MULTIPLE PRODUCTION OF JET PARTICLES IN PERIPHERAL COLLISIONS

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Peripheral collisions of high energy nucleons ($E_{lab} > 10^{12}$ ev) are considered. The Weizsäcker-Williams method was used to classify the peripheral collisions and to describe the peculiarities of each type of interaction.[5] One of the simplest cases (peripheral single-meson interaction) is calculated by perturbation methods.

THE interest in the peripheral collisions of high energy nucleons has increased considerably in recent times. This is mainly due to the fact that stars with anomalous "two hump" angular distributions have been registered and described.[1] These stars can only be interpreted as the result of a peripheral collision with formation of two excited states. The kinematics of such a process have been discussed repeatedly in the literature.[2-4] At the present moment it seems appropriate to us to consider possible versions of this interaction, assuming that the excitation of the nucleons is caused by the exchange of π mesons.

1. CALCULATION BY THE WEIZSÄCKER-WILLIAMS METHOD

In this method (which we shall call the WW method) the problem of the peripheral interaction of two nucleons is solved in two stages. In the first stage one calculates the probability for a head-on collision of a nucleon with the π meson belonging to another nucleon (or of two π mesons belonging to two different nucleons). These processes lead to the formation of an excited system with mass M*. In the second stage one computes the decay of the excited system into secondary particles.

The following processes may take place as a result of the peripheral interaction of two nucleons.

1. One-meson interaction. Only one of the nucleons gives up its meson which undergoes a central collision with the other nucleon. The recoil and the excitation of the first nucleon are neglected, i.e., it is assumed that the excitation of the first nucleon is small ($\sim \mu c^2$).

2. Single virtual $\pi\pi$ interaction. The meson belonging to one nucleon collides with the meson of the other nucleon.

3. Two-meson interaction. Each of the nucleons undergoes a central collision with the π meson of the other nucleon.

4. Double collision of virtual π mesons (for short, double $\pi\pi$ collision). The meson belonging to one of the nucleons interacts with one of the peripheral π mesons of the nucleon which it meets in its path. The other π meson of the second nucleon goes through an analogous process.

For the description of these processes by the WW method we write the function $\rho(\epsilon, \gamma, b)$ in the form*

$$\rho(\varepsilon, \gamma, b) = A^2 K_0^2\left(b\sqrt{1+(\varepsilon/\gamma)^2}\right),$$
$$\hbar = c = \mu = 1. \quad (1)$$

For $b \gtrsim r_0$ (r_0 is the smallest possible value of the impact parameter, which has the meaning of the core radius of the nucleon) this function can be approximated by the simpler function

$$\rho(\varepsilon, \gamma, b) = \begin{cases} A^2 e^{-2b}/b, & \varepsilon < \gamma \\ 0, & \varepsilon > \gamma \end{cases}. \quad (2)$$

The constant A^2 is determined (as it is usually done in the WW method[6]) by the condition of normalization to the total energy of the nucleon:†

$$\int_0^\infty\int \rho(\varepsilon, \gamma, b)\, \varepsilon d\varepsilon\, 2\pi b db = E_0 = M\gamma, \quad (3)$$

from where we have $A^2 = 2M/\gamma\pi$, where M is

*The function $\rho(\varepsilon, \gamma, b)$ has the meaning of the probability that a meson of energy ε hits a unit area which is at a distance b from a nucleon moving with velocity γ. Expression (1) has been used repeatedly in the literature.[5,6] It can be obtained by taking the meson field of the nucleon in its rest system in the form of a Yukawa potential.

†It should be noted that the WW method has an arbitrariness in this respect. The point is that neither in the form (1) nor in the form (2) can the function $\rho(\varepsilon, \gamma, b)$ be extrapolated to the region of small $b \ll r_0$. However, this region is very important in the normalization. The estimates of the cross sections for the various types of collisions obtained below, therefore, are correct only in order of magnitude.

the mass of the nucleon and E_0 its energy in the center-of-mass system (c.m.s.).

