DECAY PROCESSES IN THE DEVELOPMENT OF NUCLEAR CASCADES IN THE ATMOSPHERE

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A general method is presented for the solution of the equations describing the nuclear-cascade process in the atmosphere for initial conditions prescribed at an arbitrary depth. The fraction of energy expended by nuclear cascade showers as a result of the $\pi \to \mu + \nu$ decay is estimated.

PREVIOUS experimental data show that, in the development of air showers, a considerable fraction of the energy of the primary particle is transferred to the μ-meson component of the shower.[1-2] The large role of decay processes in the development of showers in the depth of the atmosphere as well follows from the results of reference 3.

The method of solution of nuclear cascade equations, taking into account the decay processes for an event where the initial conditions are given at the top of the atmosphere, was found in the form of a method of successive generations.[4-5] The fact that in experiments on atmospheric showers the energy of the primary is not measured directly leads to the necessity of calculating the development of showers, assuming certain initial conditions in the depth of the atmosphere. However, the usual method of successive generations is not adaptable for the solution of such problems.

Let nucleons and π mesons take part in a nuclear cascade process. The effective cross section for nuclear collisions for nucleons and π mesons is assumed to be identical. In such a case, it is convenient to measure the path of particles and the depth in the atmosphere not in units of g/cm^2 but in units of the mean free path of particles λ_0. If the atmospheric depth, measured in g/cm^2, equals z, then, in the new units, the depth corresponds to $x = z/\lambda_0$, where λ_0 is expressed in g/cm^2.

We shall consider the initial conditions to be prescribed at the depth x_0. The number of nucleons and π mesons with energy in the interval between E, E + dE at a depth x_0 is $N(E, x_0)\,dE$ and $\Pi(E, x_0)\,dE$ respectively.

The kinetic equations can be written in the form

$$\frac{\partial N(E, x)}{\partial x} = -N(E, x) + \int_0^\infty [N(E', x) W_{NN}(E', E) + \Pi(E', x) W_{\pi N}(E', E)]\,dE',$$

$$\frac{\partial \Pi(E, x)}{\partial x} = -\Pi(E, x)\left(1 + \frac{E_\pi}{Ex}\right) + \int_0^\infty [N(E', x) W_{N\pi}(E', E) + \Pi(E', x) W_{\pi\pi}(E', E)]\,dE', \quad (1)$$

where W_{NN}, $W_{N\pi}$, $W_{\pi N}$, $W_{\pi\pi}$ are the energy spectra of particles corresponding to the second index produced in a nuclear collision with a particle of energy E′, denoted by the first index; $E_\pi = Mcz_0/\tau_0\rho(z_0) = 1.4 \times 10^{11}$ ev is the critical energy of $\pi^\pm$ mesons for which the probability of a nuclear collision and of decay at the depth x = 1 are the same; $\rho(z_0)$ is the air density in g/cm^3 at depth z_0.

We represent the solution in the form of the series

$$N(E, x) = e^{-(x-x_0)} \sum_{i=0}^\infty N_i(E, x),$$

$$\Pi(E, x) = e^{-(x-x_0)} \sum_{i=0}^\infty \Pi_i(E, x). \quad (2)$$

Substituting these series in Eq. (1), we obtain

$$\sum_{i=0}^\infty \frac{\partial N_i(E, x)}{\partial x} = \sum_{i=0}^\infty G_i^{(N)}(E, x),$$

$$\sum_{i=0}^\infty \frac{\partial \Pi_i(E, x)}{\partial x} = -\sum_{i=0}^\infty \Pi_i(E, x)\frac{E_\pi}{Ex} + \sum_{i=0}^\infty G_i^{(\pi)}(E, x), \quad (3)$$

where

$$G_i^{(N)}(E,\ x)=\int_E^\infty [N_i(E',\ x)\,W_{NN}(E',\ E)$$

$$+\Pi_i(E',\ x)\,W_{\pi N}(E',\ E)]\ dE',$$

$$G_i^{(\pi)}(E,\ x)=\int_E^\infty [N_i(E',\ x)\,W_{N\pi}(E',\ E)$$

$$+\Pi_i(E',\ x)\,W_{\pi\pi}(E',\ E)]\ dE'. \qquad (4)$$

Equations (3) are identically satisfied if we require, for $i = 0$:

$$\partial N_0(E,\ x)/\partial x=0,$$
$$\partial\Pi_0(E,\ x)/\partial x=-\Pi_0(E,\ x)\,E_\pi/Ex, \qquad (5)$$

where

$$N_0(E,\ x_0)=N(E,\ x_0), \qquad \Pi_0(E,\ x_0)=\Pi(E,\ x_0),$$

and for $i > 0$:

$$\partial N_i(E,\ x)/\partial x=G_{i-1}^{(N)}(E,\ x),$$
$$\partial\Pi_i(E,\ x)/\partial x=-\Pi_i(E,\ x)\,E_\pi/Ex+G_{i-1}^{(\pi)}(E,\ x), \qquad (5')$$

where

$$N_i(E,\ x_0)=0, \qquad \Pi_i(E,\ x_0)=0.$$

Equations (5) corresponds to the physical representation of successive generations.

The solutions of Eqs. (5) and (5′) are elementary. For $i = 0$:

$$N_0(E,\ x)=N(E,\ x_0), \qquad \Pi_0(E,\ x)=\Pi(E,\ x_0)\,(x_0/x)^{E_\pi/E};$$

For $i > 0$:

$$N_i(E,\ x)=\int_{x_0}^{x} G_{i-1}^{(N)}(E,\ \xi)\,d\xi,$$

$$\Pi_i(E,\ x)=\int_{x_0}^{x}\left(\frac{\xi}{x}\right)^{E_\pi/E} G_{i-1}^{(\pi)}(E,\ \xi)\,d\xi,$$

where the integration over ξ for any generation can be carried out in final form. In the particular case $x_0 = 0$, the solutions become the well known solution of the successive generation method.

In our solution all the terms of the series are positive. The series is always convergent when the total energy of particles at x_0 is finite. The solution, however, is more complicated than for the case $x_0 = 0$, since, in order to obtain each i-th term, it is necessary, in general, to integrate i times.

However, for several cases which are of interest for the interpretation of experimental data, the role of the cascade process can be taken into account in a much simpler manner. We shall estimate the fraction of the energy of the nuclear-active component present in showers at the level of the Pamir station that is lost in the production of μ mesons and neutrinos in its further passage through the atmosphere.

According to experimental data, the form of the energy spectrum of nuclear-active particles in showers depends little on the depth, and can be represented by a power law with a differential spectrum exponent close to 2. We shall assume that the absorption of all particles in showers follows an experimental law with a mean free path of about 200 g/cm^2. Furthermore, in the depth of the atmosphere, the air density varies slowly with depth, and can be assumed to be constant, corresponding to a certain effective depth x_{eff}.

The number of $\pi^\pm$ mesons of given energy decaying per unit length is equal to

$$-(\partial\Pi(E,\ x)/\partial x)_{\rm dec}=(E_\pi/Ex_{\rm eff})\,\Pi(E,\ x), \qquad (6)$$

where x_{eff} is the effective depth in the atmosphere. The energy lost by decay of π mesons of given energy per unit path is equal to

$$-E\,(\partial\Pi(E,\ x)/\partial x)_{\rm dec}=(E_\pi/x_{\rm eff})\,\Pi(E,\ x). \qquad (7)$$

Under the assumptions made above, we have

$$\Pi(E,\ x)=\Pi(E,\ x_0)\,e^{-\mu(x-x_0)}, \qquad (8)$$

and the total energy lost by the nuclear cascade because of the $\pi \rightarrow \mu + \nu$ decay on the path from x_0 to x, is equal to

$$\mathscr{E}_{\mu,\nu}(x_0,\ x)=\frac{E_\pi}{x_{\rm eff}\mu}(1-e^{-\mu(x-x_0)})\int_0^\infty \Pi(E,\ x)\,dE, \qquad (9)$$

where x_{eff} for $\mu(x-x_0) \gg 1$ should be taken from the relation

$$x_{\rm eff}=x_0+\ln 2/\mu. \qquad (10)$$

Thus, the energy lost by the cascade in the decay process on the path from x_0 to x is determined by the total number of π mesons present in the shower at the depth x_0, in consequence of which the problem reduces to the finding of $\Pi(E, x_0)$.

