INFLUENCE OF THE MEDIUM DENSITY ON BREMSSTRAHLUNG IN ELECTRON-PHOTON SHOWERS IN THE ENERGY RANGE $10^{11} - 10^{13}$ ev


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Fifteen electron-photon showers in the energy range $10^{11} - 10^{13}$ ev recorded in emulsion stacks were examined. The energies of the primary $\gamma$ rays initiating the showers were determined by measuring the energy spectrum of cascade electrons at the depth of $2.5 - 3$ radiation units, and from the screening effect on the first pairs. The energy spectrum of pairs produced at a depth of up to $1.5$ radiation units was measured. The results are in agreement with calculations carried out taking into account the influence of multiple scattering and of the polarization of the medium on the bremsstrahlung of high-energy electrons.

1. INTRODUCTION

It has been stated earlier$^1$ that it is possible to test experimentally the effects of the medium on the bremsstrahlung$^3$ through a study of the energy spectrum of electrons in electromagnetic cascades of sufficiently high energies ($\geq 10^{12}$ ev). The polarization effect (Ter-Mikaelyan$^3$) should be felt in the emulsion for the $\gamma$-ray frequency range satisfying the condition $\hbar\omega < 7 \times 10^{-5} E$, where $E$ is the electron energy. Moreover, the expression for the radiation intensity $J$ is then $dJ \sim \omega^2 d\omega/E^2$ instead of the Bethe-Heitler relation $dJ \sim d\omega$. The multiple scattering effect (Landau and Pomeranchuk$^4$) should be felt in the emulsion for the frequency range $7 \times 10^{-5} \ll \hbar\omega/E \lesssim 2 \times 10^{-8} E/mc^2$, which leads to an expression for the intensity $dJ \sim \sqrt{\omega} d\omega/E$.

The foregoing conditions show (see also Fig. 1 of reference 2) that, for $E = 5 \times 10^{11} - 10^{12}$ ev, the influence of the medium should lead to a markedly decreased radiation probability of $\gamma$ rays with energy $\hbar\omega \lesssim 10^5$ ev. It is evident that this fact will influence the energy spectrum of cascade electrons and electron-positron pairs, this influence being the stronger the greater the cascade depth $t$.

Special calculations of electromagnetic cascades in nuclear emulsions have been carried out by us earlier using the non-asymptotic cross sections for the elementary electromagnetic processes.$^{1,2}$ In one variant of the calculations (B-H), we used the Bethe-Heitler formulas for the radiation processes, while, in the other variant (M), the radiation of high-energy electrons was calculated according to the formulas of Migdal,$^5$ which take both effects of the medium into account. The calculations have shown that, under certain conditions, the study of the energy spectra of cascade particles at small depths in electron-photon showers in the energy range $10^{11} - 10^{12}$ ev makes it possible to determine the form of the bremsstrahlung spectrum of primary electrons. Let us mention some of these conditions:

First, the energy of primary particles producing the showers should be measured with maximum possible accuracy, since the difference between the Bethe-Heitler and Ter-Mikaelyan-Landau-Pomeranchuk spectra depends very strongly on this energy.

Second, the detection efficiency for low-energy electrons and pairs should be sufficiently high, since the difference between the B-H and M spectra is especially pronounced in the soft part of the spectrum.

The experimental results should be compared with calculations carried out using the true (non-asymptotic) cross sections for the bremsstrahlung and pair production. In the interpretation of the results, one should take the large fluctuations in the number of cascade particles, leading to large statistical errors, into account.

In recent years, electron-photon showers in the energy range $10^{10} - 10^{12}$ ev have been detected in nuclear emulsions in the course of many experiments.$^6$ In several of the showers, the development in the initial stages was considerably different from the average behavior predicted by the usual cascade theories for showers produced by single $\gamma$ rays. In particular, anomalies were ob-
served in the number and in the energy spectrum of cascade particles. The hypothesis that these anomalies are characteristic for high-energy processes has been proposed.\textsuperscript{15} The most detailed study of a high-energy shower (≈7 × 10\textsuperscript{11} ev) at small depths has been carried out by Miesowicz et al.,\textsuperscript{11} who was the first to attempt to explain the experimental results on the spectrum of cascade pairs by means of the influence of the effects of the medium on the bremsstrahlung.