Let us consider the one meson collision with the help of the WW method. The probability of the collision of one nucleon with the peripheral meson of another nucleon with energy ϵ (momentum k $\sim \epsilon$) is equal to

$$dw = \int_{r_0}^{\infty} 2\pi b db \rho(\varepsilon, \gamma, b)\, \sigma(\pi, N)\, d\varepsilon.$$

Assuming that the cross section of the purely inelastic interaction of the π meson with the nucleon, $\sigma(\pi, N)$, is independent of the energy, we find

$$dw = 2M\gamma^{-1}e^{-2r_0}\sigma(\pi, N)\, d\varepsilon.$$

It is easily seen that the mass of the excited state $M^* = \sqrt{E^2 - p^2} = \sqrt{(E_0 + \epsilon)^2 - (p - k)^2}$ is equal to $M^* = 2\sqrt{\epsilon E_0}$ (here E_0 and p are the energy and the momentum of the nucleon in the c.m.s.). The probability for the formation of this state is

$$dw = \sigma(\pi, N)\, \gamma^{-2} e^{-2r_0} M^* dM^*.$$

The maximal value of M^* is $M^*_{max} = 2\gamma\sqrt{M}$. The total cross section for the process is

$$\sigma = \int_0^{2\gamma\sqrt{M}} M^* dM^* \gamma^{-2} \sigma(\pi, N)\, e^{-2r_0} = 2Me^{-2r_0} \sigma(\pi, N). \quad (4)$$

The angular distribution of the secondary particles in a one-meson collision should be symmetric in the rest system of the excited state (in the following we shall call this system the M^* system). In order to transform the angular distribution from the M^* system to the center-of-mass system of the colliding nucleons we must know the relative velocity of these systems γ_r.

The momentum of the excited state in the c.m.s. is equal to

$$p = p_0 - \varepsilon = M^* \gamma_r.$$

The energy ϵ is related to the excitation energy M^* by $\epsilon = (M^*/2)^2/M\gamma$. From this we find

$$p = M[\gamma - (M^*/2M)^2 \gamma^{-1}].$$

It should be noted that the second term on the right hand side is always considerably smaller than the first term; even for $M^* = M^*_{max} = 2\gamma\sqrt{M}$ it amounts to $1/M$ of the magnitude of the first term. Therefore $p \sim p_0$. The required velocity (or, more precisely, the quantity $\gamma_r = (1 - v_r^2)^{-1/2}$) is equal to

$$\gamma_r = p/M^* = (M/M^*)\{\gamma - (M^*/2M)^2 \gamma^{-1}\}. \quad (5)$$

The corresponding half angle of secondary particles (i.e., the angle into which half of the total number of particles are emitted) in the c.m.s. is $\theta_{1/2} = 1/\gamma_r$.

We can estimate the order of magnitude of the angle in the c.m.s. into which the excited nucleon is emitted, whereafter it decays into the secondary particles; it will be equal to $\vartheta \sim p_\perp/p$, where $p_\perp$ is the perpendicular component of the momentum. According to the uncertainty relation we have $p_\perp \approx 1/b \approx 1/\mu$ (b is the impact parameter for a peripheral collision). The maximal value of the angle ϑ is equal to

$$\vartheta_{max} = 1/b_{min}p = 1/r_0 p \approx 2\mu/p_0.$$

2. CALCULATION OF ONE-MESON COLLISIONS IN THE PERTURBATION THEORY

Let us compare the results obtained by the WW method with those obtained in the calculation of peripheral one-meson collisions by perturbation theory, assuming that the excitation is caused by the exchange of a single π meson (see reference 7, and also reference 8). The graph for this process is shown in Fig. 1. The quantities p_{01} and p_{02} are the 4-momenta of the free nucleons, q_i and p_i are the momenta of the secondary particles, and M_1 and M_2 are the masses of the intermediate excited states.

