $(\gamma\pi\pi\pi)$ vertex function near the point $t = 4\mu^2$, and the pion-nucleon coupling constant.

1. INTRODUCTION

THE theory of processes involving strongly interacting particles has recently met with considerable success by making use of the analytical properties and the unitarity of the scattering amplitudes. It was shown[1,2] that, for a number of processes, it is sufficient to know the properties of the scattering amplitudes in the region close to their nearest singularities in the transferred momentum. Okun' and Pomeranchuk[3] showed that the nearest singularities correspond to the largest impact parameters, and this fact served as a basis for a method of calculation of the amplitudes corresponding to large values of orbital angular momentum.

In the present article, this method is used for the study of peripheral collisions of photons with nucleons accompanied by meson production. The basis of the method is the dispersion relation for transferred momentum.[4] For the process in question, the relation is

$$A(s,t) = A_0(s) + \frac{a_1}{t-\mu^2} + \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{A_1(s,t')}{t'-t}dt' + \frac{a_2}{u-m^2} + \frac{1}{\pi}\int_{(m+\mu)^2}^{\infty} \frac{A_2(s,u')}{u'-u}du'. \quad (1)$$

where $A(s,t)$ is one of the invariant photoproduction amplitudes (in a general case, photoproduction is described by four amplitudes[5]), a_1 and a_2 are constants, and

$$-s = (p_1+k_1)^2 = (p_2+k_2)^2, \quad -t = (p_2-p_1)^2 = (k_1-k_2)^2,$$

$$u = 2m^2 + \mu^2 - s - t,$$

where p_1 and p_2 are the initial and final four-momenta of the nucleon, k_1 is the momentum of the photon, and k_2 the momentum of the π meson. The coefficients of the amplitude expansion in terms of Legendre polynomials

$$a_l = \int_{-1}^{1} A(s,t) P_l(\varkappa) d\varkappa$$

(where $\kappa = \cos\theta$, and θ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$ in the c.m.s.) can be expressed in terms of Legendre polynomials of the second kind

$$Q_l(\varkappa_0) = \frac{1}{2}\int_{-1}^{1} \frac{P_l(\varkappa)}{\varkappa_0 - \varkappa} d\varkappa.$$

Using the relation $t = \mu^2 + 2\omega k(\kappa - 1/v)$, where μ is the π-meson mass, ω and k are the momenta in the c.m.s. of the photon and meson respectively, and $v = k/\sqrt{k^2+\mu^2}$ is the meson velocity, we obtain

$$a_l = -\frac{a_1}{\omega k} Q_l\left(\frac{1}{v}\right) + \frac{2}{\pi}\int_{\bar\varkappa}^{\infty} A_1(s,t') Q_l(\varkappa') d\varkappa' + \ldots, \quad \bar\varkappa = \frac{1}{v} + \frac{3\mu^2}{2\omega k}. \quad (2)$$

If, for a given l, we have $Q_l(\bar\kappa) \ll Q_l(1/v)$, then the main contribution is due to the first (polar) term in the scattering amplitude. The second term is important when $a_1 = 0$. The following two terms in Eq. (1) will not be considered since they correspond to still larger arguments of the function Q_l.

2. ONE-MESON APPROXIMATION

The polar term of the photoproduction amplitude can be obtained directly from the Feynman diagram in Fig. 1, corresponding to the exchange of one virtual meson. The vertices of this diagram are determined by the physical charge[6] of the meson, e, and the nucleon-nucleon interaction constant, g. We shall normalize the amplitude F_1 so that $d\sigma = |F_1|^2 do$ in the c.m.s. We then have

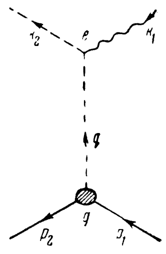

FIG. 1

$$F_1 = i\sqrt{2}\, eg \frac{m}{k\omega W}\sqrt{\frac{k}{\omega}}\, \frac{\mathbf{k}_2 \boldsymbol{\chi}}{1/v - \varkappa}\, \bar{u}_2 \gamma_5 u_1, \tag{3}$$

where χ is the unit vector of the photon polarization, ω is the photon momentum, k is the meson momentum, $W = E_1 + \omega = E_2 + k_{20}$ is the total energy (all in c.m.s.), m is the nucleon mass, $e^2 = 1/137$, and $g^2 = 0.08\,(2m/\mu)^2$; u_i are four-component spinors normalized so that $\bar{u}_i u_i = 1$. Expressing these spinors in terms of two-component unit spinors v_i, we obtain $F_1 = v_2^* R_1 v_1$, where

$$R_1 = \frac{eg}{\sqrt{2}}\sqrt{\frac{k}{\omega}}\, \frac{\sqrt{(m+E_1)(m+E_2)}}{k\omega W}\, \frac{(\mathbf{k}_2 \boldsymbol{\chi})}{1/v - \varkappa} \times \left(\frac{\sigma \mathbf{k}_2}{m+E_2} - \frac{\sigma \mathbf{k}_1}{m+E_1}\right). \tag{4}$$

