Polarization Effects in the Scattering of Electrons by Deuterons

G. V. Frolov

Radium Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 8, 1959


The differential cross section and the change of polarization of the electrons are calculated for the process of disintegration of polarized deuterons by polarized electrons, with the electromagnetic form factors of the nucleons taken into account. An expression is also obtained for the polarization of recoil deuterons from the elastic scattering of polarized electrons by unpolarized deuterons.

In a previous paper the differential cross section and the change of polarization of the electrons were calculated for the elastic scattering of polarized electrons by polarized deuterons. In the present paper we consider the differential cross section and the change of the polarization of the electrons in the disintegration of polarized deuterons by polarized electrons, and also obtain formulas for the polarization of the recoil deuterons from the elastic scattering of polarized electrons by unpolarized deuterons. The deuterons are assumed to be non-relativistic, and the admixture of D wave in the ground state of the deuteron is neglected.

Treatting the interaction of an electron with a deuteron as the interaction with the bound neutron and proton, as was done in reference 1, we have for the matrix element of the interaction:

\[ S_{II} = e^{i\theta} \langle \chi_f | S | \chi_i \rangle. \] (1)

Here \( q = p_2 - p_1 \) is the change of the four-momentum of the electron, \( \chi_f \) and \( \chi_i \) are the spin functions of the final and initial states of the deuteron, \( S = A + B (\sigma_1 + \sigma_2)/2 + C (\sigma_1 - \sigma_2)/2, \)

\[ A = I_1 (g_4 - igq/2M) - ig/2M, \quad B = I_3 [q_e g], \quad C = I_4 [q_e g], \quad g_4 = (\alpha_4 f_1 u_2), \] (2)

where \( u_2 \) and \( u_1 \) are spinors for the electron, \( I_1 = \int \overline{\xi}_{\alpha}^* (a_\alpha e^{i\varphi_{\alpha}/2} + a_\alpha e^{-i\varphi_{\alpha}/2}) \xi_{\alpha} \, \text{d}r, \)

\[ I_2 = 2 \int \overline{\xi}_{\alpha}^* (a_\alpha e^{i\varphi_{\alpha}/2} - a_\alpha e^{-i\varphi_{\alpha}/2}) \xi_{\alpha} \, \text{d}r, \]

\[ I_3 = \frac{1}{2M} \int \overline{\xi}_{\alpha}^* [(a_\alpha + b_\alpha) e^{i\varphi_{\alpha}/2} + (a_\alpha + b_\alpha) e^{-i\varphi_{\alpha}/2}] \xi_{\alpha} \, \text{d}r, \]

\[ I_4 = \frac{1}{2M} \int \overline{\xi}_{\alpha}^* [(a_\alpha + b_\alpha) e^{i\varphi_{\alpha}/2} - (a_\alpha + b_\alpha) e^{-i\varphi_{\alpha}/2}] \xi_{\alpha} \, \text{d}r, \]

\( a_\alpha, a_\alpha \) and \( b_\alpha, b_\alpha \) are the electric and magnetic form factors of the proton and neutron, \( S_{\alpha} \) and \( S_{\alpha} \) are the spin operators of the proton and neutron, \( \varphi_{KS} \) are the coordinate functions of the triplet and singlet states of the two nucleons, \( \varphi_d \) is the coordinate function of the deuteron, and \( M \) is the mass of a nucleon.

The initial state is described by a density matrix which is the direct product of the density matrices of the electron and deuteron:

\[ \rho_0 = \rho_e \otimes \rho_d, \]

where

\[ \rho_e = \frac{1}{2} \left( 1 + i \gamma \right) \eta^{(+)}(p_3), \]
\n\[ \rho_d = \frac{1}{2} \left[ 1 + \frac{1}{3} \sigma_1 \sigma_2 + \alpha (\sigma_1 + \sigma_2) + \beta_{im} (\sigma_1 \sigma_2 + \sigma_2 \sigma_1) \right], \]
\n\[ \eta^{(+)}(p) = (i \mathbf{p} - m) \gamma_4 / 2 \epsilon \] is the projection operator that selects positive-energy states. The four-vector \( \zeta_\mu = (\zeta^0, \zeta^1, \zeta^2) \) characterizes the polarization of the electron.

In the rest system \( \zeta_\mu = (\zeta^0, 0) \); in an arbitrary reference system

\[ \zeta = \zeta^0 + \zeta^1 \gamma_5 / m, \quad \zeta_\mu = i \mathbf{p}_\mu / m, \]

where \( \zeta^0 \) and \( \zeta^1 \) are the transverse and longitudinal components of the vector \( \zeta^0; \mathbf{p} \) is the momentum, \( \epsilon \) the energy, and \( m \) the mass of the electron; \( \alpha \) and \( \beta_{im} \) characterize the polarization of the deuteron; \( \beta_{im} = \beta_{mi}; S_p \beta_{im} = \beta_{ii} = 0 \). In the present case the concept of polarization also includes what is ordinarily called alignment.

