

*ANOMALIES OF ELECTRON CHARACTERISTICS OF A METAL
IN THE HIGH PRESSURE REGION*

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A peculiar "electron transition" due to variation of the topology of the Fermi surface during its continuous deformation may occur in metals at high pressures. At the point of such a transition, the electron-state density near the boundary surface, and the electron dynamics also possesses some peculiar features which lead to anomalies of the electron characteristics of the metal (thermodynamic and kinetic). The most characteristic anomalies in the vicinity of the "electron transition" point at low temperatures are investigated. The possibility of an isomorphic first-order transition near the "electron transition" point is discussed.

INTRODUCTION

In connection with the peculiar features of the energy spectrum of conduction electrons in metal, characteristic anomalies of thermodynamic and kinetic quantities can take place at low temperatures in the region of high pressures. The nature of this phenomenon consists of the following.

As is well known, the electron state density $\nu(\epsilon) = dN(\epsilon)/d\epsilon$ of conduction electrons is connected with the form of the constant energy surfaces in momentum space, $\epsilon(\mathbf{p}) = \epsilon$. Those values of the energy $\epsilon = \epsilon_k$, for which the topology of these surfaces changes [for example, an open surface changes to a closed surface by disruption of a "neck" (Fig. 1a), or a new split-off region of surface appears (Fig. 1b)], correspond to peculiarities of the surface density $\nu(\epsilon)$. At the same time the "critical" surface $\epsilon(\mathbf{p}) = \epsilon_k$ contains singular points, near which the electron dynamics has an unusual character.

Generally speaking, the value of ϵ_k is located sufficiently far from the chemical potential of the electrons ζ , and one can ascertain the presence of singular points ϵ_k only by the x-ray spectrum. However, if there exists any continuously changing parameter whose variation causes the difference $\zeta - \epsilon_k$ to pass through zero (i.e., changes the topology of the Fermi boundary surface), then the peculiarities of the spectrum density $\nu(\epsilon)$ and the electron dynamics close to the "critical" surface $\epsilon(\mathbf{p}) = \epsilon_k$ lead to singular anomalies of thermodynamic and kinetic characteristics of the electron gas in the metal.

Deformation of the lattice, and in particular

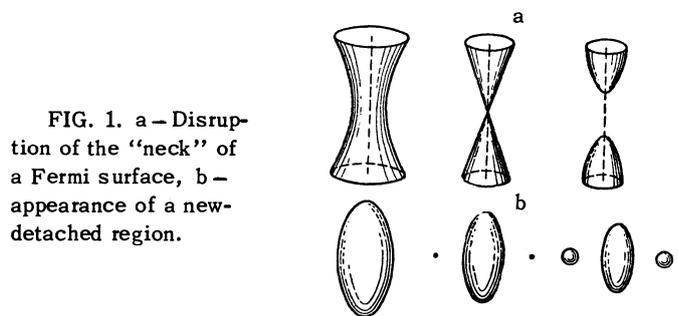


FIG. 1. a - Disruption of the "neck" of a Fermi surface, b - appearance of a new detached region.

hydrostatic compression at high pressures, can serve as such a continuous parameter. It is well known that at high pressures the anisotropy of a large number of properties is reduced, and therefore one can expect that a Fermi surface of the "corrugated cylinder" type, which is characteristic for layered structures, should go over, by gradual deformation, into a closed surface [even if the total number of electrons in the conduction band is constant (see Fig. 2a, b, c)]. It appears that other examples of variation of the topology of the Fermi surface are possible. It should be emphasized that a change of the topology of the Fermi surface is not connected with a change in the symmetry of the lattice, and therefore does not correspond to a phase transition of second order.

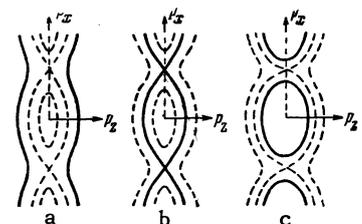


FIG. 2. Gradual transition of an open surface of the "corrugated cylinder" type into a closed band.

