LEPTONIC DECAYS OF HYPERONS WITH EMISSIONS OF PIONS

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The total probabilities for leptonic decays of hyperons with emission of a pion, \( Y \to N (Y') + l + \nu + \pi \), \( (l \) denotes an electron or \( \mu \) meson) are estimated in the case of the simplest matrix element of the universal V-A interaction for one of the possible perturbation theory diagrams.

In view of the successes of the universal theory of weak interactions\(^1\) it is of interest to study theoretically leptonic decays of hyperons. The probability of such decays was first obtained by Behrends and Fronsdal\(^2\) and Feynman and Gell-Mann,\(^1\) and the energy spectra and angular distributions by Shekhter;\(^3\)\(^4\) experimentally a few events of leptonic hyperon decays were observed.\(^5\)

In this work we consider hyperon decay processes of the type

\[ Y \to N (Y') + l + \nu + \pi, \]  

where in addition to leptons a pion is emitted:

\[ \Lambda^0 \to n + l^- + \nu + \pi^0, \quad \Lambda^0 \to p + l^- + \nu + \pi^0; \]

\[ \Sigma^- \to n + l^- + \nu + \pi^0, \quad \Sigma^- \to p + l^- + \nu + \pi^0; \]

\[ \Xi^- \to \Lambda^0 + l^- + \nu + \pi^0, \quad \Xi^- \to \Lambda^0 + l^- + \nu + \pi^0; \]

\[ \Xi^0 \to \Lambda^0 + l^- + \nu + \pi^0, \quad \Xi^0 \to \Lambda^0 + l^- + \nu + \pi^0. \]

[the letters and subscripts \( Y, N, \pi, l (\mu, e), \nu \) refer here and in the following to hyperons, nucleons, pions, charged leptons (\( \mu \) meson, electron), and neutrinos]. All these decays are described (see Okun'\(^6\)) by the universal weak interaction Hamiltonian

\[ H_{\text{inc}} = (G \sqrt{2}) (\bar{\psi}_Y \gamma^5 (1 + \gamma_5) \psi_Y) (\bar{\psi}_N \gamma^5 (1 + \gamma_5) \psi_N), \]  

(2)

where we consider the case \( CY = -CA = G/\sqrt{2} \). In the absence of a consistent theory of strong interactions we use for the description of process (1) a phenomenological matrix element. The invariant matrix element for the strongly interacting particles in this process contains eight unknown scalar functions of the invariants constructed out of the four-momenta of the various interacting particles and can be written as

\[ M_s = f_0 \gamma_5 + f_1 (\gamma_5 \gamma_\mu - \gamma_\mu \gamma_5) + f_2 \gamma_\mu + f_3 P_{\mu\nu} \]

\[ + f_4 \gamma_\mu \gamma_\nu + f_5 (\gamma_5 \gamma_\mu - \gamma_\mu \gamma_5) \gamma_\nu + f_6 \gamma_\mu \gamma_\nu + f_7 P_{\mu\nu} \gamma_5, \]  

(3)

where \( k = pt + p\nu, \ p \) being the four-momentum of the particle in question. We make use of one of the possible perturbation theory diagrams (see figure) to estimate the probability of the decay (1); the weak interaction acts at the point \( O \) and the loop represents the virtual strong interactions.

Since it is not possible at this time to determine the unknown functions \( f \) and \( g \) we consider the simplest case obtained by taking \( f_1 = g_1 = 1 \) and \( f_2, g_2 = 0 \) (\( i = 2, 3, 4 \)), the matrix element for the decay (1) becomes

\[ M = \frac{g}{2} \left( \bar{\psi}_N \gamma^5 \psi_N \frac{g}{2} \left( 1 + \gamma_5 \right) \psi_Y \right), \]  

(4)

where \( g \) is the strong interaction coupling constant \( (g^2/4\pi = 14) \). \( E \) and \( m \) are the total energy and mass of the particle and we use the system of units in which \( \hbar = c = 1 \).

