energy $E_{a}/a$. Therefore, we have the following expressions for $U_{a}$:

$$U_{a}(E_{a}, \beta) \approx \epsilon_{a} - |\epsilon_{a}| + a U(E_{a}/a, \beta). \quad (6)$$

Here $\epsilon_{a}$ is the binding energy of particles of the same type as the incident ones in the nucleus and $U(E_{a}/a, \beta)$ can be obtained from Eqs. (3) and (4). From this formula and the preceding ones it follows that bombardment of a nucleus by complex particles is advantageous with respect to obtaining high excitation energies of the remaining nucleus. From an analysis of the graph in the figure, we conclude that there is little chance of a cascade for $E_{a}/a \leq 30$ to 50 Mev and that nuclear reactions here go through formation of the compound nucleus.

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A METHOD OF EVALUATING ELECTRICAL CONDUCTIVITY

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It is well known that in the electron theory of solids the standard transport equation method meets with a number of difficulties (see, e.g., references 1-3). A number of authors have in this connection in recent years made attempts to construct an accurate theory of the transport coefficients using relaxation functions. In the present note we give a new method to evaluate the reaction of a system of particles to an external field, using quantum-mechanical Green functions. To be specific, we shall deal with the electrical conductivity, but it will be clear from the results that the method could be equally well used to evaluate any parameter characterizing the reaction of the system. We note also that to apply the general results to the Bose case one needs only change the sign in some of the intermediate equations. It is obvious that if one wants to evaluate the average current produced in the system by the action of an external electrical field it is sufficient to find the change in the "one-particle" density matrix $R_{1}(x, y; t)$. It was shown in reference 8 that this matrix is connected with the one-fermion Green function through the equation

$$R_{2}(x, y; t) = i \lim_{s \to 0} G(x, y) as x_{0} \to y_{0} = t, \quad x_{0} < y_{0} \quad (1)$$

for $x, y$, are points in four-dimensional space, the spin indices are omitted for the sake of simplicity). Let the field be characterized by the four-potential $A_{i}$ ($i = 1, 2, 3, 4 = - \varphi$, $\varphi$ is the scalar potential). If we are interested only in effects which are linear in the field we have for the change in the Green function

$$\Delta G(x, y) = (e/\hbar) \int dx dx' dx'' G_{0}(x, x'') \times \Gamma_{1}(x', z; z') G_{0}(z', y) A_{i}(z), \quad (2)$$

where $\Gamma_{1} = -iG^{-1}(z', z'')/\delta A_{1}(z)$ is the "electromagnetic" vertex part and $G_{0}$ the Green function for $A_{1} = 0$.

For a number of problems it is sufficient to restrict the discussion to the case of spatial uniformity, when ($\hbar = 1$)

$$G_{0}(x, y) = \int dp G_{0}(p)e^{-i(px-y)} ,
\Gamma_{1}(x, y; z) = (-2\pi)^{-s} \int dq dq' q^{s} \Gamma_{1}(q', q^{*}) e^{-i(q', x-z)+i(q, y-z)} \quad (3)$$

For $p \neq 0$, we shall also assume $A_{1} = A_{m} e^{-i(kz-\epsilon z)}$ ($s$ is a small positive number; if damping is present, e.g., when the electrical conductivity is finite, we can at once put $s = 0$). Equation (2) then gives

$$\Delta G(x, y) \bigg|_{s=0} = eA_{1}(y)(2\pi)^{s} \times \int dp G_{0}(p) G_{0}(p-k) \Gamma_{1}(p, p- k) e^{-i(px-y)}. \quad (4)$$

The electrical conductivity can be evaluated from this in a trivial manner.
As an illustration we shall consider the case of a constant field $A_{\alpha} = 0$, $\alpha = 1, 2, 3$ in the approximation $\Gamma_{1}(q', q'') = 1$. We shall assume (compare reference 8; the equation has a meaning also for the determination of the Green function as an ensemble average)

$$G_{0}(p) = \frac{1}{(2\pi)^{3}} \left( \frac{n_{p}}{\omega_{p} - \omega_{p} + i\gamma_{p}} \right) \frac{1 - n_{p}}{\omega_{p} - \omega_{p} - i\gamma_{p}}$$

$(\omega', \gamma', \gamma'' - \text{real}; \gamma', \gamma'' > 0)$. (5)

We must note that Eq. (5) is by no means general (in particular, this assumption means that we restrict ourselves to one — perhaps spin-degenerate band). A number of interesting cases, however, are included here. One can easily evaluate $\mathcal{G}(x, y)$ under the given conditions. Taking (1) into account we get for the change in the occupation numbers in momentum space:

(a) if $\gamma' = \gamma'' = \eta \to 0$:

$$\Delta n_{p} = - \frac{eE_{s}}{s + i\epsilon_{0}} \frac{\partial n_{p}}{\partial \epsilon_{0}};$$

(6)

(b) if $\gamma', \gamma''$ are finite and $k_{0} = 0$ (the calculation is also simple for $k_{0} \neq 0$, but leads to a more unwieldy result):

$$\Delta n_{p} = - eE_{s} \frac{\partial}{\partial \epsilon_{0}} \frac{\gamma_{p}}{\gamma_{p}} \frac{n_{p}(1 - n_{p})}{\gamma_{p}}.$$  (7)

Here $E$ is the field strength (with potential $\varphi$), $\gamma_{p} = \gamma_{p}' + \gamma_{p}'$. The right hand side of (6) agrees, as was to be expected, exactly with the "accelerating" term of the transport equation, if we understand by $s^{-1}$ the time of action of the field. The static electrical conductivity tensor has from (7) the form (in the usual units, $m$ is the true electron mass)

$$\sigma_{ab} = - \frac{e^{2}}{2m} \frac{1}{(2\pi)^{3}} \int dp p_{a} \frac{\partial}{\partial \epsilon_{0}} \frac{n_{p}(1 - n_{p})}{\gamma_{p}}.$$  (8)

In the particular case of Boltzmann statistics and isotropic model $(n_{p} \sim \exp (-\hbar \omega_{p}/kT), \omega_{p} = \omega(|p|), \gamma_{p} = \gamma(|p|))$ Eq. (7) agrees formally with the result of applying the transport equation, if we take for the "relaxation time" the quantity

$$\tau_{p} = \frac{1}{\gamma_{p}} \left[ 1 + \frac{\partial \ln \gamma_{p}}{\partial \hbar \omega_{p}} \right].$$  (9)

We emphasize, however, that Eq. (8) is valid over a far wider range than the transport equation.

1. L. E. Gurevich, Основы физической кинетики (Fundamentals of Physical Kinetics) Gostekhizdat, 1940.