It should be noted that Eq. (3) does not exhibit the transversality property. However, the longitudinal part of Eq. (4) does not contain a pole.

In order to find the amplitude corresponding to a state of the nucleon-meson system with a given angular momentum j and parity $(-1)^{l+1}$, it is necessary to expand $R_1 v_1$ in terms of spherical spinors[7]

$$R_1 v_1 = \sum_{jlM} a_{jlM}^{(1)}\, \Omega_{jlM}(\mathbf{k}_2/k). \tag{5}$$

Using Eq. (4) and the explicit expressions for Ω_{jlM} we obtain

$$a_{jlM}^{(1)} = -\frac{eg\sqrt{\pi}}{\omega W}\sqrt{\frac{k}{\omega}}\sqrt{(m+E_1)(m+E_2)} \times \Big\{\frac{\omega}{m+E_1}\sqrt{\frac{l(l+1)}{2l+1}}\,(-1)^{1/2-\nu}\, C_{l\lambda^{1}/_{2}\nu}^{jM} \times \Big[Q_{l-1}\Big(\frac{1}{v}\Big) - Q_{l+1}\Big(\frac{1}{v}\Big)\Big] + \frac{k}{m+E_2}\sqrt{\frac{l'(l'+1)}{2l'+1}}\, C_{l'\lambda^{1}/_{2}\nu}^{jM}\Big[Q_{l'-1}\Big(\frac{1}{v}\Big) - Q_{l'+1}\Big(\frac{1}{v}\Big)\Big]\Big\}, \tag{6}$$

where $l' = 2j - l$; $\lambda = \pm 1$ correspond to right- and left-handed polarizations of the photon; $\nu = \pm 1/2$; and $C_{l\lambda 1/2\nu}^{jM}$ are the vector addition coefficients. The indices λ and ν can assume only one value for a given M: for $M = \pm 3/2$, $\lambda = \pm 1$, $\nu = \pm 1/2$; for $M = \pm 1/2$, $\lambda = \pm 1$, $\nu = \mp 1/2$. The values of the coefficients $a_{jlM}^{(1)}$ are given in Table I.

The polar term of the type discussed is contained only in the photoproduction amplitude of one type. Firstly, it is absent in the neutral π-meson production amplitude. Secondly, it corresponds to only one of the four possible types of the amplitude variation with the polarization of the nucleons and of the photon. [It can be seen from Eq. (3) that, e.g., F_1 vanishes if the meson moves in the direction of motion of the photon.] In order to find the other amplitudes, and also to estimate the accuracy of the one-meson approximation, it is necessary to analyze the integral term in Eq. (1). An analogous study in the case of nucleon-nucleon scattering was carried out by Galanin, Grashin, Ioffe, and Pomeranchuk.[8]

3. TWO-MESON APPROXIMATION

In order to study the integral term in Eq. (1), it is necessary to make use of the fact that, for $t > 4m^2$, $s < 0$, and $u < 0$, the quantity A(s, t) represents the amplitude for the production of a

TABLE I. Values of the coefficients $a_{jlM}^{(1)}$ in units of $10^{-4} eg/\mu$

	ω, Mev	152.5	178.7	244.5
	$1/v$	2	1.5	1.2
$a_{l+1/2,\,l,\,-3/2}^{(1)} = -a_{l+1/2,\,l,\,3/2}^{(1)}$	$l=2$	137	282	479
	$l=3$	32.6	93.5	215
	$l=4$	8.14	32.8	104
	$l=5$	2.08	11.9	51.3
$-a_{l-1/2,\,l,\,-3/2}^{(1)} = -a_{l-1/2,\,l,\,3/2}^{(1)}$	$l=2$	298	607	1020
	$l=3$	56.7	161	365
	$l=4$	12.7	51.0	158
	$l=5$	3.06	17.3	73.8
$-a_{l-1/2,\,l,\,-1/2}^{(1)} = -a_{l-1/2,\,l,\,1/2}^{(1)}$	$l=2$	85.9	159	214
	$l=3$	24.0	63.1	120
	$l=4$	6.55	24.6	65.7
	$l=5$	1.78	9.54	35.8
$-a_{l+1/2,\,l,\,-1/2}^{(1)} = a_{l+1/2,\,l,\,1/2}^{(1)}$	$l=2$	88.6	163	220
	$l=3$	23.5	62.0	117
	$l=4$	6.27	23.6	62.7
	$l=5$	1.67	8.98	33.7