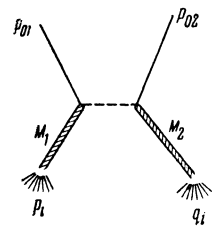

FIG. 1

According to the general rules the total probability for the process is equal to

$$dw = \sum_{n,n'} \int \frac{1}{(k^2 + \mu^2)^2} [1]^2 [2]^2 \prod_i^n d^3p_i \prod_j^{n'} d^3q_j \delta \times \left(p_{01} + p_{02} - \sum^n p_i - \sum^{n'} q_j \right), \quad (6)$$

where $k = p_{01} - \Sigma p_i = -(p_{02} - \Sigma q_j)$; [1] and [2] are matrix elements of the interaction of the meson with the nucleon, leading to the formation of n and n′ secondary particles, respectively. The probability dw can also be expressed in the form

$$dw = \sum_{n,n'} \int \frac{1}{(k^2 + \mu^2)^2} d^4P_1 [1]^2 [2]^2 \prod_i^n d^3p_i \prod_j^{n'} d^3q_j \times \delta\left(p_{02} + k - \sum q_j\right) \delta\left(p_{01} - k - \sum p_i\right), \quad (7)$$

where $P_1 = \Sigma p_i$. Now

$$\sum_n \int [1]^2 \prod_i^n d^3p_i \delta \left(p_{01} - k - \sum p_i\right) = w_{\pi N}(-k,\ p_{01}),$$

$$\sum_{n'} \int [2]^2 \prod_j^{n'} d^3q_j \delta \left(p_{02} + k - \sum q_j\right) = w_{\pi N}(k,\ p_{02}), \tag{8}$$

where $w(k, p_{01})$ is the total probability of the interaction of the π meson with the nucleon. In general the quantity $w(k, p_{01})$ depends on the energy of the colliding particles in the c.m.s.; the sum of their energies is equal to the mass of the excited state M_1 (or M_2). Hence $w(k, p_{02}) = w(M_2)$.

We shall assume further that these quantities are constant for large energies (M_1 or $M_2 \gg M$) and independent of M_1 and M_2. Then

$$dw \sim d^4P_1 w_{\pi N}^2 / (k^2 + \mu^2)^2. \tag{9}$$

It is easily shown that

$$d^4P_1 = (P_1/2E_0)\, d\Omega_{P_1} M_1 dM_1 M_2 dM_2, \tag{10}$$

where $d\Omega_{P_1} = \sin\vartheta\, d\vartheta\, d\varphi$ and ϑ is the angle between P_1 and p_{01}.

Let us examine the denominator of (9) in the c.m.s. of the colliding nucleons. For sufficiently large energies, when $E_{01} = E_{02} \gg M_1, M_2, M$, we can expand all quantities into a series in powers of M/E, which yields

$$k^2 + \mu^2 = 4\,\{\mu^2/2 + E_{01}E_1(1 - \cos\vartheta) + \varkappa/2\}^2,$$

where*

$$\varkappa = (M_1^2 - M^2)(M_2^2 - M^2) / 4E_{01}^2 + (M_1^2 + M_2^2)\, M_1^2 M_2^2 / (2E_{01})^4.$$

We note that $\kappa \ll \mu^2$ for M_1, $M_2 \ll \mu E_{01}/M$. We then obtain for the probability†

$$dw \sim \frac{w_{\pi N}^2 2\pi \sin\vartheta\, d\vartheta\, M_1 dM_1 M_2 dM_2 P_1}{8\,\{\mu^2/2 + E_{01}E_1(1 - \cos\vartheta) + \varkappa/2\}^2 E_{01}}. \tag{9a}$$

In the region of small angles, $(1 - \cos\vartheta) \ll \mu^2/2E_{01}^2$, expression (9a) is large because the denominator is small ($\sim\mu^2$); as the angle increases the expression (9a) decreases rapidly, approximately like ϑ^{-3}. This is the region of peripheral collisions, as the perpendicular component of the momentum transfer is here of the order μc.

