decay, then one would expect $F$ to be an order of magnitude smaller than $G$. An analogous quenching takes place in the form factor responsible for the $K_{\mu 2}$ decay.\(^6\)

Here is a table of hyperon decay probabilities calculated on the assumption of an $A-V$ interaction only with a constant $F = 0.1G$. The results of the calculation using the exact formula\(^10\) (the decay probabilities given in reference 10 for $F = G$ are somewhat high due to a mistake in the coefficient) are for all practical purposes the same as those obtained from an approximate formula; for example for the decay $\Lambda^0 \rightarrow p + \mu^- + \bar{\nu}$ one may use

$$W = \frac{F^2}{15m^5} (m_\Lambda - m_p)^5 (m_p/m_\Lambda)^7 \Phi \left( m_p/m_\Lambda \right)^2 \left( \Phi(x) = (1 - 4.5x - 4x^2) \sqrt{1 - x} \right) + \frac{10}{3} x^2 \ln \frac{1 + \sqrt{1 - x}}{1 - \sqrt{1 - x}}$$

(7)

(for the electron modes $x \ll 1$ and $\Phi \approx 1$). It is seen from the table that the product $W_T$ ($\tau =$ experimental hyperon lifetime), which gives the fraction of leptonic decays relative to the total number of decays, for $F = 0.1G$ is of the order of $2 \times 10^{-4}$ for $\Lambda^0$ and $10^{-3}$ for $\Sigma^-$ and $\Xi^-$ (in the last case the estimate is complicated by the absence of exact data on $\Xi^-$ lifetime). In view of the fact that the number of $\Lambda$ and $\Sigma$ decays investigated so far is much less than $1/W_T$, the absence of leptonic modes among them is not surprising.

In the table are shown hyperon decay probabilities calculated on the assumption of an $A-V$ interaction only with a constant $F = 0.1G$. The results of the calculation using the exact formula\(^10\) (the decay probabilities given in reference 10 for $F = G$ are somewhat high due to a mistake in the coefficient) are for all practical purposes the same as those obtained from an approximate formula; for example for the decay $\Lambda^0 \rightarrow p + \mu^- + \bar{\nu}$ one may use

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$W$</th>
<th>$10^9 \tau$</th>
<th>$W_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda^0 \rightarrow p + \mu^- + \bar{\nu}$</td>
<td>5.8 x 10^4</td>
<td>0.277</td>
<td>1.6 x 10^{-4}</td>
</tr>
<tr>
<td>$\Lambda^0 \rightarrow p + \mu^+ + \bar{\nu}$</td>
<td>9.4 x 10^4</td>
<td>0.277</td>
<td>2.6 x 10^{-4}</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n + \mu^- + \bar{\nu}$</td>
<td>3.4 x 10^4</td>
<td>0.167</td>
<td>5.7 x 10^{-4}</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n + \mu^+ + \bar{\nu}$</td>
<td>1.5 x 10^4</td>
<td>0.167</td>
<td>2.5 x 10^{-4}</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow \Lambda^0 + \mu^- + \bar{\nu}$</td>
<td>1.2 x 10^4</td>
<td>-1</td>
<td>1.2 x 10^{-3}</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow \Lambda^0 + \mu^+ + \bar{\nu}$</td>
<td>3.2 x 10^4</td>
<td>-1</td>
<td>3.2 x 10^{-4}</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow \Sigma^0 + \mu^- + \bar{\nu}$</td>
<td>1.4 x 10^4</td>
<td>-1</td>
<td>1.4 x 10^{-4}</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow \Sigma^0 + \mu^+ + \bar{\nu}$</td>
<td>2.1 x 10^4</td>
<td>-1</td>
<td>2.1 x 10^{-5}</td>
</tr>
</tbody>
</table>

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241

ON THE ROTATION OF THE PLANE OF POLARIZATION OF ELASTIC WAVES IN A MAGNETICALLY POLARIZED MEDIUM

K. B. VLASOV and B. Kh. ISHMUKHAMIETOV

Institute for the Physics of Metals, Academy of Sciences, U.S.S.R.

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Let us consider the propagation of plane elastic waves in a magnetically polarized medium (i.e., one located in a constant, uniformly polarized magnetic field $H_0$, or one which contains a constant, uniform magnetization polarization $I_0$) with uniaxial symmetry. Let us study the case in which a constant polarizing field $H_0$ is oriented along the axis of symmetry, which we shall take to be the axis $x_3$. Neglecting magneto-mechanical effects (i.e., magnetostriction and gyromagnetic effects) the non-equilibrium elastic processes are described by the relations: \(^1\)

$$\sigma_{ij} = c_{ijk}^\phi x_j + c_{ijk}^\phi x_k, \quad \sigma_k = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2,$$

$$\omega_k = (\partial u_i/\partial x_j - \partial u_j/\partial x_i)/2, \quad (1)$$

where $\sigma_\phi = c_{ijk}^\phi = c_{ijk}^\phi$ are the components of the mechanical stress tensor, $u_1$ are the components of the displacement vector, and the non-zero components of the dynamic elastic modulus tensor, under the given conditions, are $c_{fg}$ and $c_{fg}$ (which depend on $H_0$ or $I_0$), given in reference 1. Here $f$, $g$, and $q$ are the customary symbols for index pairs in the theory of elasticity.