nucleon-antinucleon pair in a photon-meson collision: $\gamma + \pi \rightarrow N + \bar{N}$. In that range, $A_1(s, t) = \operatorname{Im} A(s, t)$, and, in order to find this, one can use the unitarity relation

$$2 \operatorname{Im} \langle N\bar{N} | \gamma\pi \rangle = \sum_n \langle n | N\bar{N} \rangle \langle n | \gamma\pi \rangle^*,$$

where $\langle b | a \rangle$ denote the elements of the scattering matrix $T = -i(S-1)$, which differ from the corresponding amplitudes by normalization factors. The function $A_1(s, t)$ calculated from this relation is analytic, and can therefore be continued in the region $t < 4m^2$. In the region $4\mu^2 < t < 9\mu^2$, the only state n is the two-meson state, and the unitarity condition is represented by the diagram in Fig. 2. In this diagram, the vertices represent the analytical continuation of the amplitudes of annihilation $N\bar{N} \rightarrow \pi\pi$ or of scattering $\pi N \rightarrow \pi N$, and of the amplitude of photoproduction of mesons on mesons $\gamma\pi \rightarrow \pi\pi$. Let us introduce the notation

$$\langle N\bar{N} | \gamma\pi \rangle = (2\pi)^4 \delta^{(4)} (p_1 + k_1 - p_2 - k_2)\, mB / 2\sqrt{E_1 E_2 \omega k_{20}},$$
$$\langle \pi\pi | N\bar{N} \rangle = (2\pi)^4 \delta^{(4)} (p_1 + f_1 - p_2 - f_2)\, mB^{(\pi)} / 2\sqrt{E_1 E_2 \mathcal{E}_1 \mathcal{E}_2}$$
$$\langle \pi\pi | \gamma\pi \rangle^* = (2\pi)^4 \delta^{(4)} (k_1 + f_2 - k_2 - f_1)\, B^{(\gamma)} / 4\sqrt{\mathcal{E}_1 \mathcal{E}_2 \omega k_{20}}.$$

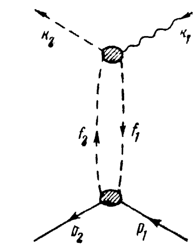

FIG. 2

where p_1 and p_2 are the nucleon four-momenta, E_1 and E_2 the nucleon energies, k_1 is the photon four-momentum, k_2 is the meson four-momentum, ω and k_{20} are the photon and meson energies respectively, f_1 and f_2 are the four-momenta of the intermediate mesons, and $\mathcal{E}_1$ and $\mathcal{E}_2$ their energies. The unitarity condition can now be written in the form

$$B_1 = \frac{1}{32\pi^2} \sqrt{\frac{t - 4\mu^2}{t}} \int B^{(\gamma)} B^{(\pi)} do,$$
$$B_1 = \operatorname{Im} B \text{ for } t > 4m^2,\ s < 0,\ u < 0, \tag{7}$$

where do is the solid angle element in the c.m.s. corresponding to the annihilation channel $\mathbf{p}_1 = \mathbf{p}_2$, $\mathbf{f}_1 = \mathbf{f}_2$, $\mathbf{k}_1 = \mathbf{k}_2$. Introducing the invariants

$$z = \frac{\nu}{t - \mu^2} \sqrt{\frac{t}{t - 4m^2}}, \quad z_1 = \frac{\nu_1}{\sqrt{(t - 4m^2)(t - 4\mu^2)}},$$
$$z_2 = \frac{\nu_2}{t - \mu^2} \sqrt{\frac{t}{t - 4\mu^2}}$$