The differential cross section for inelastic scattering

The differential cross section for disintegration of the deuteron which corresponds to the relative momentum \( \kappa \) of the nucleons in the final state has the following form:

\[ \frac{\text{d}\sigma}{\text{d}\Omega} = \frac{1}{4 \pi} \left( \frac{e^2}{4 \pi} \right)^3 \left[ \frac{S_p \eta^{(+)}(p_3) S_p S^+}{1 + \xi \sin^2 (\theta/2)} \right] \sin^4 (\theta/2) \left( 2 \pi \right)^2 \frac{\text{d}\kappa}{(2\pi)^2}. \] (3)
Here $\delta$ is the angle of scattering of the electron, and $\xi = \epsilon_1/M$.

For a prescribed scattering angle the energy spectrum of the electrons has a sharp peak. Therefore for the range of energies that makes the main contribution to the cross section, $q$ does not change to any important extent. Regarding $q$ as constant, one can easily carry out the integration over $k$, using in an approximate way the completeness of the functions $\varphi_{KB}$ and $\varphi_{KT}$. Performing also the summation over the spin variables of the nucleons and the electron, we have

$$
\int \frac{d\sigma}{(2\pi)^3} S_p (p_1^+ (p_2) S_{p_2} e^{-iS}) = \frac{m_{I_3}^2 \cos^2 \frac{\theta}{2} - \frac{m_{I_3}^2}{m_{I_3}^2}}{2} (I_{I_3}^2 + I_{I_4}^2) + \frac{im_{I_3}^2}{2m_{I_3}^2} (I_{I_3}^2 - I_{I_4}^2) \beta_{lm} \left( \epsilon_2 e_2 \cos \frac{\theta}{2} (p_{1l} p_{2m} + p_{2l} p_{1m}) \right)
$$

and

$$
- \epsilon_2 (\epsilon_1 + \epsilon_2 \sin^2 \frac{\theta}{2}) p_{1m} p_{2l} - \epsilon_1 (\epsilon_1 + \epsilon_2 \sin^2 \frac{\theta}{2}) p_{2m} p_{1l}.
$$

Here

$$
|I_{I_3}|^2 = \left( \frac{d\sigma}{(2\pi)^3} \right) |I_{I_3}|^2 = a_p^2 + a_n^2 + 2a_p a_n f (2q) - (a_p + a_n)^2 f^2 (q),
$$

$$
|I_{I_4}|^2 = \frac{1}{2M} (a_p + a_n + 2a_p a_n) f^2 (q),
$$

$$
|I_{I_4}|^2 = (2M)^2 (a_p + a_n + 2a_p a_n)^2 f^2 (q),
$$

$$
|I_{I_4}|^2 = (2M)^2 (a_p + a_n + 2a_p a_n)^2 - (a_p + a_n + 2a_p a_n) f (2q).
$$

The form factor of the deuteron is

$$
f (q) = \int \rho_{d} (q) e^{i\phi q r} dr = \int_0^{\infty} u^2 \sin \frac{q r}{2} \frac{dr}{q r},
$$

where $\phi_d = u/(4\pi)^{1/2}$. In particular, if $\psi_d = (\gamma/2\pi)^{1/2} e^{i\gamma r}/r$, the form factor is

$$
f (q) = \frac{\gamma}{q} \tan^{-1} \frac{q}{ \frac{\gamma}{q}}.
$$

From these expressions it can be seen that the difference $|I_{I_3}|^2 - |I_{I_4}|^2$ involves only terms containing $f^2 (q)$ and $f (2q)$. These quantities are small for the changes of momentum that are of interest here, and therefore the contribution to the cross section from the "tensor" polarization is also small.

The final expression for the differential cross section for disintegration accompanied by scattering of the electron by the angle $\delta$ can be conveniently written with the following choice of coordinate axes:

$$
k = p_1/p_1, \quad n = [p_1 \times p_1]/|p_1 \times p_1|, \quad l = [k n].
$$

We get

$$
d\sigma /d\Omega = \left( d\sigma /d\Omega \right)_0 (1 - N N_0),
$$

where

$$
N_0 = |I_{I_3}|^2 + \frac{m_{I_3}^2}{m_{I_3}^2} (2|I_{I_3}|^2 + |I_{I_4}|^2),
$$