In the region of large angles, $(1 - \cos\vartheta) \gg \mu^2/2E_{01}^2$, the above calculation is not valid, since one cannot restrict oneself any longer to first order perturbation theory. Integrating over the region of small angles from zero to $\vartheta \sim \mu/p_{01}$, we obtain

$$dw \sim w_{\pi N} \frac{dM_1 M_2 dM_2 P_1 \mu^2}{p_{01}^2 E_{01}(2\mu^4 + 3\varkappa\mu^2 + \varkappa^2)}. \tag{11}$$

Let us investigate the denominator of this expression. We have

$$2\mu^4 + 3\mu^2\varkappa + \varkappa^2 = (\mu^2 + \varkappa)(2\mu^2 + \varkappa). \tag{12}$$

We note that, when the excitations M_1 and M_2 are of the same order of magnitude, expression (12) increases rapidly, starting from

$$M_1 \sim M_2 = (2E_{01}^2\mu^2)^{1/4} \approx \sqrt{M\mu\gamma}. \tag{13}$$

The quantity (13) is considerably smaller than M_{max}^*, which appears in the WW method.

Let us now consider the case when one of the nucleons is not excited at all ($M_1 = M$), whereas the other is strongly excited ($M_2 \gg M$). It is seen that the denominator of (11) stays almost constant up to

$$M_2 = [16E_{01}^2(\mu/M)^2]^{1/4}$$

and then increases very rapidly as M_2 increases. Therefore the quantity $M_{2\,max}$ (defined in the same way as M_{max}^* in the WW method) is equal to

$$M_{2\,max} = 2E_{01}\sqrt{\mu/M} = 2\gamma\sqrt{M\mu}$$

and agrees with M_{max}^* of the WW method. We note that we obtain the same result in the case when the excitation of the first nucleon is not equal to zero, but small, i.e., when $M_1 - M \sim \mu$.

It is seen from the preceding discussion that the perturbation calculation on the basis of the graph of Fig. 1 is not completely equivalent to the WW method. The point is that in the WW method one always assumes that one of the nucleons gives up its meson and the other nucleon captures it. One should therefore expect that the investigation of the matrix element corresponding to the graph of Fig. 2, where $t_1 < t_2$ always, is more closely related to the WW method. Indeed, the probability for the strong excitation of the nucleon with the

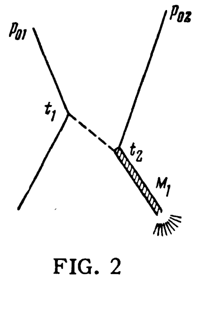

FIG. 2

*In the last term we made use of $M \ll M_1 + M_2$, i.e., we have assumed that one of the nucleons is excited appreciably.

†We use the sign $\sim$ in front of this expression, since we do not intend to determine the total cross section for the peripheral collisions, but are interested only in its dependence on M_1, M_2, and ϑ.

momentum p_{02} calculated on the basis of this graph has the denominator $E_{01} - E_1 - \sqrt{\mu^2 + (p_{01} - p_0)^2}$, which increases rapidly as soon as the excitation of the first nucleon $(M_1 - M)$ becomes greater than μ. This can be seen easily by calculating the matrix element of graph 2 in the rest system of the first nucleon. Thus any strong symmetric excitation of the nucleons is in this case not very probable. The same can be said about the graph in which $t_1 > t_2$ always. However, the matrix element corresponding to the graph of Fig. 1, which was considered above, is the sum of the matrix elements for the graph 2 (where $t_1 < t_2$) and for the analogous graph with $t_1 > t_2$. The probability equals the sum of the component probabilities plus an interference term. This term makes possible the excitation of the two nucleons up to $M_1 \sim \sqrt{M\mu\gamma}$. However, when one nucleon is excited more strongly than the other, the interference term appears to be unimportant. It is essential that the interference is not taken account of in the WW method, so that the symmetric excitation of the nucleons cannot be treated by the WW method.

