Thus for sufficiently large $\tau \omega_{\text{max}}$ the wave front of an electromagnetic shock wave consists of a circularly polarized oscillation with a variable frequency.

*The case of a nonlinear relation between the electric displacement $D$ and field $E$ can be treated similarly, as well as the case of nonlinearity with respect to both the electric and the magnetic fields.

1This circumstance (for electromagnetic waves) was first pointed out and utilized by I. G. Kataev.

**In a stationary wave the field components (which, in general, are not transverse) have the form $f(z-vt)$ where the velocity $v=\text{const.}$

***We note that the value for the velocity of the shock wave determined from (2) and (4) coincides with that from (7).


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ON THE HEAT CONDUCTIVITY AND ATTENUATION OF SOUND IN SUPERCONDUCTORS

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We have previously calculated the electronic heat conductivity $\kappa_E$, of superconductors and the phonon conductivity $\kappa_P$, determined by the scattering of phonons by electrons. It will be shown here that from the theoretical temperature dependence of $\kappa_E$ and $\kappa_P$ found, we can explain, to a considerable extent, all the relationships in the existing experimental data on the heat conductivity of superconductors.

According to our earlier paper $\kappa_P$ can be expressed as:

$$\kappa_P^4 = \kappa_E^4 F(T)/F(T_0),$$

$$F(T) = -8 (6^4 - 5^4)(e^6 - 1)^{-1}$$

$$+ 6\zeta(3)(e^6 + 1) - \sum_{n=1}^{\infty} s^n \exp(-2bs)$$

$$\times (4b^3s^3 + 4bs + 2) + 6\zeta(4)(e^6 - 1)$$

$$- (e^6 - 1) \sum_s s^n \exp(-2bs)$$

$$+ 12b^3s^3 + 12bs + 6) + 32b(e^6 - 1)^{-1}$$

$$- a^4 \sum_s s \exp(-2bs) \left[ -s(2b-a) \right] + 6 \sum_s s^n \exp(-2bs),$$

$$a = 2b - 0.16, \quad \zeta(s) = \sum_{n=1}^{\infty} n^{-s}. \quad (1)$$

In the normal state $\kappa_P^2 = \text{const} \cdot T^2$; $b = \Delta(T)/kT$, where $\Delta(T)$ is the energy gap, and $\kappa_E/\kappa_P$ depends only on $T$ and $T/kT$. For comparison with experiment one must use a specimen with sufficient impurity concentration for $\kappa_E$ to be small. In Fig. 1 the theoretical curve is drawn according to Eq. (1) and the experimental points are for an In-Tl alloy measured by Sladek. If $(T_K - T)/kT$ is not very small, $\kappa_E$ is not appreciably affected by the electron-phonon interaction.

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As can be seen from Fig. 1, the conductivity $\kappa_P$ increases exponentially as $T \to 0$, owing to the increase in phonon mean free path with decreasing scattering by electrons. At sufficiently low temperatures the lattice thermal resistance due to electron scattering, $1/\kappa_{PE}$, becomes less than the resistance due to scattering by lattice defects and crystal boundaries, $1/\kappa_{PD}$ ($\kappa_{PD}$ is the same as $\kappa_{PD}$ in a normal metal). Since the resulting lattice conductivity is $\kappa_{P} = \kappa_{PE}/(\kappa_{PE} + \kappa_{PD})$, we get $\kappa_P \approx \kappa_{PD}$ at still lower temperatures. $\kappa_{PD}$ usually decreases according to a power law $\sim T^2$ at low temperatures. For temperatures
such that $k_{pd} \sim k_{pe}$, the lattice conductivity should then have a maximum (see curve 1 of Fig. 2). Such a maximum was found in experiments on Pb + 10% Bi.\footnote{B. T. Gellikman, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1042 (1958), Soviet Phys. JETP 7, 721 (1958).}

The electronic heat conductivity varies in quite a different way, because of the reduction in the number of electronic excitations, as was shown by Ge'ilikman.\footnote{B. T. Gellikman and V. Z. Kresin, Dokl. Akad. Nauk SSSR 123, 259 (1958).} $\kappa_e$ first decreases slowly and then exponentially with decreasing temperature (see curve 2 of Fig. 2).

\[ \frac{\gamma_e}{\gamma_h} = \frac{x - \ln [(e^{b+x}+1)(e^{b+1})^{-1}] + D(x) (2b-x+2\ln [(e^{b}+1)(e-b-1)^{-1}])}{\ln [(e^b+1)/2]}, \]

where $x = \hbar \omega / kT$; $D(x) = \begin{cases} 1, & x \geq 2b \\ 0, & x < 2b \end{cases}$

For $x \ll 1$ this gives $\gamma_e/\gamma_h = 2/(e^b+1)$, which agrees with the expression previously obtained by Bardeen, Cooper, and Schrieffer.\footnote{J. L. Olsen and C. A. Renton, Phil. Mag. 44, 776 (1953).}

We thank Academician L. D. Landau for valuable advice.

\footnote{*There is a misprint in the final formula of reference 2.}


\footnote{\textsuperscript{3}R. J. Sladek, Phys. Rev. 97, 902 (1955).}


\footnote{\textsuperscript{6}N. V. Zavaritskiǐ, J. Exptl. Theoret. Phys. by measurements on Al and Zn,\textsuperscript{6} Sn,\textsuperscript{7,9} In,\textsuperscript{7} and Pb.\textsuperscript{8}}

Only at very low temperatures do we have $\kappa_e < \kappa_p$ and $\kappa \approx k_{pd}$. In very impure specimens $k_p \gg \kappa_e$ and $\kappa \approx \kappa_p$ at all temperatures.\footnote{S. J. Laredo, Proc. Roy. Soc. 229A, 473 (1955).}

For intermediate cases of not very pure superconductors, $\kappa_e$ is the main component near $T_K$, so that $\kappa$ falls with decreasing temperature. At sufficiently low temperatures $k_p$ becomes larger than $\kappa_e$, and $\kappa$ is then determined by curve 1 of Fig. 2.

Such a temperature dependence was found in experiments on Sn, Hg and Pb,\textsuperscript{9-11} while de Haas and Rademakers\textsuperscript{11} and Mendelssohn and Olsen\textsuperscript{5} found a maximum in $\kappa$, related to the maximum in $k_p$ (the collected experimental data are contained in Shoenberg's book\textsuperscript{12}).

Let us now examine the coefficient $\gamma$ of absorption of sound in superconductors, due to electronic excitations, when the frequency is $\omega \gg 1/\tau$, where $\tau$ is the relaxation time. The absorption due to phonons is, under these conditions, the same as in a normal metal.

From a consideration of the probabilities of absorption of a sound quantum and of the reverse process, we obtain for the ratio $\gamma_s / \gamma_h$:

\[ \frac{\gamma_s}{\gamma_h} = \frac{x - \ln [(e^{b+x}+1)(e^{b+1})^{-1}] + D(x) (2b-x+2\ln [(e^{b}+1)(e-b-1)^{-1}])}{\ln [(e^b+1)/2]}, \]

where $x = \hbar \omega / kT$; $D(x) = \begin{cases} 1, & x \geq 2b \\ 0, & x < 2b \end{cases}$

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