$$
N = 2I_{I_3} (\xi^2) \tan \frac{\theta}{2} [(\mathbf{a k}) \epsilon_2 \sin \theta - (\mathbf{a l}) (\epsilon_1 - \epsilon_2 \cos \theta)]
$$

and

$$
\tan \frac{\theta}{2} [(\mathbf{a k}) \epsilon_1 + \epsilon_2] [(\mathbf{a k}) (\epsilon_1 - \epsilon_2 \cos \theta) + (\mathbf{a l}) \epsilon_2 \sin \theta]
$$

$$
+ (|I_{I_3}|^2 - |I_{I_4}|^2) \sin^2 \frac{\theta}{2} \beta_{lm} \left[ r_{k n} \left( 1 + \tan^2 \frac{\theta}{2} (\epsilon_1 + 2\epsilon_2 \epsilon_2 + \epsilon_2^2 \cos^2 \theta) + 4M \epsilon_{k n} \epsilon_2 \epsilon_1 \epsilon_2 \sin \theta \right) + 4M \epsilon_{k n} \epsilon_2 \epsilon_1 \epsilon_2 \sin \theta \right],
$$

where

$$
\xi_2 = \text{the polarization of the electrons in inelastic scattering}.
$$

The basic expression for the final polarization $\xi_2$ of the electrons from disintegration of polarized deuterons by electrons of polarization $\xi_1$ is as follows:

$$
\xi_2 = \text{the change of polarization of the electrons in inelastic scattering}.
$$
\( \zeta_5 = \frac{(e_5/m) \text{Sp} (T_{\sigma t}(\sigma_5^{+}k_5), m_5S_{\sigma t}^{+}+(\mu_{5/2})\text{d}x)}{\text{Sp}(\sigma_5^{+}) (p_5 \text{Sp} S_{\sigma t})} \). \tag{9}

To determine the polarization of those electrons that have been scattered through the angle \( \vartheta \), independent of the final state of the nucleons, we must carry out an integration over \( \kappa \). This is done in just the same way as before.

We present the result:

\[
\begin{align*}
\zeta_7 &= k\{C_1(\zeta_7)|k + C_{12}(\zeta_7n) + C_{13}(\zeta_7) + D_{12}(ak) + D_{13}(a)\} \\
&+ n\{C_{21}(\zeta_7n) + C_{22}(\zeta_7)|n + D_{21}(ak) + D_{22}(a)\}.
\end{align*}
\]

The coefficients appearing in the formula (10) have the following values:

\[
\begin{align*}
C_{11} &= \frac{\cos \theta}{N_0 - N} \left[ |I_1|^2 + \frac{q_1^2}{2} (|I_2|^2 + |I_4|^2) \right] (1 + 2 \tan^2 \frac{\vartheta}{2}) \\
&- 2s_2 \tan^2 \frac{\vartheta}{2} (|I_1|^2 - |I_4|^2) \beta \mu \left[ i\gamma_n \sin \theta (s_1 - s_2 \cos \theta) + 2s_2 k \left( s_1 - \frac{1}{2} s_2 \sin^2 \theta \right) + 2I_2 \left( 1 + \sin \frac{\vartheta}{2} \right) (s_1 + s_2 \sin^2 \theta) \right] \\
&- l, \beta m \left( s_1 + s_2 \cos \theta \right) \right], \\
C_{12} &= \frac{s_2 \tan^2 \frac{\vartheta}{2}}{N_0 - N} \left[ |I_1|^2 + \frac{q_1^2}{2} (|I_2|^2 + |I_4|^2) \right] \beta \mu \left[ 2n, \beta m \left( s_1 + s_2 \cos \theta \right) - \frac{1}{2} s_2 \sin^2 \theta \right] \\
&- e_2 \cos^2 \frac{\vartheta}{2} - n, k \sin \theta (s_1 + s_2) \right], \\
D_{11} &= \frac{-c_2 \tan \frac{\vartheta}{2}}{N_0 - N} \left[ |I_1|^2 + \frac{q_1^2}{2} (|I_2|^2 + |I_4|^2) \right] \times \tan \frac{\vartheta}{2} (s_1 - s_2 \cos \theta) \\
&\times \sin \frac{\vartheta}{2} \left( s_1 - s_2 \cos \theta \right) \right], \\
D_{12} &= \frac{c_2 \tan \frac{\vartheta}{2}}{N_0 - N} \left[ |I_1|^2 + \frac{q_1^2}{2} (|I_2|^2 + |I_4|^2) \right] \times \sin \frac{\vartheta}{2} \left( s_1 - s_2 \cos \theta \right) \\
&+ C_{21} \sin \theta, \\
C_{22} &= -C_{22} \sin \theta, \\
D_{21} &= -D_{21} \tan \theta, \\
D_{22} &= -D_{22} \sin \theta. \\
\end{align*}
\]