The simple virtual $\pi\pi$ collision can be described in perturbation theory by the graph of Fig. 3. The analog of this process in electrodynamics is the pair formation by an electron on a nucleus.* However, it is more convenient to consider this process with the help of the WW method and not by perturbation theory.

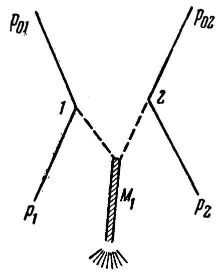

FIG. 3

The probability for this interaction is

$$w = \int \sigma(\pi, \pi)\, \rho_1(\varepsilon_1, \gamma, b_1)\, \rho_2(\varepsilon_2, \gamma, b_2)\, d\varepsilon_1\, d\varepsilon_2\, dS,$$

where the integration goes over the plane perpendicular to the line of motion of the nucleons, excluding the regions where b_1 and b_2 are smaller than r_0; b_1 and b_2 are the distances from a given point to the centers of the nucleons. Using the approximation (2) we find

$$w = \frac{2M^2}{\pi}\left\{e^{-4r_0} + (1 - 4r_0)\,\mathrm{Ei}\,(4r_0)\right\}\sigma(\pi, \pi). \tag{14}$$

The probability for the production of an excited state with proper energy $M^* = 2\sqrt{\epsilon_1\epsilon_2}$ is equal to

$$dw(M^*) = f\int_0^{\gamma} \sigma(\pi, \pi)\,(2M/\pi\gamma)^2\, \varepsilon_1^{-1}\, d\varepsilon_1\, M^*\, dM^*$$
$$= \sigma(\pi, \pi)\,(2M^2/\pi^2\gamma^2)\ln\gamma \cdot M^*\, dM^*\, f, \tag{15}$$

where

$$f = \int dS e^{-2(b_1+b_2)}/b_1 b_2 = \frac{\pi}{2}\left[e^{-4r_0} + (1 - 4r_0)\,\mathrm{Ei}\,(4r_0)\right].$$

It should be emphasized that the production of two strongly excited systems is impossible in one-meson collisions; this can occur only through the exchange of two mesons.

A two-meson collision takes place when a peripheral meson belonging to one of the nucleons interacts with the other nucleon and at the same time the meson of the second nucleon interacts with the first one. The Feynman graph corresponding to this process is shown in Fig. 4. The calculation of this process by the WW method was done in reference 5; here we shall quote only some of the results.

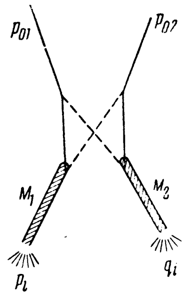

FIG. 4

1. The probability for the double interaction is somewhat smaller than the probability for the simple interaction, but is still of the same order of magnitude (their ratio is ½).

2. The probability that the excitation of the nucleons in a double πN interaction is the same (and, hence, the angular distribution of the secondary particles is symmetric in the c.m.s.) depends weakly on the energy and is equal to about ½ in the energy region (in the laboratory system) $E_{lab} \approx 10^{12} - 10^{14}$ ev.

*Pair production in the collision of an electron with a nucleus can be calculated by the WW method in two ways. One can expand the field of the electron in terms of quanta and then use the known cross section for pair production by a quantum on a nucleus, $\sigma(\gamma, N)$. Or one can expand the fields of the electron and of the nucleus in terms of quanta and then use the cross section for pair production in the collision of two quanta, $\sigma(\gamma, \gamma)$. The results of these two calculations must be identical, since both methods correspond to the same graph (Fig. 3). In the first case the WW method is employed in the point 1, and in the second case in the points 1 and 2. The calculation presented above [using the cross section $\sigma(\pi, \pi)$] corresponds to the second method.