However, similar anomalies have not been detected in later experiments\textsuperscript{12-14} in which the showers were studied more systematically. Thus, the published results on the spectrum of cascade particles in high-energy showers are contradictory. The majority of the published material can, unfortunately, not be used in examining the problem of the shape of the bremsstrahlung spectrum of high-energy electrons, since the basic conditions stated above have not been satisfied.

In the present article, the results of a systematic study of 15 electron-photon showers in the energy range 10\textsuperscript{11} - 10\textsuperscript{13} ev are presented. Preliminary results on five showers in the energy range 3 × 10\textsuperscript{11} - 2 × 10\textsuperscript{12} ev have already been published earlier.\textsuperscript{16}

2. GENERAL CHARACTERISTICS OF THE SHOWERS

In the course of the experiment, six emulsion stacks with a total volume of about 10 liters were used. The stacks were irradiated at 20 - 27 km altitude. General data on the four stacks analyzed up to 1958, and on the position of the five investigated showers in the stacks, have been reported in references 16 and 17. The two new stacks (α and β) were composed of layers with dimensions 10 × 20 × 0.04 cm\textsuperscript{3}, and had a volume of 1.4 and 3.1 liters respectively.

The R-NIKFI emulsion was mainly used. The grain density in relativistic electron tracks amounted to 30 - 35 grains per 100 μ, i.e., considerably greater than in the Ilford G-5 emulsion. This facilitated greatly the detection and analysis of the showers.

Showers with energy of ≈10\textsuperscript{12} ev were usually detected in scanning the plates with the naked eye on white background, along the well-developed part of the cascade at a depth of 2 - 3 radiation lengths. Only those showers produced by single isolated (not connected with jets) photons, and which had a path in one layer not less than a few millimeters, were selected for analysis.

The energy \( E_\gamma \) of the primary photons producing the showers was determined from the number of cascade electrons with energy higher than \( E_c = 300 \text{ Mev} \) at the depth of 2.5 - 3 \( t_0 \), where \( t_0 = 29 \text{ mm} \) is the radiation length in the emulsion. The method used was analogous to that described by Pinkau\textsuperscript{13} and Miesowicz et al.\textsuperscript{11} (For details, see references 16 and 17.) The lateral electron distribution function of Guzhavin and Ivanenko\textsuperscript{18} was used for the calculation of the number of particles traveling within a circle with radius \( ρ \) (in contrast with experiments 11, 13, 16, and 17, in which the calculations of Eyges and Fernbach\textsuperscript{13} were used).

Numerical integration of the results of Guzhavin and Ivanenko\textsuperscript{18} yielded the function \( η(s, ρ/t_0) \) (where \( s \) is the "age" parameter determined by the energy of the primary electron \( E_0 \) and the values of \( ε \) and \( t \)), which gives the ratio of the number of electrons with energy \( > ε \), whose tracks are inside a circle with radius \( ρ \) around the shower axis, to the total number of electrons \( N(ε, E_0, t) \) with energy \( > ε \) at a given depth \( t \) (see Table I).

Values of the energy \( E_\gamma \) (for 12 out of 15 showers) determined from the curves \( N(ε, E_0, t) \) for electrons, calculated by the Monte Carlo method\textsuperscript{2} and from corresponding curves of Janossy,\textsuperscript{20} are given in Table II. The difference between the results for the five showers\textsuperscript{16,17} presented previously and the data in Table II is due mainly to a change in the lateral corrections inherent in the function \( η \). In addition, those curves \( N(ε, E_0, t) \) which have been used in the present analysis are based on greater statistical material\textsuperscript{2} and are somewhat different from the corresponding curves used earlier.\textsuperscript{16}

In finding \( E_\gamma \) from the calculated curve \( N(ε, E_0, t) \), the errors in \( E_\gamma \) were determined

<table>
<thead>
<tr>
<th>( x )</th>
<th>( s = 0.4 )</th>
<th>( s = 0.5 )</th>
<th>( s = 0.6 )</th>
<th>( s = 0.7 )</th>
<th>( x )</th>
<th>( s = 0.4 )</th>
<th>( s = 0.5 )</th>
<th>( s = 0.6 )</th>
<th>( s = 0.7 )</th>
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<td>0.34</td>
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<td>0.79</td>
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TABLE II. Results of the measurement of energy $E_Y$ (in $10^{11}$ ev)