(which correspond in the c.m.s. to the cosine of the angles between the vectors $\mathbf{p}_1, \mathbf{k}_1$; $\mathbf{p}_1, \mathbf{f}_1$; $\mathbf{f}_1, \mathbf{k}_1$) where, moreover,

$$\nu = PK, \quad \nu_1 = Pf, \quad \nu_2 = fK$$
$$(P = p_1 + p_2,\ K = k_1 + k_2, \quad f = f_1 + f_2)$$

we can write Eq. (7) in the following form:

$$B_1 = \frac{1}{32\pi^2} \sqrt{\frac{t - 4\mu^2}{t}} \iint B^{(\gamma)} B^{(\pi)} \times (1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2)^{-1/2} dz_1\, dz_2, \tag{8}$$

where the integration is carried out over the area of the ellipse determined by the zero of the expression under the radical sign.

The structure of the amplitude $B^{(\gamma)}$ is uniquely determined by the condition that it should be a pseudoscalar quantity containing linearly the photon polarization vector χ and depending on the vectors K, f, and $q = k_1 - k_2$. The only such quantity is

$$B^{(\gamma)} = e_{\alpha\beta\lambda\mu} \chi_\alpha q_\beta K_\lambda f_\mu A^{(\gamma)} (\nu_2, t), \tag{9}$$

where $A^{(\gamma)}$ is an invariant.

The amplitude $B^{(\pi)}$ has the following general form:

$$B^{(\pi)} = \bar{u}_2 \{ iA^{(1)} (\nu_1, t) + \hat{f} A^{(2)} (\nu_1, t) \} u_1. \tag{10}$$

In Eq. (9) and (10) we have omitted the dependence on isotopic variables. This dependence is determined by the fact that two intermediate mesons should be found in the state with isospin $T = 1$. In fact, for $T = 2$, the meson could not have been emitted by nucleons; for $T = 0$, they are in an even-charge state and therefore could not have been emitted by a photon with its consecutive transformation into a π meson (which also has an even charge). For $T = 1$, all three mesons are in an odd-charge state, and the change in the isotopic state of the nucleon is determined by the charge of the emitted meson.

We thus have

$$B_1(\nu, t) = \frac{1}{32\pi^2} \sqrt{\frac{t - 4\mu^2}{t}}\, e_{\alpha\beta\lambda\mu} \chi_\alpha q_\beta K_\lambda \int A^{(\gamma)} (\nu_2, t) f_\mu \times \bar{u}_2 [iA^{(1)} (\nu_1, t) + \hat{f} A^{(2)} (\nu_1, t)]\, u_1 do. \tag{11}$$

The quantity B_1 is used for calculating an integral of the type of Eq. (2), in which the region near the lower limit $t = 4\mu^2$ makes the important contribution. Since, in Eq. (11), the interval of integration over ν_1 $(-1 \leq z_i \leq 1)$ is of the order of $\sqrt{t - 4\mu^2}$, we can use, instead of the functions which do not have singularities near this region, the first terms of their expansion at the point $\nu_i = 0$. From these considerations, we shall make the substitution

$$A^{(\gamma)}(\nu_2,\ t)\approx A^{(\gamma)}(0,\ t),\qquad A^{(1)}(\nu_1,\ t)\approx A^{(1)'}(0,\ t)\,\nu_1$$

($A^{(1)}(0, t) = 0$ because the function $A^{(1)}$ is odd with respect to ν_1.) Such an approximation is, however, not applicable for the function $A^{(2)}(\nu_1, t)$, since that function has no pole for $\nu_1 = \pm(t-2\mu^2)$. Separating the polar term explicitly, let us write $A^{(2)}$ in the form

$$A^{(2)}(\nu_1,\ t)=A^{(p)}(\nu_1,\ t)+A^{(3)}(0,\ t),$$

$$A^{(p)}(\nu_1, t)=4\pi g^2\left(\frac{1}{t-2\mu^2-\nu_1}+\frac{1}{t-2\mu^2+\nu_1}\right). \quad (12)$$

Thus

$$B_1=\frac{1}{32\pi^2}\sqrt{\frac{t-4\mu^2}{t}}\,e_{\alpha\beta\lambda\mu}\chi_\alpha q_\beta K_\lambda A^{(\gamma)}$$

$$\times(0,\ t)\bar{u}_2\Big\{iA^{(1)'}(0,\ t)\int f_\mu\nu_1 do$$

$$+A^{(3)}(0,\ t)\int f_\mu\hat{f}\,do+\int f_\mu\hat{f}A^{(p)}(\nu_1,\ t)\,do\Big\}u_1. \quad (13)$$

In view of the fact that the integrand in Eq. (13) does not depend on ν_2, we have

$$\int do\ldots=2\pi\int_{-1}^{1}dz_1\ldots.$$

and the integration can be carried out easily.