On the other hand, the primitive lattice can cease to be thermodynamically stable up to the approach to the "critical" Fermi surface, so that the phase transition of first order takes place before the anomaly connected with the variation of topology.* However, since the time of electron "attuning" in the deformation of the lattice is shorter than the time of realignment of the lattice in the phase transition, similar anomalies can be observed even in short-lived metastable states. Therefore, in what follows, the problem of stability or metastability of a state close to the point of variation of the topology of the Fermi surface can be disregarded.

Another parameter — the concentration of impurities or components in an alloy — would appear to change the chemical potential ζ in the Fermi surface strongly; however, for an irregular lattice, the very concept of a Fermi surface loses its exact meaning, owing to the absence of translational symmetry; therefore the singularity of $\nu(\epsilon)$ is smeared out and is practically absent.

We shall investigate below the properties of the metal near the singular point of an "electron transition" (variation of the topology of the Fermi surface). We shall begin with the case of zero temperature, at which the thermodynamic characteristics possess singularities connected with the singularities $\nu(\epsilon)$. These singularities fade away with increase in temperature; the corresponding estimates are given at the end of the first section.

1. ANOMALIES OF THERMODYNAMIC QUANTITIES

The state density is determined by the equation

$$\nu(\epsilon) = (2\pi)^{-3}V \int_{\epsilon(\mathbf{p})=\epsilon} d\Omega_{\mathbf{p}} / |\nabla_{\mathbf{p}} \epsilon|. \tag{1}$$

V is the volume of the metal and $d\Omega_{\mathbf{p}}$ is the element of surface area of $\epsilon(\mathbf{p}) = \epsilon$ in \mathbf{p} -space. Close to the singular point \mathbf{p}_0 on the "critical" surface, $\epsilon(\mathbf{p})$ has the form

$$\epsilon(\mathbf{p}) = \epsilon_k + p'^2 \varphi(\mathbf{n}), \quad \mathbf{p} - \mathbf{p}_0 = \mathbf{p}' = p' \mathbf{n}, \tag{2}$$

where \mathbf{n} is a unit vector.

In the simplest case, when there is no degeneracy at the singular point $p' = 0$, we obtain by suitable choice of axes

$$\epsilon(\mathbf{p}) = \epsilon_k \pm p_1'^2 / 2m_1 \pm p_2'^2 / 2m_2 \pm p_3'^2 / 2m_3. \tag{2a}$$

If the signs of $p_i'^2$ are identical, then a new surface recess appears at the point $\mathbf{p} = \mathbf{p}_0$; if these signs are different, then the neck is disrupted at this point (see Fig. 1,a, b).

It is easy to demonstrate that the state density close to the point ϵ_k is $\nu(\epsilon) = \nu_0(\epsilon) + \delta\nu$, where $\nu_0(\epsilon)$ is a smooth function and $\delta\nu$ differs from zero only on one side of the point ϵ_k — on the side where the number of recesses of the surface $\epsilon(\mathbf{p}) = \epsilon$ is large:

$$\delta\nu = \begin{cases} 0; \\ \alpha |\epsilon - \epsilon_k|^{1/2}, & \alpha \sim N / \zeta^{3/2} > 0. \end{cases} \tag{3}$$

In what follows, to be specific, we shall assume that the number of surface recesses increases for $\epsilon > \epsilon_k$. In this case, for a total number of states $N(\epsilon) = N_0(\epsilon) + \delta N$, we have

$$\delta N = \begin{cases} 0, & \epsilon < \epsilon_k \\ \frac{2}{3} \alpha (\epsilon - \epsilon_k)^{3/2}, & \epsilon > \epsilon_k. \end{cases} \tag{4}$$

Correspondingly, we have for the thermodynamic potential Ω ,

$$\Omega(\zeta) = \Omega_0(\zeta) + \delta\Omega, \quad \delta\Omega = - \int_0^{\infty} \frac{\delta N d\epsilon}{1 + \exp[(\epsilon - \zeta)/T]}$$