In order to carry out a numerical estimate, we assume, according to experimental data, that nuclear-active particles in the shower are essentially nucleons, the number of which at the depth x and in the energy interval E, E + dE is equal to

$$N(E,\ x)\,dE=Ae^{-\mu(x-x_0)}E^{-2}dE, \qquad E_1<E<E_2. \qquad (11)$$

If we assume a mean free path of nuclear-active particles in the air equal to $\lambda_0 = 80$ g/cm^2, then the absorption mean free path of the shower λ = 200 g/cm^2 corresponds to the value

$$\mu = 0.4. \quad (12)$$

We assume further that $W_{\pi N}(E', E) \equiv 0$, i.e., we neglect the production of fast high-energy nucleons in nuclear collisions of π mesons.

Substituting Eq. (11) into Eq. (1), we obtain

$$\mu = 1 - \int_{E}^{E_2} (E/E')^2 W_{NN}(E', E)\, dE. \quad (13)$$

Equation (13) satisfies the value $\mu = 0.4$ in the region* $E \ll E_2$ if, in the collision of a nucleon of energy E' with the nucleus, the energy spectrum of secondary nucleons can be described by a homogeneous function of E/E', and if the energy of secondary nucleons amounts to $0.6\,E'$. In particular, the equation is satisfied if we assume that $W_{NN}(E', E)\,dE = \delta(E - 0.6\,E')\,dE$, i.e., that the colliding nucleon conserves the fraction of energy equal to 0.6 of the primary energy, and that no additional nucleons are produced.[6] We shall assume that, in nuclear collisions of nucleons, $0.4\,E'$ is transferred on the average to π mesons, and out of this energy, $2/3$ goes to $\pi^{\pm}$ mesons. Substituting the solutions for $N(E, x)$ and $\Pi(E, x)$ written in the form

$$N(E, x) = N(E, x_0)\, e^{-\mu(x-x_0)},$$
$$\Pi(E, x) = \Pi(E, x_0)\, e^{-\mu(x-x_0)},$$

[which corresponds to Eqs. (8) and (11)] into the second of Eqs. (1), we obtain an integral equation from which we find $\Pi(E, x_0)$:

$$\Pi(E, x_0) = \frac{1}{1 - \mu + E_\pi / E x_0} \int_{E}^{E_1} [N(E', x_0)\, W_{N\pi}(E', E) + \Pi(E', x_0)\, W_{\pi\pi}(E', E)]\, dE'. \quad (14)$$

We have carried out a numerical estimate of the energy fraction of the nuclear-active component of the shower lost in the $\pi^{\pm} \to \mu^{\pm} + \nu$ decay in the lower third of the atmosphere (from the depth of 655 g/cm^2 corresponding to the altitude of the Pamir station to sea level). To explain the dependence of this fraction on the mechanism of the elementary act of the nuclear interactions, the calculations were carried out for different assumptions regarding the character of the elementary act and, consequently, the form of the functions $W_{N\pi}$ and $W_{\pi\pi}$. In the first approximation, which is more in line with present ideas, the assumed model has been close to the statistical model of Fermi-Landau. In the second assumption, a substantially lower multiplicity was assumed.

The energy spectrum of the nuclear-active component at the altitude of 655 g/cm^2 was assumed in the form $E^{-2}\,dE$ for the energy range from E_1 to E_2, and the estimates were carried out for different values of the upper limit of the spectrum E_2 of nucleons in the shower. The lower limit of the energy spectrum of nucleons was assumed to be equal to $E_1 = 3 \times 10^9$ ev. All produced particles were assumed to be π mesons, out of which one third were π^0 mesons. Thus, two variants were considered.

1. It was assumed that the total multiplicity of produced mesons in a collision between a nucleon of energy E' with the nucleus of an air atom is equal to

$$n(E') = 4\,(E'/2Mc^2)^{1/4}. \quad (15)$$

A total of 40% of the energy of the incident nucleons is transferred to the meson in the collisions. The energy spectrum represents two monoenergetic meson groups, and one quarter of the mesons obtain 0.75 of all energy, while the remaining three quarters obtain 0.25 of the total energy transferred to mesons. In collisions of mesons with the nucleus, the number of produced mesons is determined by the same relation (15), but the whole energy is transferred to mesons. The energy spectrum of meson-produced mesons has a form analogous to the spectrum of mesons produced by a nucleon.