The amplitude $A^{(\gamma)}$ in Eq. (13) represents the physical amplitude for the photoproduction of mesons on mesons. Unfortunately, it is not possible to say anything definite about its value. An analysis of the meson photoproduction on nucleons and a comparison with the results of the present calculation may enable us to find its value. It should be noted that, in effect, our results will depend on the value of $A^{(\gamma)}$ at the one point $t = 4\mu^2$, $\nu = 0$.

The quantities $A^{(1)'}$ and $A^{(3)}$ represent the amplitudes for the scattering of mesons on nucleons in a non-physical region as a function of both variables t and ν_1. They were calculated in reference 8 by using the dispersion relations and experimental data. It should be noted that the procedure of analytical continuation of $A^{(1)}$ and $A^{(3)}$ from the physical region $t < 0$ used in reference 8 can be carried out only up to the region close to $t = 4\mu^2$. Our results will therefore be correct only to that extent in which the region close to the lower limit will be the only important one in integrals (2). The results obtained in reference 8 are as follows:

$$A^{(1)'}(0.4\,\mu^2)=-0.029g^2/m\mu^2,$$

$$A^{(3)}(0.4\mu^2)=0.025g^2/2\mu^2.$$

These coefficients are small, and, since the integrals next to them in Eq. (13) are proportional to $(t-4\mu^2)^{3/2}$ the contribution of these terms to B_1 will be small. We may, consequently, limit ourselves to the polar term in meson scattering $A^{(p)}$. Thus

$$B_1=\frac{1}{4}g^2\sqrt{\frac{t-4\mu^2}{t}}\,e_{\alpha\beta\lambda\mu}\chi_\alpha q_\beta K_\lambda A^{(\gamma)}(0,\ t)$$

$$\times\bar{u}_2\int_{-1}^{1}dz_1 f_\mu\hat{f}\left(\frac{1}{t-2\mu^2-\nu_1}+\frac{1}{t-2\mu^2+\nu_2}\right)u_1. \quad (14)$$

Carrying out the integration in Eq. (14), we obtain the following expression for B_1

$$B_1=\frac{1}{4}g^2\,\frac{A^{(\gamma)}(0.4\mu^2)}{2i\mu m\,[(1+x)(1-\varepsilon^2-\varepsilon^2x)]^{1/2}}\,e_{\alpha\beta\lambda\mu}\chi_\alpha q_\beta K_\lambda\bar{u}_2$$

$$\times(c_1\gamma_\mu+c_2P_\mu\hat{P})\,u_1, \quad (15)$$

where

$$c_1=4i\mu^2\Big\{-\frac{\varepsilon(1+2x)\,[(1-\varepsilon^2-\varepsilon^2x)\,x]^{1/2}}{2(1-\varepsilon^2-\varepsilon^2x)}$$

$$+x\tan^{-1}\frac{2\,[x(1-\varepsilon^2-\varepsilon^2x)]^{1/2}}{\varepsilon(1+2x)}$$

$$+\frac{\varepsilon^2}{4}\frac{(1+2x)^2}{(1-\varepsilon^2-\varepsilon^2x)}\tan^{-1}\frac{2\,[x(1-\varepsilon^2-\varepsilon^2x)]^{1/2}}{\varepsilon(1+2x)}\Big\},$$

$$c_2=\frac{i\varepsilon^2}{1-\varepsilon^2-\varepsilon^2x}\Big\{-\frac{3\varepsilon(1+2x)\,[x(1-\varepsilon^2-\varepsilon^2x)]^{1/2}}{2(1-\varepsilon^2-\varepsilon^2x)}$$

$$+x\tan^{-1}\frac{2\,[x(1-\varepsilon^2-\varepsilon^2x)]^{1/2}}{\varepsilon(1+2x)}$$

$$+\frac{3\varepsilon^2}{4}\frac{(1+2x)^2}{1-\varepsilon^2-\varepsilon^2x}\tan^{-1}\frac{2\,[x(1-\varepsilon^2-\varepsilon^2x)]^{1/2}}{\varepsilon(1+2x)}\Big\}.$$