4Furuichi, Kodama, Sugahara, and Yonezawa, Progr. Theoret. Phys. (Japan) 18, 64 (1956).
9Bruin, Holthuizen and Jongeijans, Nuovo cimento 9, 422 (1958).
LETTERS TO THE EDITOR

Let us consider the propagation of elastic waves along the direction of the polarizing field \( H_0 \). Taking the solution of the equations of the theory of elasticity \( \mu \partial u / \partial x \) in a form proportional to \( \exp \{ i (\omega t - kx) \} \), and neglecting absorption and the relativistic corrections to Ohm’s law for the currents in a moving medium, we obtain the proper frequencies

\[
\omega^{(1)} = k (c_{44} / \rho)^{1/2}, \quad \omega^{(2, 3)}
\]

Here it has been assumed that

\[
c_{44} = -c_{44} = i \omega c_{44}; \quad c_{44} = c_{44}^* = i \omega c_{44}^*;
\]

and the notation \( B = c_{44}^* - c_{44}^* \) has been introduced. The frequency \( \omega^{(1)} \) corresponds to a longitudinal wave, and \( \omega^{(2)} \) and \( \omega^{(3)} \) correspond to two circularly polarized transverse waves; i.e., \( (u_j^1 / u_2^1)^{2, 3} = \pm 1 \).

From Eq. (2) it follows that in the propagation of plane polarized transverse elastic waves, travelling along the direction of a constant magnetic field, one may expect a rotation of the plane of polarization of the elastic waves through an angle

\[
\varphi = x H_0 x = B k^{(0)} x / 2 (pc_{44})^{1/2},
\]

where \( k^{(0)} = (k^{(1)} + k^{(2)}) / 2 \).

We estimate the possible magnitude of the rotational constant \( \chi \), based on the microscopic model which has been proposed. We assume that the elastic medium is a metal, and use the free-electron model. According to Pippard, when a transverse wave passes through a metal it sets up alternating electric fields and currents in it, determined by the displacements. By working out the Lorentz force acting on the resulting current \( (J_\epsilon - N\epsilon) \), where \( J \) and \(-\epsilon\) are the electric currents due to electrons and ions respectively, it can be shown that the quantity

\[
B = -N\epsilon (1 - g) H_0 / c (k^2 + g\delta^2),
\]

acts like a component of the elastic modulus tensor. Here \( k^2 = 4\pi\omega\rho_0^2 \tau / c^2 \); when \( \omega \tau < 1 \) the quantity \( g \) is equal to \( g \approx 1 - (k\tau)^2 / 5 \) if \( k\tau \ll 1 \), and \( g = 3\pi / (4k\tau) \) if \( k\tau >> 1 \); \( c \) is the speed of light, \( N \) is the number of free electrons in one cubic centimeter, and \( 1 \) and \( \tau \) are the length and time of the free path. Note that in deriving the expression (4) no account was taken of the effect of the constant magnetic field \( H_0 \) in calculating the magnitude of the electric current. Hence it may be used either if the radius of curvature of the electron orbits in the magnetic field \( r_0 >> l \), or if the current due to electrons can be neglected in comparison with the ion currents. It follows from the work of Pippard that the latter conditions holds if \( k^2 > k_b^2 \) and \( kl >> 1 \). Furthermore, we have not taken into account the effect of the magnetic field on the distribution function (these considerations are discussed in reference 3).

Generally speaking, the constant \( B \) may itself be complex, as can be seen from Eq. (4), for example. Hence in calculating \( \varphi \) or \( \chi \) the real part of \( B \) must be used. The imaginary part of \( B \) gives the different absorption coefficients for the left-hand and right-hand circularly polarized waves. Consequently in the transmission of linearly polarized waves we should expect, in addition to a rotation of the plane of polarization, the appearance of some ellipticity (circular magnetic dichroism of transverse elastic waves). The axial ratio of the ellipse in this case is expressed in the following form:

\[
b / a = \pm \tanh Im \left( B k^{(0)} x / [2 (pc_{44})^{1/2}] \right).
\]

The greatest value of the rotational constant \( \chi \) is obtained under the conditions \( k^2 > k_b^2 \) and \( kl >> 1 \). In this case, according to Eqs. (3) and (4), if we assume that \( N \sim 10^{22} \text{ cm}^{-3} \), \( \rho \sim 10^5 \text{ cm}^{-3} \), and \( v_\tau \sim 10^5 \text{ cm sec}^{-1} \), we obtain \( \chi = Ne/(2pc_{44}) \sim 10 \text{ rad cm}^{-1} \text{oe}^{-1} \), where \( v_\tau = (c_{44}/\rho)^{1/2} \).

It is interesting to note that the propagation of transverse waves along the \( x_1 \) axis, with their plane of polarization parallel to the \( x_2 \) axis, must be accompanied by the propagation of a longitudinal wave. In this case \( u_j^1 / u_2^1 = -2i\omega (c_{16} - c_{16}^*) / (c_{11} - c_{12}) \). In the free electron model which has been used for the metal, \( c_{16}^* - c_{16}^* = -B \).

In conclusion the authors would like to express their thanks to S. V. Vonsovskii for his continued interest in the work.

3 K. B. Vlasov, Физика металлов и металлоделение (Phys. of Metals and Metal Research) 7, 447 (1959).

Translated by D. C. West

242