Thus, it was assumed that

$$W_{N\pi}(E', E)\, dE = \frac{2}{3}\left(\frac{E'}{2Mc^2}\right)^{1/4}\left[\delta\left(E - \frac{0.75 \cdot 0.4E'}{(E'/2Mc^2)^{1/4}}\right) + 3\delta\left(E - \frac{0.25 \cdot 0.4E'}{3\,(E'/2Mc^2)^{1/4}}\right)\right],$$

$$W_{\pi\pi}(E', E)\, dE = \frac{2}{3}\left(\frac{E'}{2Mc^2}\right)^{1/4}\left[\delta\left(E - \frac{0.75\,E'}{(E'/2Mc^2)^{1/4}}\right) + 3\delta\left(E - \frac{0.25E'}{(E'/2Mc^2)^{1/4}}\right)\right]. \quad (16)$$

The calculation of the energy spectrum of mesons according to formula (14) by means of successive approximations gives fast convergence.

2. In addition, a calculation was carried out under the assumption of an essentially different mechanism of π-meson production. It was assumed that the multiplicity of meson production is constant: $n(E') = 3$. The production functions $W_{N\pi}$ and $W_{\pi\pi}$ were assumed to be of the form

*Since Eq. (13) cannot be correct in the range where the condition $E \ll E_2$ is not satisfied, the absorption coefficient will, in fact, depend both on E and on x, increasing with both E and x. However, in calculating the energy lost because of the $\pi \to \mu + \nu$ decay, neglect of the dependence of μ on E and x leads to an overestimate of this energy by certainly not more than 10%, as has been shown by check estimates.

$$W_{N\pi}(E', E)\,dE = 2\delta(E - 0.4E'/3),$$
$$W_{\pi\pi}(E', E)\,dE = 2\delta(E - E'/3). \qquad (17)$$

The calculation was carried out only for $E_2 = 10^{12}$ ev. The results are given in the table for both variants. The number in the table gives the relative fraction of the energy lost for the $\pi \rightarrow \mu + \nu$ decay in the passage of the shower from $z_0 = 655$ g/cm^2 to $z = 1033$ g/cm^2, i.e., $\mathscr{E}_{\mu\nu}(x - x_0)/E_0$, where $\mathscr{E}_{\mu\nu}$ is given by Eq. (9) and

$$E_0 = \int_{E_1}^{E_2} E\,[N(E, x_0) + \Pi(E, x_0)]\,dE.$$

At an intermediate stage of the calculations we obtained the fraction of all nuclear-active particles consisting of π mesons. This fraction clearly depends sharply on the energy range. For the energy of 5 Bev and for different variants it amounts to 11 – 13% increasing, in the energy range of 100 Bev, to 30 – 40% (slow increase with increasing E_2).

Thus, as a result of the calculations, it was found that, for the energy spectrum of the nuclear-active shower component of the type $E^{-2}dE$ at mountain altitudes, a considerable fraction of the energy of that component (of the order of 50%) should be lost for the production of μ mesons and neutrinos, and thus, were lost for the development of the cascade. Such a conclusion, clearly, is almost independent of the mechanism of the elementary act of nuclear collision, which is indicated by the similarity of results of the calculations of the first and second variants.

E_2	1	2
10^{12}	0.57	0.55
10^{13}	0.5	—
10^{14}	0.42	—

[1] Dobrotin, Zatsepin, Rozental', Sarycheva, Khristiansen, and Éĭdus, Usp. Fiz. Nauk **49**, 185 (1953).

[2] K. Greisen, Progress in Cosmic Ray Physics, (North Holland Publishing Co., Amsterdam, 1958) Vol. 3.

[3] S. I. Nikol'skiĭ and A. A. Pomanskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 618 (1958), Soviet Phys. JETP **8**, 430 (1959).

[4] G. T. Zatsepin and I. L. Rozental', Dokl. Akad. Nauk SSSR **99**, 369 (1954).

[5] U. Haber-Schaim and G. Yekutieli, Nuovo cimento **11**, 172 (1954).

[6] Vernov, Grigorov, Zatsepin, and Chudakov, Izv. Akad. SSSR, Ser. Fiz. **19**, 493 (1955), Columbia Tech. Transl. p. 445.

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