<table>
<thead>
<tr>
<th>Shower</th>
<th>From Monte-Carlo method</th>
<th>According to Janossy</th>
<th>From Eq. (3)</th>
<th>From Eq. (4)</th>
</tr>
</thead>
<tbody>
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<td>$\beta$-213</td>
<td>$16.5 \pm 1.4$</td>
<td>$2.4 \pm 0.7$</td>
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<td>$13.1$</td>
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<td>$\beta$-212</td>
<td>$2.0 \pm 0.5$</td>
<td>$1.5 \pm 0.3$</td>
<td>$7.5$</td>
<td>$7.5$</td>
</tr>
<tr>
<td>D-84</td>
<td>$6.1 \pm 2.1$</td>
<td>$2.3 \pm 0.6$</td>
<td>$0.7$</td>
<td>$0.7$</td>
</tr>
<tr>
<td>O-209</td>
<td>$6.1 \pm 2.1$</td>
<td>$2.3 \pm 0.6$</td>
<td>$0.7$</td>
<td>$0.7$</td>
</tr>
<tr>
<td>E-53</td>
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<td>$2.3 \pm 0.6$</td>
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<td>$0.7$</td>
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<td>a-79</td>
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<td>$2.3 \pm 0.6$</td>
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<td>$2.3 \pm 0.6$</td>
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<td>D-89</td>
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<td>$2.3 \pm 0.6$</td>
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<td>O-317</td>
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<td>$2.3 \pm 0.6$</td>
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<tr>
<td>E-29</td>
<td>$6.1 \pm 2.1$</td>
<td>$2.3 \pm 0.6$</td>
<td>$0.7$</td>
<td>$0.7$</td>
</tr>
<tr>
<td>E-39</td>
<td>$6.1 \pm 2.1$</td>
<td>$2.3 \pm 0.6$</td>
<td>$0.7$</td>
<td>$0.7$</td>
</tr>
</tbody>
</table>

3. MEASUREMENT OF THE ENERGY OF PRIMARY PAIRS FROM THE SCREENING EFFECT

The energies of the primary pairs for $E_Y > 3 \times 10^{11}$ ev were also determined from the decrease in the grain density in the pair tracks near the vertex, due to the mutual screening of the electron and positron fields. This screening effect had first been calculated by Chudakov\textsuperscript{21} using purely classical considerations, and was then treated by other authors\textsuperscript{22,23} among them Burkhardt\textsuperscript{23} using quantum mechanics. The possibility of using the screening effect for the determination of the energy of separate pairs of $E_Y \geq 3 \times 10^{11}$ ev was confirmed experimentally by Wolter and Miesowicz,\textsuperscript{24} by Varfolomeev et al.,\textsuperscript{17} and by Iwadare,\textsuperscript{25} who, in addition to the measurement of the ionization energy losses of the pairs, carried out independent measurements of the pair energy. Since the various available theoretical formulas for the ionization due to pairs, as given by different authors, differ considerably from each other in the numerical values of the constants, it is reasonable to determine these from experimental results\textsuperscript{17,24,25}.

In a classical treatment, the ratio of the specific ionization losses of a pair $I$ to the losses of two separate electrons $2I_{pl}$ can be written in the form

$$R = A + B \ln r^2,$$

where $I_{pl}$ is the ionization at the plateau of the ionization-loss curve, $r$ is the distance between the trajectories of the electron and the positron, and $A$ and $B$ are constants (at least for $R < 0.9$) which depend on the medium.

The distance $r$ is determined by the initial opening angle of the pair $\theta$ and by the multiple scattering of the electron and positron over a distance $x$ from the pair vertex. The angle $\theta$ and the deflection due to multiple scattering, and consequently also $R$, depend statistically on the pair energy $E_Y$.