(T is the temperature in ergs) or, setting $\epsilon - \epsilon_k = x$, $\zeta - \epsilon_k = z$:

$$\delta\Omega = - \frac{2\alpha}{3} \int_0^{\infty} \frac{x^{3/2} dx}{1 + \exp[(x - z)/T]}. \tag{5}$$

At low temperatures, $T \ll |z|$, we obtain

$$\delta\Omega = \begin{cases} -(\sqrt{\pi}/2) \alpha T^{1/2} \exp(-|z|/T), & z < 0, \\ -\frac{4}{15} \alpha z^{3/2} - (\pi^2/6) \alpha T^2 z^{1/2}, & z > 0. \end{cases} \tag{6}$$

For $T = 0$,

$$\delta\Omega = \begin{cases} 0, & z < 0, \\ -\frac{4}{15} \alpha z^{3/2}, & z > 0. \end{cases} \tag{7}$$

This means that the second derivatives of Ω at the point of "electron transition" $z = 0$ have a vertical break, while the third derivatives go to infinity as $z^{-1/2}$.

Inasmuch as the number of conduction electrons must be considered constant (at least in the vicinity of the point $\zeta = \epsilon_k$), it is useful to employ the free energy $F(T, \nu)$ per electron. In this case the volume ν is a parameter connected with the applied pressure; $\epsilon_k = \epsilon_k(\nu)$ is a direct function of the volume ν and the chemical potential ζ is also a function of ν by virtue of the constancy

*Evidently, such is the case in the so-called isomorphic transition, where the parameters of the lattice undergo a discontinuous change in one and the same type of structure (see below, page 1133).

of the number of particles:*

$$N(\zeta, v) = 1. \quad (8)$$

Denoting by v_k the volume at which the variation of the topology of the Fermi surface begins (i.e., $z = \zeta - \epsilon_k = 0$), we have

$$N_0(\epsilon_k(v_k), v_k) = 1. \quad (9)$$

From (8) and (9), we get

$$z = \gamma(v - v_k), \quad \gamma = -(\partial N_0 / \partial v + v_0 d\epsilon_k / dv) / v_0. \quad (10)$$

Writing the free energy F in the form

$$F = F_0 + \delta F,$$

where F_0 is the smooth part of the free energy plotted against the density $v_0(\epsilon)$, it is easy to be convinced that δF is quantitatively equal to the irregular contribution $\delta\Omega$, expressed in the variables v and T . Thus δF is given by Eqs. (6) and (7), where one must set $z = \gamma(v - v_k)$. Then, as $T \rightarrow 0$

$$\delta \frac{C}{T} = -\delta \frac{\partial^2 F}{\partial T^2} = \begin{cases} 0, & z < 0, \\ (\pi^2/3) \alpha z^{1/2}, & z > 0; \end{cases} \quad (11)$$

$$\delta \frac{1}{T} \frac{\partial p_e}{\partial T} = -\frac{1}{T} \delta \frac{\partial^2 F}{\partial T \partial v} = \begin{cases} 0, & z < 0, \\ (\pi^2/3) \alpha \gamma z^{-1/2}, & z > 0. \end{cases} \quad (12)$$

$$\delta \frac{\partial p_e}{\partial v} = -\delta \frac{\partial^2 F}{\partial v^2} = \begin{cases} 0, & z < 0, \\ \alpha \gamma^2 z^{1/2}, & z > 0; \end{cases} \quad (13)$$

The expression (11) represents the anomaly of the electron specific heat C for $z > 0$:

$$C/T = (C/T)_0 \{1 + \alpha z^{1/2} / v_0\}.$$

The expression (12) gives the anomaly of the coefficient of electron compressibility. The total pressure in the metal is made up of the electron pressure and the pressure p_L of the lattice "core" of the metal, which frequently compensates it. However, inasmuch as the lattice part of the compressibility $\partial p_L / \partial v$ is continuous at the point $z = 0$,[†] then the peculiarity of the total coefficient of compressibility is given by the same expression (see Fig. 3a).