Let us denote by F_2 the amplitude of the process in the two-meson approximation, which is determined in the c.m.s. from the relation $d\sigma = |F_2|^2 do$. The discontinuity in the amplitude ΔF_2 near the point $t = 4\mu^2$, arising when t traverses a discontinuity along the real axis from the point $t = 4\mu^2$ to infinity, can, using Eq. (15), be written in the following form (δ is the isotopic index):

$$\Delta F_2^\delta=-\frac{g^2}{4\pi}\frac{mA^{(\gamma)}(0{,}4\mu^2)}{W}\sqrt{\frac{k}{\omega}}\,\varepsilon T_\delta\bar{u}_2\Big\{\frac{W}{m}(\mathbf{k}_1[\chi\times\mathbf{k}_2])\,J_2$$

$$-[\omega\gamma[\mathbf{k}_2\times\chi]+\gamma_0\mathbf{k}_1[\chi\times\mathbf{k}_2]+k_{20}\gamma[\chi\times\mathbf{k}_1]]\,J_1\Big\}u_1, \quad (16)$$

where

$$J_1=-\frac{\varepsilon}{2}\sqrt{x}+x\tan^{-1}\frac{2\sqrt{x}}{\varepsilon}+\frac{\varepsilon^2}{4}\tan^{-1}\frac{2\sqrt{x}}{\varepsilon},$$

$$J_2=-\frac{3\varepsilon}{2}\sqrt{x}+x\tan^{-1}\frac{2\sqrt{x}}{\varepsilon}+\frac{3\varepsilon^2}{4}\tan^{-1}\frac{2\sqrt{x}}{\varepsilon}.$$

Expressing u_1 and u_2 through two-component spinors v_1 and v_2, we obtain

$$\Delta F_2^\delta=-\frac{g^2}{4\pi}\frac{mA^{(\gamma)}(0.4\mu^2)\,\varepsilon}{W}\sqrt{\frac{k}{\omega}}\,T_\delta\,\frac{\sqrt{(m+E_1)(m+E_2)}}{2m}$$

$$\times v_2^*\Big\{-\frac{W}{m}(\mathbf{k}_1[\mathbf{k}_2\times\chi])\,J_2\Big[1-\frac{(\mathbf{k}_1\mathbf{k}_2)}{(m+E_1)(m+E_2)}$$

$$- i \frac{\sigma[\mathbf{k}_2 \times \mathbf{k}_1]}{(m+E_1)(m+E_2)}\Big]$$

$$+ J_1 \Big[ib\sigma\chi + i\frac{\mathbf{k}_2\chi}{m+E_2}\sigma(\omega\mathbf{k}_2 - k_{20}\mathbf{k}_1) + \Big(1 + \frac{\omega}{m+E_1}$$

$$+ \frac{k_{20}}{m+E_2} + \frac{\mathbf{k}_2\mathbf{k}_1}{(m+E_1)(m+E_2)}\Big)\mathbf{k}_1[\mathbf{k}_2 \times \chi]$$

$$+ i\frac{\mathbf{k}_1[\mathbf{k}_2\times\chi]}{(m+E_1)(m+E_2)}\sigma[\mathbf{k}_2\times\mathbf{k}_1]\Big]\Big\} v_1,$$

$$b = (\omega\mathbf{k}_2\mathbf{k}_1 - k_{20}\omega^2)/(m+E_1) + (k_{20}\mathbf{k}_2\mathbf{k}_1 - \omega k^2)/(m+E_2). \tag{17}$$

Let us write F_2 in the form $F_2 = v_2^* R_2 v_1$. The amplitude corresponding to the state with angular momentum j and parity $(-1)^{l+1}$ is given by the expression

$$a_{jlM}^{(2)} = \int \Omega_{jlM}^* \left(\frac{\mathbf{k}_2}{k}\right) R_2 v_1 do, \tag{18}$$

where the integration is carried out over the angles determining the direction $\mathbf{k}_2$. Assuming that $l \gg 1$ and $l\xi \gg 1$ ($\xi = \mu/\omega < 1$), we bring the integral over $d\kappa$ to the form

$$\int_{-1}^{1} P_l(\varkappa) R_2 d\varkappa = \frac{2\mu^2}{\pi i\omega k} Q_l(\varkappa_2) \int_0^\infty \exp(-L_l x)\Delta R_2 dx. \tag{19}$$

where κ_2 corresponds to the point $t = 4\mu^2$, and

$$L_l = (l+1)F = 4\xi(l+1)[16 + 9\xi^2 + 6\xi^2/(1+\sqrt{1+\xi^2/\varepsilon^2})]^{-1/2}.$$