For the mean value of $R$, we can write $\bar{R} = A + B \ln \bar{r}^2$, where the averaging, in general, should be carried out taking into account the distribution of the energy $E_Y$ between the components of the
pair, the distribution of the multiple scattering deflection at a given distance \( x \), and the distribution of the angle \( \theta \) for a given energy distribution \( E_\gamma \). Sacrificing some accuracy, we assume that the average value of \( \ln r^2 \) can be replaced by \( \ln \bar{r}^2 \). We furthermore write

\[
\bar{r}^2 = \theta^2 x^2 + \frac{k x^2}{E_\gamma} \left[ \frac{1}{a^2} + \frac{2}{(1-i\alpha)^2} \right],
\]

where \( a \) is the ratio of the energy of one of the electrons of the pair to the total pair energy \( E_\gamma \), and \( k \) is the multiple scattering constant determining the mean-square deflection of the electron in space after traversing a path of length \( x \) from the tangent to its trajectory at the point \( x = 0 \). From the data of Rossi\(^{28}\) and of Pickup and Voyvodic,\(^{27}\) it can be assumed that \( k = 6.15 \text{ Mev rad/cm}^2 \).\(^*\)

If we are interested in the most probable value of \( R \) for a given value of \( E_\gamma \), it is not necessary to average over all possible angles \( \theta \), but it is sufficient to consider the angle \( \theta \) equal to the most probable angle \( \delta = (4m^2/E_\gamma) F(a) \).\(^{28}\) After averaging the expression for \( r^2 \) over \( a \) in the limits of \( 0.5 - 0.1 \), using the pair distribution function with respect to the variable \( a \) for the case of full screening\(^{28}\) and the values \( F(a) \) from the article by Borsellino,\(^{28}\) we obtain

\[
\bar{r}^2 = 1.6 \left( \frac{2x/E_\gamma}{1+140x} \right)^2 (1+140x),
\]

where \( x \) is given in centimeters, and \( E_\gamma \) in Mev. For the mean value of \( R \), we can therefore write

\[
\bar{R} = A' + B \ln \left( \left( \frac{2x/E_\gamma}{1+140x} \right)^2 (1+140x) \right),
\]

where \( A' \) and \( B \), as before, are constants.

Experimental values of \( R_1 \) are shown in Fig. 1 as a function of \( z = \ln \left( \left( \frac{2x/E_\gamma}{1+140x} \right)^2 (1+140x) \right) \) (where \( x \) is given in cm, and \( E_\gamma \) in Mev). Data of references 17, 24, and 25, and results of measurements of the pair \( \alpha = 79 \) have been used. The constants \( A' \) and \( B \) have been determined from experimental points in the range \( R \leq 0.9 \) by the least-squares method, and the following expression has been obtained as a result

\[
\bar{R} = 0.167 \{45.0 + \ln \left( \left( \frac{2x/E_\gamma}{1+140x} \right)^2 (1+140x) \right) \}. \quad (3)
\]

The standard deviation of experimental points \( R_1 \) from the corresponding values \( \bar{R} \) given by Eq. (3) is \( \sim 20\% \). It has been assumed that the statistical distribution of the ratio \( R_1/\bar{R} \) is independent of \( \bar{R} \). The curves for \( R_{\max} = 1.20 \bar{R} \) and for \( R_{\min} = 0.80 \bar{R} \), where \( \bar{R} \) is given by Eq. (3), can be used for an estimate of the errors in the determination of the energy \( E_\gamma \) of individual pairs by means of Eq. (3).

It is evident that the errors in the determination of \( E_\gamma \) from the experimental values of \( R_{2\gamma} \), averaged over an interval of \( x \), depend also on the length of this interval \( x \). An increase in the interval length leads to a decrease in the relative statistical fluctuations of the ionization losses of the pair, and thus to a decrease in the error of \( R_{2\gamma} \). On the other hand, with an increase of \( x \), the statistical spread of the values \( R_{2\gamma} \) around the curve \( C \) increases. Unfortunately, the ex-
The experimental material presently available is, so far, insufficient for choosing an optimal value of $x$. The pair energy $E_\gamma$ found from ionization measurements is given in Table II. For the first eight pairs, the values of $R_{E_\gamma}$ were determined by measuring both grain ($n$) and gap ($g$) density. For each pair, and for each variation of the ionization measurement method ($n$ and $g$), two intervals were used, $0 - x_1$ and $0 - x_2$. The values of $x_1$ and $x_2$ were chosen from the conditions $R(x_1) = 0.4 - 0.6$ and $R(x_2) = 0.8 - 0.9$. Thus, for each of the eight pairs, four values each of $E_1(\alpha)$, of max $E_1(\alpha)$, and of min $E_1(\alpha)$, were obtained, where $\alpha = n, g$ indicates the method of measurement, and $i = 1, 2$ the interval $x_1$ or $x_2$. The results were averaged and the errors calculated in the way described below.