The decay probability is calculated from the usual formula

\[ W = (2\pi)^3 \int \left| \left< M \right> \right|^2 d^4p_N d^4p_\nu d^4p_\pi d^4p_\mu \left< \left< \pi_\nu - p_N - p_\pi - p_\mu - p_\nu \right> \right> \]

where we make use of the method of Dalitz\(^7\) (see also Okun' and Shebalin\(^8\)) to reduce the integration over \( d^4p_N, d^4p_\pi, d^4p_\mu, d^4p_\nu \) to the invariant integration over \( d^4Q^*, d^4R^* \) \( (Q^* = p_1 + p_\mu, R^* = p_\nu - p_\mu) \) followed by the integration over \( d^4Q, d^4R \) \( (Q = p_N + p_\pi, R = p_N - p_\pi) \).

Neglecting the electron mass in comparison with the masses of all other particles and assuming that the condition \( \Delta m Y \ll 1 \) is satisfied, where \( \Delta = m_Y - (m_N + m_\pi) \), we find for the decays (1) with the emission of an electron or \( \mu \) meson respectively the following expressions for the total
The lifetimes \( \tau_0 \), \( \tau_\mu \) for the cascade hyperon were calculated from Eqs. (5) and (6) with the nucleon mass replaced by the mass of the lambda particle; the crossed out entries refer to reactions energetically forbidden.

\[
\begin{array}{cccc}
\Lambda^0 \rightarrow p + \pi^+ + \pi^- & \tau_0, \text{sec} & 29 & - \\
\Lambda^0 \rightarrow n + \pi^+ + \pi^- & \tau_0, \text{sec} & 76 & - \\
\Sigma^+ \rightarrow n + \pi^+ + \pi^- & \tau_0, \text{sec} & 0.48 & 3.15 \\
\Sigma^0 \rightarrow p + \pi^+ + \pi^- & \tau_0, \text{sec} & 0.57 & 0.79 \\
\Sigma^- \rightarrow p + \pi^+ + \pi^- & \tau_0, \text{sec} & 0.77 & 0.93 \\
\Xi^- \rightarrow \Lambda^0 + \pi^+ + \pi^- & \tau_0, \text{sec} & 0.0033 & - \approx 10 \\
\Xi^- \rightarrow \Lambda^0 + \pi^+ + \pi^- & \tau_0, \text{sec} & 0.0034 & - \approx 10 \\
\end{array}
\]

probabilities:

\[
W \approx \frac{2G^2 (q^4/M^4)}{1 - 5.7 \cdot 10^{-12} \cdot \frac{m_N}{2m_Y}} \frac{\sqrt{m_\eta}}{2m_Y} \left[ \frac{V_{m_\eta}}{V_{m_N}} \right] \frac{\Delta_{1/2} + 3m_\eta^2}{2m_Y} K^6 \]

where

\[
K^6 = \left[ (\Delta^4 - \frac{1}{2} m_\eta^2 \Delta^2 + \frac{1}{24} m_\eta^4) \Delta^2 + \frac{1}{2} m_\eta^2 \Delta - 4 \Delta^3 \right] \frac{1}{2} \Delta^2
\]

It follows from a comparison of theory and experiment for hyperon leptonic decays (see Shekhter\(^4\)) that the effective weak interactions coupling constant is approximately an order of magnitude smaller than its usual value \( G = 10^{-5}/m_p \) (mp — mass of the proton). This fact may be due to a renormalization of the \( C_V \) and \( C_A \) coupling constants due to the strong interactions. Following Shekhter\(^4\) we take \( G = 10^{-6}/m_p \). The table shows the hyperon lifetimes \( \tau_0 \) and \( \tau_\mu \), corresponding to electron and \( \mu \)-meson decays with the emission of a pion, as calculated from Eqs. (5) and (6) using the above value for the coupling constant. For purposes of comparison we also list in the table the

\[\Delta \approx m_Y - m_\eta - m_\mu \approx m_\eta \]

\[\Delta^* \approx m_Y - (m_\eta + m_\mu + m_\nu) = m_\eta - m_\nu.\]

It should be noted that the transition to the limit \( m_\mu = 0 \) is not allowed in Eqs. (5) and (6), since they were derived on the assumption \( 2m_\mu \gg \Delta \) or \( \Delta^* \).


\(^8\) L. B. Okun' and E. P. Shebalin, JETP 37, 1775 (1959), Soviet Phys. JETP 10, 1252 (1960).

Translated by A. M. Bincer

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