A small region of integration near the lower limit is important in Eq. (19). It is therefore necessary to know the amplitude in the vicinity of the point $t = 4\mu^2$. The size of the effective integration interval is $\kappa \sim 1/L_l$ or $t - 4\mu^2 \sim 4\mu^2/L_l \ll 4\mu^2$. Let us substitute Eq. (19) into Eq. (18) and integrate over $d\varphi$ using Eq. (17). In the expression thus obtained, there remain integrals of the type

$$\int_0^\infty \exp(-L_l x) J_i dx.$$

Substituting the explicit expression for J_1, J_2, we obtain

$$\int_0^\infty \exp(-L_l x) J_1 dx = -\frac{\pi\varepsilon}{8L_l}\left(\frac{2}{\sqrt{\pi L_l}} - \frac{\varepsilon}{2} + \frac{2\varepsilon}{3\sqrt{\pi}}\zeta_l\right),$$

$$\int_0^\infty \exp(-L_l x) J_2 dx = -\frac{3\pi\varepsilon}{8L_l}\left(\frac{4}{3\sqrt{\pi L_l}} - \frac{\varepsilon}{2} + \frac{8\varepsilon}{9\sqrt{\pi}}\zeta_l\right). \tag{20}$$

The final result is expanded in terms of the parameter $\zeta_l = \epsilon\sqrt{L_l}/2$, which means that we consider $l \ll 4/\epsilon^2 \approx 180$.

Thus, the final expression for $a_{jlM}^{(2)}$ is

$$a_{l+1/2,\,l,\,-3/2}^{(2)} = -a_{l+1/2,\,l,\,3/2}^{(2)} = A\beta(1/v)\Phi_1(1/v),$$
$$a_{l-1/2,\,l,\,-3/2}^{(2)} = a_{l-1/2,\,l,\,3/2}^{(2)} = A\beta(1/v)\Phi_2(1/v),$$
$$a_{l+1/2,\,l,\,-1/2}^{(2)} = -a_{l+1/2,\,l,\,1/2}^{(2)} = A\beta(1/v)\Phi_3(1/v),$$
$$a_{l-1/2,\,l,\,-1/2}^{(2)} = a_{l-1/2,\,l,\,1/2}^{(2)} = -A\beta(1/v)\Phi_4(1/v). \tag{21}$$

Where the following notation is used:

$$A = \frac{\mu^2\varepsilon g^2 A^{(\gamma)}(0, 4\mu^2)}{4\sqrt{2}\,\pi}(T_\delta^{(+)}A^{(+)} + T_\delta^{(-)}A^{(-)} + T_\delta^{(0)}A^{(0)}),$$

$$\beta = (\varepsilon/4FW)[\pi k(m+E_1)(m+E_2)/\omega]^{1/2},$$

$$\Phi_1 = \sqrt{l(l+1)(l+2)}\Big\{\frac{3W}{m}\frac{1}{2l+1}(S_{l-1} - S_{l+1}) - (1+h_1) \times \frac{1}{2l+1}(N_{l-1} - N_{l+1}) - \frac{3W}{m}h_1h_2\frac{1}{2l+3}(S_l - S_{l+2}) - h_2(1+h_1)\frac{1}{2l+3}(N_l - N_{l+2})\Big\}, \tag{22}$$

$$\Phi_2 = \sqrt{(l-1)l(l+1)}\Big\{\frac{3W}{m}\frac{1}{2l+1}(S_{l-1} - S_{l+1}) - (1+h_1) \times \frac{1}{2l+1}(N_{l-1} - N_{l+1}) - \frac{3Wh_2}{m}\frac{h_1}{2l-1}(S_{l-2} - S_l) - h_2(1+h_1)\frac{1}{2l-1}(N_{l-2} - N_l)\Big\}, \tag{23}$$

$$\Phi_3 = \sqrt{l+1}\Big\{\frac{3W}{m}\frac{l}{2l+1}(S_{l-1} - S_{l+1}) - (1-h_1)\frac{l}{2l+1} \times (N_{l-1} - N_{l+1}) + 2\Big(h_1 + \frac{k_{20}}{m+E_2}\Big)N_{l+1} - \frac{3W}{m}h_1h_2\frac{l+2}{2l+3}(S_l - S_{l+2}) + h_2(1-h_1)\frac{l+2}{2l+3}(N_l - N_{l+2}) - \frac{2}{k}(k_{20}h_1 + kh_2)N_l\Big\}, \tag{24}$$