First, the mean logarithmic values of $E_i$, max $E_i$ and min $E_i$, were determined separately for the first and second segments of the pair track (logarithmically averaged over the symbol $\alpha$). Then, by averaging the values of $E_i$ over $i$, the average values of $E_\gamma$ given in Table II were obtained. The errors $E_\gamma$ were determined as standard deviations of max $E_i$ and min $E_i$ from $E_i$. The described method of averaging was chosen since the results of the measurements by the various methods over the same segments are not independent. The deviations max $E_i$ and min $E_i$ from $E_i$ are basically due to multiple scattering and not to errors in measuring $n$ and $g$. On the other hand, the measurements over various segments can be regarded as only weakly correlated.

For the remaining five pairs given in Table II ($E_\gamma \sim 10^{14}$ ev), only the measurements of $n$ over one segment $x_2$ were used.

For comparison, the values of $E_\gamma$ are also given in Table II, as determined by the relation

$$R = \frac{1}{12.9} \left\{ 43.6 + \ln \left[ \frac{\sqrt{2 \pi}}{E_\gamma} \right] (1 + 140x) \right\},$$

which follows from the calculations of Burkhardt, if one takes the multiple scattering into account and averages the results by the same method as described above.

Comparison of the data given in Table II shows that, in the majority of cases, a satisfactory agreement is obtained between the values of $E_\gamma$ as determined from the shower development, and from the screening effect using Eq. (3).

The measurements of $n$ and $g$ on the first pairs of the showers $\beta-212$ and $\beta-213$ are shown in Figs. 2 and 3. The energy of these showers was determined from the screening effect only. This was, first of all, because of the lack of calculated cascade curves for energies $E_\gamma \sim 10^{13}$ ev. For such high energies, the influence of the medium can already be felt on the spectrum of cascade electrons with energies of $\sim 10^8 - 10^9$ ev (especially at small depths $t$). The usual cascade curves can, therefore, not be used. In addition, especially large fluctuations in the number of cascade particles at high $E_\gamma$ lead to very large errors in the determination of $E_\gamma$ from the shower development by the method described above.

4. MEASUREMENT OF THE ENERGY SPECTRA OF THE PAIRS

Further analysis of the showers consisted of the following: the vertices of the pairs produced...
at the depth $\leq 1.5 t_{0}$ were carefully searched for. The region inside the radius $p = 150 \mu$ around the shower axis was investigated.* In the majority of cases, the search was repeated several times by various observers in order to reduce to a minimum the possibility of overlooking low-energy pairs.

The pairs lying outside the radius $p = 150 \mu$ or at an angle greater than $10 (mc^2/E_\gamma)$ 
$\times \ln (E_\gamma/mc^2)$ to the shower axis, where $E_\gamma$ is the pair energy, were regarded as part of the background and thus not related to the shower.†

After the spatial position of the shower had been reconstructed, the energy of the pair electron was measured from multiple scattering. Measurements of the multiple scattering angle were carried out using a MBI-8M microscope with a glass guiding rail insuring low table noise. Additional head isolation of the light source made it possible to lower the thermal noise. During the measurements, the binocular attachment was rigidly connected to the base of the microscope by means of a metal frame. A guard shield was placed between the observer and the photographic plate. The microscope table was placed on rubber shock absorbers, and was loaded with a ballast of about 200 kg. As a result, the general noise in the measurements of the mean absolute values of second differences amounted to 0.13 $\mu$ over a cell of 250 $\mu$, and to 0.20 $\mu$ over a cell of 500 $\mu$. (The noise was measured on proton and electron tracks of $\sim 10^{12}$ ev.) A cell of 250 $\mu$ was mainly used.

For a number of cells $n = 15 - 20$, the relative error in the determination of the electron energy is, according to Expong, 30 given by the expression

$$\sigma = \frac{1.12}{\sqrt{n}} \left[ \frac{9}{16} + \frac{3\lambda^2}{36} + \frac{35\lambda^4}{360} \right]^{1/2},$$

where $\lambda$ is the ratio of the measured second difference to the value of the general noise, amounting to $\sim 20 - 30\%$ up to an energy of $(5 - 7) \times 10^8$ ev, which, for the purpose of the present work, was fully satisfactory. In separate cases, either a cell of 500 $\mu$, or the relative multiple scattering method, was used for a determination of the pair energy. In several cases, because of unfavorable conditions for multiple scattering measurements, the pair energy was determined from the opening angle according to the Borsellino formula. 28 In any case, it can be assumed that the pair energy was measured with a sufficient accuracy up to $10^8$ ev.