Finally, as seen from (13), the strongest singularity is possessed by the thermal coefficient of pressure $\partial p / \partial T$ (Fig. 3b). Considering that at low pressures it is the electron part of the thermal coefficient of pressure that plays the funda-

*In the case of several intersecting zones, $N(\zeta, v)$ is the total number of particles in all the zones.

†It may turn out that the part of the "lattice" binding energy, brought about by the conduction electrons, has a singularity at the point $z = 0$ of the same character as that of Ω_0 ; this would give an insignificant contribution to $\partial p / \partial v$, which does not change any of the results qualitatively.

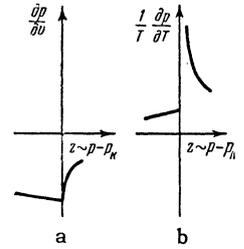


FIG. 3. a - Anomaly of the coefficient of compressibility close to the singular point $z = 0$, b - anomaly of the thermal coefficient of the pressure near the point $z = 0$.

mental role, we have for the smooth part of $\partial p / \partial T$:

$$(\partial p / \partial T)_0 = AT, \quad A = (\pi^2/3) \partial v_0 / \partial v.$$

Thus, in the region of the anomaly,

$$\left(\frac{\partial p}{\partial T}\right)_v = AT \left\{1 + \alpha \gamma z^{-1/2} / 2 \frac{\partial v_0}{\partial v}\right\}, \quad z > 0. \quad (14)$$

The same sort of singularity is possessed by the coefficient of thermal expansion $(\partial v / \partial T)_p = -(\partial p / \partial T)_v / (\partial p / \partial v)_T$:

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{AT}{\alpha_0} \left\{1 + \alpha \gamma z^{-1/2} / 2 \frac{\partial v_0}{\partial v}\right\}, \quad \alpha_0 = -\left(\frac{\partial p}{\partial v}\right)_0. \quad (15)$$

To be able to consider the anomalies in all the formulas (6) - (15) on a pressure scale, it suffices to assume that $p - p_k = -\kappa_0(v - v_k)$ and, consequently,

$$z = \gamma(v - v_k) = (\gamma / \alpha_0)(p_k - p). \quad (16)$$

In this case, it is convenient to estimate the coefficients γ , γ / κ_0 and the critical pressure p by expressing them in terms of the original difference of the energy $z_0 = (\zeta - \epsilon_k)_0$ (for zero pressure), and also of the critical deformation $(v_0 - v_k) / v_0$, for which the transition

$$|z| = |z_0| |p - p_k| / p_k, \quad p_k = \alpha_0 |v_0 - v_k|,$$

$$|\gamma| = |z_0| / |v_0 - v_k| \quad (17)$$

exists.

Naturally, the expressions (17) are only estimates, since the linear relations between p , $v - v_0$ and $z - z_0$ are violated in the large interval of deformations. If we set $|v_0 - v_k| / v_0 \sim 0.05$ to 0.1 for critical deformation, then $p_k \sim 5 \times 10^4$ to 10^5 kg/cm².*

Equations (11) - (15) are valid for $T \ll |z|$. Thus, for finite temperatures, the singularities of all the thermodynamic quantities are lessened. The width of the temperature lessening of the

*It can be thought that less symmetric deformations (for example, uniform compression or tension) greatly change the geometry of the Fermi surface and require smaller stresses. In particular, violation of the initial symmetry of the crystal for such appropriately small deformations can lead to a splitting of the surface at the points of unavoidable self-intersection (for terms of identical symmetry) and lines of accidental intersection (for terms of different symmetry).

anomaly will be $\Delta z \sim T$; on the pressure scale this gives

$$\Delta p / p_k = T / |z_0| = T / |\zeta - \varepsilon_k|_{p=c^*} \quad (18)$$

As follows from Eq. 12, the negative quantity $(\partial p / \partial v)_0 = -\kappa_0$ obtains a positive increment $\sim z^{1/2}$ in the region of the anomaly. If as a result, $p(v)$ ceases to be a monotonic function and becomes positive at some point $\partial p / \partial v$, then an isomorphic phase transition of first order takes place with a discontinuity in the volume. Writing the expansion of $\partial p / \partial v$ for small $z > 0$ and $T \rightarrow 0$:

$$\partial p / \partial v = -\kappa_0 + \alpha \gamma^2 z^{1/2} - \kappa_1 z, \quad (18a)$$

we establish the fact that the region of instability ($\partial p / \partial v > 0$) can exist for the condition $\alpha \gamma^2 > 2\sqrt{\kappa_0 \kappa_1}$, and lies to the right of the point $z = 0$ in the interval

$$\alpha \gamma^2 - \sqrt{(\alpha \gamma^2)^2 - 4\kappa_0 \kappa_1} < 2\kappa_0 z < \alpha \gamma^2 + \sqrt{(\alpha \gamma^2)^2 - 4\kappa_0 \kappa_1}. \quad (19)$$

Here the point of "electron transition," $z = 0$, lies itself in the region of metastability or stability.

Since the singularity at the point $z = 0$ is reduced at finite temperatures, one should not call

the "electron transition" at the point $z = 0$ a phase transition; in this connection we always speak of "anomalies" at the point $z = 0$, although in the terminology of Ehrenfest one could tentatively call such anomalies at $T = 0$ "transitions of the $2\frac{1}{2}$ order," since the second derivatives of the thermodynamic potentials have a singularity $\sim z^{1/2}$, and the third derivatives have a singularity $\sim z^{-1/2}$.

To conclude this section, we determine the anomaly of spin paramagnetism at $T = 0$. In an external magnetic field, the spin magnetic moment will be

$$M = \mu \{N(\zeta + \mu H) - N(\zeta - \mu H)\},$$

$$N_+ + N_- = \text{const},$$

while the magnetic susceptibility $\chi = \partial M / \partial H$ has the form

$$\chi = \{v(\zeta + \mu H) + v(\zeta - \mu H)\} \mu^2 = \chi_0$$

$$+ \{\delta v(z + \mu H) + \delta v(z - \mu H)\} \mu^2,$$

$$\chi_0 = \mu^2 \{v_0(\zeta + \mu H) + v_0(\zeta - \mu H)\},$$

$$\delta v(z) = \begin{cases} 0, & z < 0, \\ \alpha z^{1/2}, & z > 0. \end{cases}$$

Then

$$\delta \chi = \begin{cases} 0, & z + \mu H < 0, \quad z - \mu H < 0, \\ \alpha(z + \mu H)^{1/2}, & z + \mu H > 0, \quad z - \mu H < 0, \\ \alpha(z - \mu H)^{1/2}, & z + \mu H < 0, \quad z - \mu H > 0, \\ \alpha(z + \mu H)^{1/2} + \alpha(z - \mu H)^{1/2}, & z + \mu H > 0, \quad z - \mu H > 0 \end{cases} \quad (20)$$

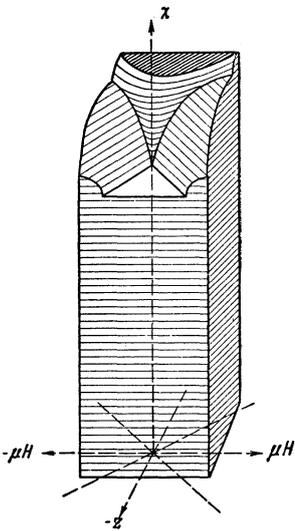


FIG. 4. Anomaly of the paramagnetic susceptibility close to the point $z = 0$.

(see Fig. 4).

However, the actual picture is somewhat further complicated by the superposition of quantum diamagnetism, which we have not taken into account here.