From these expressions it can be seen that the "tensor" polarization of the deuteron gives a small contribution to the polarization of the scattered electron. It is more convenient to express the final polarization in the coordinate system connected with the scattered electron:

\[
\zeta_7 = (e_7\beta \mu)_k, \quad \zeta_7 = (e_7\beta \mu)_n, \quad \zeta_7 = (e_7|p_7 \beta \mu|p_7).
\]

\( \zeta_7 \) is the longitudinal component; \( \zeta_2 \) is the transverse component perpendicular to the plane of the reaction; \( \zeta_3 \) is the transverse component in the plane of the reaction. For the incident electron

\[
\zeta_7 = (\zeta_7, k), \quad \zeta_7 = (\zeta_7, n), \quad \zeta_7 = (\zeta_7, l).
\]

In this notation the result can be written in the following way:

\[
\zeta_7 = \left[ C_{11} \zeta_7 |k + (\alpha k) D_{11} + (\alpha l) D_{13}\right]/\cos \theta, \\
\zeta_7 = \left[ C_{21} \zeta_7 |l + C_{22} \zeta_7 |l \right] + C_{23} \zeta_7 |l, \\
\zeta_7 = \left[ C_{23} \zeta_7 |l - C_{23} \zeta_7 |l \right], \quad \zeta_7 = \zeta_7 |l = 0.
\]

It can be seen that in the scattering the longitudinal and transverse components change independently of each other. If the deuteron is originally unpolarized, we get

\[
\zeta_7 = \left((\alpha k) D_{11} + (\alpha l) D_{13}\right)/\cos \theta, \quad \zeta_7 = \zeta_7 |l = 0.
\]

Longitudinally polarized electrons are produced. Comparison of these expressions with experimental data can give additional information about the form factors of nucleons.

**POLARIZATION OF THE RECOIL DEUTERONS FROM ELASTIC SCATTERING**

The following expressions are obtained for the quantities \( \alpha \) and \( \beta \) that characterize the polarization of the recoil deuterons from elastic scattering of polarized electrons by unpolarized deuterons:

\[
\begin{align*}
(ak) &= -\frac{4s_2 \sin^2 \frac{\vartheta}{2} (a_p + a_n + a_p + a_n)}{3M_R} \left( \zeta_7 |k \right) \left( a_p + a_n \right) \\
&- \frac{s_2 \left( a_p + a_n + a_p + a_n \right)}{2M_R} \left( 2 \tan^2 \frac{\vartheta}{2} + 5 \sin^2 \frac{\vartheta}{2} \right), \\
(\alpha) &= \frac{2s_2 \sin^2 \frac{\vartheta}{2} \tan \frac{\vartheta}{2} \left( a_p + a_n + a_p + a_n \right)}{3M_R} \left( \zeta_7 |k \right) \left( a_p + a_n \right) \\
&\times \left( 2 + \xi \right) + \frac{s_2 \left( a_p + a_n + a_p + a_n \right)}{2M_R} \left( 2 - \xi \sin^4 \frac{\vartheta}{2} \right), \\
\beta_{\mu \mu} k k &= \frac{q^2 \left( a_p + a_n + a_p + a_n \right)}{4l^2 M_R^2} \sin^2 \left( 1 + 2 \tan^2 \frac{\vartheta}{2} \right) \\
&- \frac{\xi \sin^4 \frac{\vartheta}{2} \mu}{2}, \\
\beta_{\mu \mu} l m &= \frac{q^2 \left( a_p + a_n + a_p + a_n \right)}{4l^2 M_R^2} \tan \left( 2 + \xi \cos \theta \right), \\
\beta_{\mu n} m &= -\beta_{\mu n} m, \quad \beta_{\mu m} n &= -\beta_{\mu m} n, \quad \beta_{\mu m} m = -\beta_{\mu m} m.
\end{align*}
\]

Here

\[
\begin{align*}
R_0 &= \left( a_p + a_n \right)^2 - \frac{q^2 \left( a_p + a_n + a_p + a_n \right)}{6M^2} \left( 1 - \frac{2}{3} \cos^2 \frac{\vartheta}{2} \right). 
\end{align*}
\]
The other notations are the same as before.

From these expressions it can be seen that the average value $\frac{1}{2}\alpha$ of the spin of the recoil deuteron lies in the plane of the scattering. $\alpha$ is zero if the incident electrons are unpolarized, but the quantities $\beta_{im}$ are different from zero even in this case.

Therefore experiments to determine the "tensor" polarization of the recoil deuterons are in a certain sense the simplest of the polarization experiments on the elastic scattering of electrons by deuterons.

Translated by W. H. Furry