The showers $\beta-212$ and $\beta-213$ appear as solid strands of electron tracks over a relatively long path length (up to $\sim 1 t_{0}$). In their analysis, our aim was to determine the number of pairs with energy $> 1$ Mev produced at the depth $\leq 1.0 t_{0}$ and $\leq 1.5 t_{0}$ respectively.

5. RESULTS AND DISCUSSION

The influence of the medium should make itself strongly felt in the total number of pairs produced at small depths. The total number of pairs $N (> \epsilon)$ with total energy greater than $\epsilon = 1.5$ Mev,* produced at depths $\leq 1.0 t_{0}$ and $\leq 1.5 t_{0}$ is shown in Figs. 4 and 5 as a function of the primary electron energy. The calculated curves (1 and 2)

\[\text{Number of pairs } N (> \epsilon) \text{ with energy greater than } \epsilon = 1 - 2 \text{ Mev produced on the average at a depth } \leq 1.0 t_{0} \text{ per primary electron with energy } E_0. 1' - \text{calculated curve for the average of } N \text{ in the B-H variant}; 2' - \text{calculated curve for the average value of } N \text{ in the M variant}; \bullet - \text{experimental values for showers listed in Tables II and III}; \Delta - \text{results of Miosic et al.}^{11} \circ - \text{experimental data averaged over several showers.}

\[\text{Number of pairs } N (> \epsilon) \text{ with energy greater than } \epsilon = 1 - 2 \text{ Mev produced on the average at a depth } \leq 1.0 t_{0} \text{ per primary electron with energy } E_0. 1' - \text{calculated curve for the average of } N \text{ in the B-H variant}; 2' - \text{calculated curve for the average value of } N \text{ in the M variant}; \bullet - \text{experimental values for showers listed in Tables II and III}; \Delta - \text{results of Miosic et al.}^{11} \circ - \text{experimental data averaged over several showers.}

*Both the experimental and the theoretical numbers of pairs with energy $1 - 5$ Mev are relatively small. The value of $\epsilon$ can, therefore, be taken as $1 - 2$ Mev.
The energy points give the average number of pairs per primary electron.

The shower E-53 is assumed to be produced by one electron since it develops along the track of one electron. The energy of the second electron of the primary pair is < 10^{11} ev.

In addition to the experimental results on 15 showers investigated in the present experiment (round points in Figs. 4 and 5), the data of Miesowicz et al. on one shower with energy of ~ 7 \times 10^{11} ev (Figs. 4 and 5), and data on one shower with energy of ~ 10^{12} ev analyzed by the Budapest and Prague groups* (Fig. 5), have also been used.

As has already been mentioned above, the values of \( E_{\gamma} = 2 E_0 \) determined only from the screening effect (Table II) have been used for the showers D-39 and {3-212. The energy of shower E-39 has been estimated only from the screening effect because of the unfavorable position of the shower in the stack.

At energies \( E_{\gamma} \sim 10^{12} \) ev, one can consider both methods of measurements of \( E_{\gamma} \) equally accurate, and the final values of the energy of showers D-84, O-209, and E-53 were therefore determined by averaging the results from Table II.

Because of large statistical errors both in the number of pairs \( N \) and in the values of the shower energy \( E_{\gamma} \), it was advisable to average the results over separate groups with close values of \( E_{\gamma} \). In Figs. 4 and 5, the results for the showers with energy \( E_{\gamma} > 1.8 \times 10^{11} \) ev, collected in six groups, are represented as large light circles. In view of the fact that, according to curves 1 and 2, the number of pairs \( N \) depends almost linearly on \( \ln E_0 \), the averaging has been done in coordinates \( N, \ln E_0 \). The errors indicated in Figs. 4 and 5 (similarly to other figures in the present article) are statistical errors.