2. ANOMALIES OF GALVANOMAGNETIC CHARACTERISTICS

The anomalies of the kinetic coefficients (for example, the coefficient of heat conduction and electrical conductivity, viscosity, sound absorption, etc.) are connected both with the singularities of the state density $\nu(\epsilon)$ and of the thermodynamic quantities, and with the singularities of the electron dynamics on the critical surface. So far as the first part of the phenomenon is concerned, this leads, as also in the case of thermodynamic quantities, to singularities of the type $|z|^{-1/2}$ and $|z|^{1/2}$. For example, inasmuch as the absorption coefficient of sound χ_1 is connected with the derivative of the sound velocity c with respect to the density, while $c^2 \sim \partial p / \partial v$, we can expect that

$$\delta \chi_1 \sim \delta(\partial^2 p / \partial v^2) \sim |z|^{-1/2}.$$

In the present paper we shall not consider the detailed theory of anomalies of kinetic coefficients in the vicinity of the transition point $z = 0$. We

shall only investigate the case in which the entire effect is fundamentally determined by a change in the electron dynamics. This takes place for the electrical resistance in strong magnetic fields if an open surface at the point $z = 0$ is converted into a closed one (or vice versa).

For definiteness, we shall assume that the initial open surface of the "corrugated cylinder" type at the point $z = 0$ goes over into closed regions by means of a disruption of the neck (see Fig. 2a, b). As was shown in references 1 and 2, the asymptotic character of the electrical resistance in a strong magnetic field depends materially on the topology of the Fermi surface, and is determined by the presence or absence of open trajectories of the electrons. The latter are obtained by the intersection of the Fermi surface with the plane perpendicular to the magnetic field ($p_z = \text{const.}$, z is the direction of the magnetic field). For closed surfaces there are no such open trajectories, and the resistance in strong fields generally tends toward saturation* (to ρ_∞ ; in order of magnitude, $\rho_\infty \sim \rho_0$). For an open surface of the corrugated-cylinder type, open trajectories are obtained for the direction of the magnetic field perpendicular to the axis of the cylinder; this leads to an unlimited growth of the resistance: $\rho \sim H^2$. We shall investigate precisely this case.

We begin the calculation of the resistance in the vicinity of the point $z = 0$ with the determination of the conductivity tensor σ_{ik} . For $z > 0$, by assumption, the surface is closed. This means that the conductivity has the form¹

$$\sigma_{ik}^{(0)} = \begin{vmatrix} \gamma^2 a_{xx} & \gamma a_{xy} & \gamma a_{xz} \\ \gamma a_{yx} & \gamma^2 a_{yy} & \gamma a_{yz} \\ \gamma a_{zx} & \gamma a_{zy} & a_{zz} \end{vmatrix}, \quad \gamma = H_0/H \ll 1, \quad (21)$$

where H_0 is the characteristic magnetic field, in which the period of precession of the electron is equal to its path length; the a_{ik} have the order of magnitude of the conductivity in the absence of the magnetic field.

For $z < 0$, a contribution is added to the value of $\sigma_{ik}^{(0)}$ from the open trajectories. Choosing the x axis along the corrugated cylinder, we can write down the dispersion law close to the singular point p_0 on the surface $\epsilon(\mathbf{p}) = \epsilon_k$ in the form

$$\epsilon = \epsilon_k + p_x'^2/2m_1 - p_y'^2/2m_2 - p_z'^2/2m_3, \quad \mathbf{p}' = \mathbf{p} - \mathbf{p}_0. \quad (22)$$

The contribution $\delta\sigma_{ik}$ from the open trajectories is proportional to its relative share,

*The case in which the electrons and holes are equal in number is an exception.

i.e., the thickness of the layer of the open trajectories Δp_z (Δp_z is the width of the neck in the corrugated cylinder in Fig. 2).

According to (22),

$$\Delta p_z = m_3^{1/2} |z|^{1/2}.$$

Thus, taking into account the form of the conductivity tensor in the presence of open trajectories along the x axis [see reference 1, Eq. (29) and reference 2, Eq. (16)], we have

$$\delta\sigma_{ik} = \left| \frac{z}{\zeta} \right|^{1/2} \begin{vmatrix} \gamma^2 b_{xx} & \gamma b_{xy} & \gamma b_{xz} \\ \gamma b_{yx} & b_{yy} & b_{yz} \\ \gamma b_{zx} & b_{zy} & b_{zz} \end{vmatrix}. \quad (23)$$