3. The angular distribution of the secondary particles in the c.m.s. consists of two parts (corresponding to the two excited nucleons). However, if the decay of the excited nucleons conforms to the Landau theory, both these angular distributions are quite smeared out, so that their separation is practically impossible.

4. The coefficient of anelasticity for collisions of this type should be of order unity.

The double $\pi\pi$ interaction (see Fig. 5) is differ-

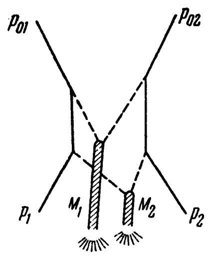

FIG. 5

ent from the preceding case both in principle and in practice. Firstly, it can occur only if the amplitude of the two-meson state is represented in the peripheral field of the nucleon with sufficiently large weight. The frequency with which these cases occur, therefore, gives information not only about the average strength of the meson field at the periphery of the nucleon, but also about the functional form of this field. Secondly, the coefficient of anelasticity for these collisions must be considerably smaller than for a head-on or double πN collision; the maximal value occurs when the first nucleon gives up to mesons having the maximal energy $\epsilon \sim \gamma\mu$ in the c.m.s. In this case* $K_{max} \approx 2\mu/M \approx 0.3$. These cases should be observed in the form of "stars" having a small coefficient of anelasticity with an angular distribution which has two maxima† (i.e., the function $dN/d\lambda$, $\lambda = -\ln \tan \theta$, contains two maxima).

It should be noted that the interpretation of these "stars" is considerably more complicated than in the case of "stars" with a large coefficient of anelasticity even in the case of a symmetric angular distribution it is here in general not possible to determine the energy of the primary particle from the angular distribution of the secondary ones.

*The average value of the coefficient of anelasticity is $K \sim 0.5\, K_{max} \sim 0.15$. However, under the experimental conditions, the average value for the observed stars should be somewhat smaller.

†If the energy of the colliding mesons is the same in the c.m.s., the double $\pi\pi$ interaction gives an angular distribution with only one maximum. In this case it is impossible to differentiate it from the one-meson interaction.

In order to predict the number of secondary particles and their angular distribution in the c.m.s., one must adopt some hypothesis for the mechanism of their creation. In our case the use of the Fermi-Landau theory is less justified than in the case of NN and πN interactions.* Therefore, the question whether one should use some other model (for example, the Heisenberg model, which assumes maximal energy dissipation) or the Fermi-Landau model can be decided only by experiment. Some experimental data quoted in reference 2 were evaluated in reference 9. It follows from these data that in the case of the double $\pi\pi$ interaction the process is apparently described more adequately by the Heisenberg theory than by the Landau theory.

Finally, it is appropriate to make some remarks on the manner in which the peripheral meson field of the nucleon affects the collision of a nucleon with a nucleus. This problem has been considered by one of the authors,[5] who found the following results:

1. Owing to the presence of the peripheral field, a strong excitation of the nucleus is possible, much stronger than the excitation studied by Heitler and Terraux.[11] The probability for this excitation decreases with increasing energy.

2. If the meson present in the peripheral field of the nucleon has sufficient energy, it will interact independently of the nucleon, which will give rise to the appearance of an accompanying shower. The shower as a whole will become asymmetric: a large part of the particles will fly "backwards" in the c.m.s., i.e., in the opposite direction of the primary nucleon.†

It was found in reference 5 that: a) the number

*The point is that the Fermi-Landau theory is appropriate for the description of classical processes. For a head-on collision of two nucleons and propagation along their impact waves (according to reference 10) we have for the action $S \approx p\Delta = M\gamma(1/\mu\gamma) = M/\mu \gg 1$.