An analysis of the calculations showed that the distribution of showers produced by two electrons with energies \( E_0 \) with respect to the number of pairs \( N(2E_0) \) at a depth \( t \) can be described by Poisson's law with an average value of \( k \) if, for the independent variable, we take the value \( Q(2E_0)/N(2E_0) \), where \( N(2E_0) = Q(2E_0) \) is the average number of pairs with energy > \( \epsilon \).

The factor \( k \) depends, in general, on \( E_0, \epsilon, \) and \( t \). For \( \epsilon = 1-10 \) Mev, and \( E_0 = 10^{10}-10^{12} \) ev, we can take \( k = 3 \) for the depth 1.0 \( t_0 \) and \( k = 6 \) for the depth 1.5 \( t_0 \).

The standard deviations of the number of pairs at the depth \( t \) in a shower produced by two electrons can, therefore, be calculated from the relation \( \sigma = N/\sqrt{k} \).

The integral energy spectrum of pairs produced at a depth \( \leq 1.5 t_0 \) in the three showers D-84, O-209, and E-53 is shown in Fig. 6. In the calculation, the data have been averaged for one primary electron. Theoretical curves correspond to a logarithmic mean energy of primary electrons \( E_0 = 8.3 \times 10^{11} \) ev. Analogous results for three showers with energy \( E_{\gamma} \sim 3 \times 10^{11} \) ev (D-44, I-109, and \( \alpha \)-79) are given in Fig. 7. Theoretical curves are calculated for the energy \( E_0 = 1.5 \times 10^{11} \) ev.

The comparison of experimental and theoretical results has been carried out only for the number and energy spectra of pairs, but not for electrons. This was because of the following reasons: according to the calculations carried out by us, the differences between the two variants of the calculations

*The authors are grateful to Dr. E. Fenyves and Prof. V. Petrizlka for supplying data on this shower.\(^{11}\)
B-H and M is markedly greater for the spectrum of pairs than for the spectrum of electrons, at the same finite depth.

Moreover, more reason exists for using the results of the one-dimensional calculations for pairs rather than for electrons. As a result of scattering, the length of the low-energy electron path can be substantially different from the path along the shower axis $t$, which has not been taken into account in the calculations. Finally, experimental errors in detecting pairs are much smaller than for single electronic tracks in the section $t_I$. Slow electrons reaching a given depth can, because of scattering, be found at considerable distances from the shower axis. As a result, the tracks may be missed in scanning. The probability of such omissions is considerably lower when detecting pairs. It is also much simpler to exclude background tracks which are not related to the shower in scanning for pairs.

It follows from the study of the radial distribution of pair vertices in showers that the pairs are mainly concentrated in the region of small $p$ ($\lesssim 10\mu$). The mean value of $p$ decreases with increasing $E_{\gamma}$. Consequently, the largest experimental errors due to the omission of pairs belonging to the shower should occur in the study of showers with the smallest values of $E_{\gamma}$ (in our case $\sim 10^{11}$ ev). However, the number of pairs satisfying an accepted selection criterion in five showers with energy of $\sim 10^{11}$ ev (see Figs. 4 and 5), are at least not smaller than the theoretically expected number. This indicates the absence of systematic negative experimental errors. One can consider that the experimental errors are at least such that they do not lead to a marked lowering of the total number of pairs as compared with statistical errors.

As can be seen from Figs. 4 - 7, the experimental points are concentrated close to curve 2, which takes the effect of the medium on radiation into account. A statistical analysis of the results shown in Figs. 4 and 5 has been carried out in order to obtain some quantitative estimate of the deviation of the experimental data from curves 1 and 2, using various significant criteria. Only the data for showers with energy $E_{\gamma} > 1.8 \times 10^{11}$ ev have been used. In the first type of analysis, the sign of the deviation of the experimental points from curve 1' in Figs. 4 and 5 has been taken into account. The values of the probabilities that, for a given point distribution, the curve 1' is really the median curve, is given in Table III.

The deviations have been assessed by the $\chi^2$ criterion. The statistical distribution of the compared values around theoretical averages $y$ should, in such a case, follow a normal law. In order that this condition be satisfied, it is necessary to compare the experimental values $N_i$ averaged over groups of two to three showers (see Figs. 4 and 5).