Retaining the principal terms in γ and z , we can write for the total conductivity (for $z < 0$):

$$\sigma_{ik} = \sigma_{ik}^{(0)} + \delta\sigma_{ik} = \begin{vmatrix} \gamma^2 a_{xx} & \gamma a_{xy} & \gamma a_{xz} \\ \gamma a_{yx} & \gamma^2 a_{yy} + \left| \frac{z}{\zeta} \right|^{1/2} b_{yy} & \gamma a_{yz} + \left| \frac{z}{\zeta} \right|^{1/2} b_{yz} \\ \gamma a_{zx} & \gamma a_{zy} + \left| \frac{z}{\zeta} \right|^{1/2} b_{zy} & a_{zz} \end{vmatrix}, \quad (24)$$

where $b_{ik} = b_{ik}(z)$. Then, for the resistance along the x axis,

$$\rho = (\sigma^{-1})_{xx} = \frac{a_{yy} a_{zz} - a_{yz} a_{zy}}{|a_{xy}|^2 a_{zz}} + \frac{b_{yy}(z)}{|a_{xy}|^2} \left| \frac{z}{\zeta} \right|^{1/2} \left(\frac{H}{H_0} \right)^2. \quad (25)$$

To make clear the dependence of b_{yy} on $z = \zeta - \epsilon_k$, it is necessary to take into account that the velocities v_x and v_y are very small ($v \perp \sim \mathbf{p}'$) on open trajectories close to the singular point $\mathbf{p}' = 0$, and that the time of motion close to this point on the curve $\epsilon = \epsilon_k$ diverges logarithmically [see reference 3, Eq. (3.15)]:

$$T \sim \int dp_x / v_y \sim \ln(\epsilon - \epsilon_k).$$

Therefore, the fundamental contribution to the mean velocity \bar{v}_y is given by the motion near this point; this leads to the following estimate of \bar{v}_y :

$$\bar{v}_y \sim v / \ln(|z|/\zeta),$$

v = velocity of the electron far from the singularity.

Inasmuch as the contribution to the conductivity σ_{yy} made by the open trajectories contains the factor \bar{v}_y^2 (see reference 2), then

$$b_{yy} \sim a / \ln(\zeta/|z|). \quad (26)$$

Finally, we obtain

$$\frac{\rho}{\rho_0} = \begin{cases} A + B \frac{|z|^{1/2}}{\zeta^{1/2}} \left(\frac{H}{H_0} \right)^2 / \ln \frac{\zeta}{|z|}, & z < 0, \\ A, & z > 0, \end{cases} \quad (27)$$

where A and B are constants of the order of unity.

The anomaly of the pressure derivative of the resistance is especially sharp:

$$\frac{1}{\rho_0} \delta \frac{d\rho}{d\rho} = \frac{Bz_0}{2|z|^{1/2} \zeta^{1/2} \rho_k} \left(\frac{H}{H_0} \right)^2 \left| \ln \frac{\zeta}{|z|} \right|. \quad (28)$$

Attention must be given to one characteristic circumstance: while the anomalies of the thermodynamic quantities are located in the region $z > 0$ (i.e., to the right of the point of "electron transition," $z = 0$), the anomaly of the resistance lies in the region $z < 0$ (i.e., to the left of the point $z = 0$).*

As follows from the estimates (18), all the anomalies are more marked at low temperatures; however, the effect exists even in the region of very high temperatures. As far as the anomalies of resistance in a magnetic field are concerned,

*A much weaker anomaly of the resistance due to thermodynamic parameters takes place also for $z > 0$.

the entire effect is essentially a low temperature one, since it takes place only when the radius of curvature of the trajectory of the electron in the magnetic field is much less than its mean free path.

¹Lifshitz, Asbel, and Kaganov, JETP 31, 63 (1956), Soviet Phys. JETP 4, 41 (1957).

²I. M. Lifshitz and V. G. Peschanskiĭ, JETP 35, 1251 (1958), Soviet Phys. JETP 8, 875 (1959).

³I. M. Lifshitz and M. I. Kaganov, Usp. Fiz. Nauk 69, 419 (1959), Soviet Phys.—Uspekhi 2, 831 (1960).

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