$$\Phi_4 = \sqrt{l}\Big\{\frac{3W}{m}\frac{l+1}{2l+1}(S_{l-1} - S_{l+1}) - (1-h_1)\frac{l+1}{2l+1} \times (N_{l-1} - N_{l+1}) - 2\Big(h_1 + \frac{k_{20}}{m+E_2}\Big)N_{l-1} - \frac{3W}{m}h_1h_2\frac{l-1}{2l-1}(S_{l-2} - S_l) + h_2(1-h_1)\frac{l-1}{2l-1}(N_{l-2} - N_l) + \frac{2}{k}(k_{20}h_1 + kh_2)N_l\Big\},$$

$$S_l = \frac{1}{l+1}Q_l(\varkappa_2)\left(\frac{4}{3\sqrt{\pi L_l}} - \frac{\varepsilon}{2} + \frac{8\varepsilon}{9\sqrt{\pi}}\zeta_l\right),$$

$$N_l = \frac{1}{l+1}Q_l(\varkappa_2)\left(\frac{2}{\sqrt{\pi L_l}} - \frac{\varepsilon}{2} + \frac{2\varepsilon}{3\sqrt{\pi}}\zeta_l\right),$$

$$h_1 = \omega/(m+E_1), \quad h_2 = k(m+E_2). \tag{25}$$

The values of $T_\delta^{(+)}$, $T_\delta^{(-)}$, and $T_\delta^{(0)}$ are shown in Table III, which is taken from reference 5. The values of $\beta(1/v)\Phi_n(1/v)$ ($n = 1, 2, 3, 4$) are presented in Table II.

CONCLUSION

A rough estimate of the two-meson amplitude can be made if we assume the cross section for the photoproduction of mesons on mesons to be

TABLE II.

	ω, Mev	152.5	178.7	244.5		ω, Mev	152.2	178.7	244.5
	$1/v$	2	1,5	1,2		$1/v$	2	1,5	1,2
$\beta\,\Phi_1 \cdot 10^{-6}$	$l=3$	53.7	298	2850	$\beta\,\Phi_3 \cdot 10^{-6}$	$l=3$	45.5	254	2380
	$l=4$	3.71	32.9	525		$l=4$	3.31	29.3	463
	$l=5$	0.289	4,08	109		$l=5$	0.265	3.75	99.0
$\beta\,\Phi_2 \cdot 10^{-7}$	$l=3$	—105	—533	—4470	$\beta\,\Phi_4 \cdot 10^{-6}$	$l=3$	46.2	267	2210
	$l=4$	1.42	18.8	362		$l=4$	3.38	28.2	398
	$l=5$	0.447	7.19	206		$l=5$	0.258	3.45	81.8

TABLE III.

	$\gamma+p\rightarrow\pi^0+p$	$\gamma+n\rightarrow\pi^0+n$	$\gamma+p\rightarrow\pi^++n$	$\gamma+n\rightarrow\pi^-+p$
$T^{(+)}$	1	1	0	0
$T^{(-)}$	0	0	$\sqrt{2}$	$-\sqrt{2}$
$T^{(0)}$	1	—1	$\sqrt{2}$	$\sqrt{2}$

$\sim e^2/\mu^2$. In that case, $A^{(\gamma)}(0.4\mu^2) \sim 4\pi e/\mu^3$, and the two-meson amplitude already makes a negligible contribution for $l \gtrsim 3$ as compared with the one-meson amplitude.

Eq. (21) contains three constants $A^{(+)}$, $A^{(-)}$, and $A^{(0)}$. In fact, the photoproduction process is characterized by a single constant in each particular case. For the photoproduction of π^0 mesons on protons, $\gamma p \rightarrow \pi^0 p$, this constant is $A^{(+)} + A^{(0)}$; for the $\gamma n \rightarrow \pi^0 n$ process it is $A^{(+)} - A^{(0)}$; and for the processes $\gamma p \rightarrow \pi^+ n$ and $\gamma n \rightarrow \pi^- p$ the constant is $\sqrt{2}\,(A^{(0)} + A^{(-)})$ and $\sqrt{2}\,(A^{(0)} - A^{(-)})$ respectively. $A^{(+)}$, $A^{(-)}$, and $A^{(0)}$ can be determined from the study of several different processes.

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