In the beginning, let us assume that the errors of the measurements of $E_{\gamma}$ can be neglected. The results for each of the showers are independent of each other, and we can therefore consider them as samples from a general set of theoretically possible values $N$ for chosen (fixed) values $x = \ln E_{\gamma}$. According to the addition theorem for the $\chi^2$ distribution for $m$ experimental (group) values of $N_i$, the sum of $\chi^2$ can be written as

$$\chi^2 = \sum_{i=1}^{m} \frac{(N_i - y(x_i))^2}{\sigma_i^2},$$

where $N_i$ is the average experimental group value of the number of pairs, $y(x)$ is the theoretical curve, and $\sigma_i$ are the standard deviations of the theoretically possible values $N_i$ from $y(x_i)$. Since $y(x)$ and $\sigma_i$ are known, the sum has $m$ degrees of freedom.

In order to take the errors in the measurements of $E_{\gamma}$ into account, the results of the confluent analysis2 carried out by Klepikov and Sokolov33 have been used. The errors of $E_{\gamma}$ and $N_i$ can be regarded, in first approximation, as non-correlated. The theoretical curves $y(x)$ can, with good accuracy, be approximated over each segment by a straight line, so that $y'' = 0$. The sum of $\chi^2$ can then be represented as

$$\chi^2 = \sum_{i=1}^{m} \frac{(N_i - y(x_i))^2}{\sigma_i^2 + (y''(x_i))^2},$$

where $\delta_i$ are the errors of $(\ln E_{\gamma})_i$. The probability that the resulting value of the sum of $\chi^2$ is not smaller than the observed value, assuming the

<table>
<thead>
<tr>
<th>Method of analysis and criterion</th>
<th>Neglecting errors in $E_{\gamma}$</th>
<th>Taking errors in $E_{\gamma}$ into account</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis of the error sign</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1.0\sigma$</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>$1.5\sigma$</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>$1.0\sigma + 1.5\sigma$</td>
<td>3.1</td>
<td>3.1</td>
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<tr>
<td>$\chi^2$ criterion</td>
<td></td>
<td></td>
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<tr>
<td>$1.0\sigma$</td>
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<td>90</td>
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<tr>
<td>$1.5\sigma$</td>
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<tr>
<td>$1.0\sigma + 1.5\sigma$</td>
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<td>96</td>
</tr>
<tr>
<td>$U$-criterion</td>
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<td>0.7</td>
</tr>
<tr>
<td>$1.0\sigma$</td>
<td>1.1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table III. Results of a statistical analysis of the data on the number of pairs in showers (in percent)
correctness of the curves 1 and 2, respectively, is
given in Table III.

Finally, an analysis was carried out averaging
the results at one point (U - criterion\textsuperscript{25}). The
averaging was carried out in coordinates \(N, \ln E_0\).
Data on the number of pairs in 11 showers at the
depth of 1.0 \(t_0\) and in 12 showers at the depth of
1.5 \(t_0\) (\(E_\gamma > 1.8 \times 10^{11}\) ev) have been used. The
relative deviations \(u = (\bar{N} - \bar{y}(E))/\sigma_{av}\) were
determined, and corresponding deviations calculated
taking into account the errors of \(E_0\)

\[
u = \frac{\bar{N} - \bar{y}(E)}{\sqrt{\varepsilon_{av}^2 + (\sigma_{av} E)^2}},
\]

where \(\sigma_{av}\) is the statistical error of the average
number of pairs \(\bar{N}, \bar{y}(E)\) is the theoretically ex­
pected value of the average, and \(\delta_{av}\) is the sta­
tistical error of the average value of \(\ln E_0\). Data
given in Table III represent the probability that,
in the present experiment, the values of \(u\) will
not be smaller than the observed values.

The above statistical analysis of the results,
carried out using three significant criteria, points
to the incorrectness of curve 1. At the same time,
a contradiction with curve 2 is not discovered.
Thus, one can assume that at least a qualitative
proof of the influence of the medium on brems­
strahlung has been obtained. For a quantitative
test of the Migdal formulas, it is necessary to in­
crease the statistical material. The experimental
possibilities, together with the results of the cal­
culations,\textsuperscript{2} justify considering the above-described
method of analyzing electron-photon showers as
one of the most suitable ones for a further study
of the effects of the medium on bremsstrahlung
of high-energy electrons.

In conclusion, the authors express their grati­
tude to D. M. SamoIlovich and her collaborators
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