This process can therefore be considered classical and there is no objection to the application of the Fermi-Landau theory in this case. For a head-on πN collision the action is of the same order of magnitude as for an NN collision. However, it can be easily shown that for a $\pi\pi$ collision (when it is also treated hydrodynamically) the magnitude of the action is smaller: S = 1. This indicates that the $\pi\pi$ interaction is a quantum process and can therefore follow different laws than the NN or πN interactions.

† In view of the asymmetry of the angular distribution, the energy can in this case not be determined by the angles. The shower has to be symmetrized first, i.e., the accompanying shower has to be separated out. The observation of asymmetric showers and the problem of their symmetrization has been discussed earlier by Takibaev.[12]

of particles in the accompanying shower, N_{acc}, is on the average one half of that in the main shower, N_{main}, and b) the half angle of the accompanying shower, $\theta_{1/2\,acc}$, (in the laboratory system) is on the average four times larger than $\theta_{1/2\,main}$. The latter means that in the coordinates $dN/d\lambda$, λ the maxima of the angular distributions are located at a distance $\Delta\lambda = \log 4 = 0.6$.

We quote here the results of a comparison of these estimates with the experimental data.[12,13] The data of Gurevich, Mishakova, Nikol'skiĭ, and Surkova[13] indicate that the probability for a strong excitation of the nucleus does indeed decrease as the energy goes up. The portion (α) of stars caused by nucleons and containing more than 10 black and gray tracks is for different intervals of the energy E_{lab} equal to

E_{lab}, Mev =	10^{10} — 10^{11}	10^{11} — 10^{12}	10^{12} — 10^{13}
α =	0.8	0.43	0.4

For a comparison of the theoretical data on the accompanying collisions with experiment we made use of the summed histograms of six stars of reference 13 (147 tracks; see Fig. 6) and of six stars of reference 12 (139 tracks; see Fig. 7). All these stars are the result of collisions of energetic nucleons ($E_{lab} \sim 10^{12}$ ev) with nuclei (the stars contained black and gray tracks).

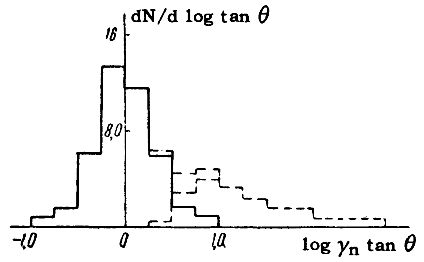

FIG. 6

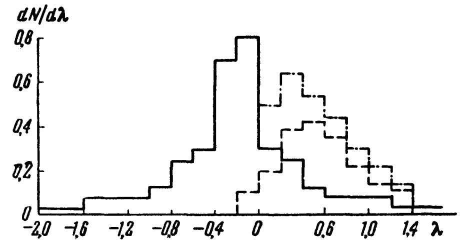

FIG. 7

The symmetrization and separation of the accompanying showers was carried out in the following way. It was assumed that the maximum of the histogram (in the $dN/d\lambda$, λ coordinates) coincides with the maximum of the main shower. The total number of particles in the main shower was taken to be twice the number of particles lying to the left of the maximum. The solid line in Figs. 6 and 7 represents the histogram of the collisions, whereas the dashed line refers to the accompanying collisions. It follows from these histograms that $N_{main}/N_{acc} = 2.3$ (reference 13) and $= 1.8$ (reference 12). The distance between the maxima is $\Delta\lambda = 0.8$ (reference 13) and 0.7 (reference 12). In our opinion, these data are in satisfactory agreement with the theoretical estimates.

It should be noted that the comparison above is not sufficiently accurate for the following reasons: a) not all asymmetric showers obtained in reference 13 were used, but only those which have a clearly defined asymmetry; b) we used summed histograms, and additional asymmetries may have been introduced in their composition because the position of the maximum could not be determined accurately.

In conclusion the authors take this opportunity to express their gratitude to Prof. E. L. Feĭnberg for fruitful discussions and interest in this work and to Prof. I. I. Gurevich and his collaborators for showing us their data.

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