RANGE-ENERGY RELATION FOR 660-Mev PROTONS

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The mean range of protons in copper was determined. The proton energy was computed from the measured angle of Vavilov-Cerenkov radiation emitted by protons in Plexiglas. The corrected range of $(658 \pm 2)$-Mev protons in copper was found to be $257.6 \pm 1.2$ g/cm$^2$. Assuming the ionization potential to be independent of the velocity, the calculated value of $I_{Cu}$ is $305 \pm 10$ ev. The stopping power relative to copper was also measured for H, Be, C, Fe, Cd and W.

INTRODUCTION

Bethe and Livingston's expression for the average ionization losses of charged particles contains the average ionization potential $I$ of the substance traversed by the particles. In the theory of ionization losses it is assumed that the average ionization potential $I$ depends only on the atomic properties of the medium and is in principle independent of the incident particle's velocity. However, considerable experimental evidence indicates that the average ionization potential tends to decrease with increasing particle velocity, at least for heavy elements ($Z > 13$).

Lindhard and Scharff used the Thomas-Fermi statistical model to show that for different energies the energy losses of charged particles in substances with large $Z$ is a function of only the parameter $x$ and is given by

$$- \frac{dE}{dx} = \frac{4\pi N (2e)^3}{m\alpha} L(x), \quad x = \frac{\alpha}{Z_0}, \quad \alpha_0 = \frac{e^2}{\hbar} .$$

(1)

As in the Bethe-Blloch theory, $L(x)$ is a logarithmic relation for $x > 100$, in which region the specific form of $L(x)$ proposed by Lindhard and Scharff provided a sufficiently good description of the experimental data available at the time and to some extent confirmed Sachs and Richardson's idea that $I$ might be dependent on energy.

Caldwell subsequently analyzed all available experimental data on the determination of ionization potentials of different elements for different energies.

### Table I. Summary of ionization potentials in ev

<table>
<thead>
<tr>
<th>Element</th>
<th>$Z$</th>
<th>From Caldwell for 18 Mev*</th>
<th>From Bloembergen and van Heerden for 60 Mev</th>
<th>From Thomson for 270 Mev</th>
<th>From Mather and Segre for 340 Mev</th>
<th>From Bakker and Segre for 340 Mev</th>
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<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>18.0</td>
<td>162±5</td>
<td>151.9</td>
<td>1136±100</td>
<td>810.7±12</td>
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<tr>
<td>Li</td>
<td>3</td>
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<td>151.9</td>
<td>147.9±3</td>
<td>810.7±12</td>
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<tr>
<td>Be</td>
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<td>34.0</td>
<td>147.9±3</td>
<td>309.9±3</td>
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<tr>
<td>C</td>
<td>6</td>
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<td>309.9±3</td>
<td>309.9±3</td>
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<td>N</td>
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<td>74.6</td>
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<tr>
<td>O</td>
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<td>Sn</td>
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<tr>
<td>Ta</td>
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<td>881</td>
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<tr>
<td>W</td>
<td>74</td>
<td>810.7±12</td>
<td>810.7±12</td>
<td>810.7±12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Based on the experimental work of Sachs and Richardson.8
incident energies, making all essential corrections. The result was strong modification of the experimental curve for $x < 100$. Caldwell concluded that there is no basis at present for assuming that $I$ depends on velocity; he was of the opinion that for high energies we must use higher ionization potentials than those given by Bakker and Segrè, which are approximately 132 ev.

In the present paper we shall present measurements of the range-energy relation for 660-Mev protons in copper.

EXPERIMENT

The mean energy of our external synchrocyclotron beam was 660 Mev, with a spread of ±4 Mev. The beam was formed by a collimator of 20 mm diameter. The instruments for precise measurement of proton energy and range were placed in line on a special truss behind a four-meter reinforced concrete shield 17 m from the exit window of the accelerator vacuum chamber. The experimental arrangement is shown in Fig. 1.

MEASUREMENT OF MEAN PROTON ENERGY

The mean proton energy was determined by Mather's method which was described in reference 9. In the present paper we shall give the experimental results with only a brief description of the procedure. The method is based on the sharply defined directivity of Vavilov-Cerenkov radiation, with the semiangle of the radiation cone given by

$$\cos \theta (\lambda) = 1/n (\lambda) \beta,$$

where $n (\lambda)$ is the refractive index of the medium for a given wavelength $\lambda$ and $\beta = v/c$ is the particle velocity.

Because of the continuous spectrum of Vavilov-Cerenkov radiation the form of the function $n (\lambda)$ must be taken into account in determining velocities from $\cos \theta (\lambda) = 1/n (\lambda) \beta$. Mather developed an ingenious scheme for measuring the emission angle of a given wavelength. In virtue of $n (\lambda)$ the blue portion of the radiation spectrum is located on the outside of the radiation cone while the red portion is located inside. When a portion of the cone of divergent light from the radiator passes through an achromatic prism we obtain an almost parallel light beam which can be focused by a lens into a narrow band of white light with its center of gravity around the position of a given wavelength. The vertex angle $\alpha$ of the achromatic prism is determined from the extinction equation of first order dispersion $d\psi/d\lambda = 0$ written for the case of perpendicular emission from the radiator and minimum prism angle of deviation, where $\psi (\lambda)$ is the direction in which the radiation is viewed after emerging from the prism. We now give this equation, which appears with a misprint in Mather's paper:

$$\frac{db}{d\lambda} = \frac{d\psi}{dn} \frac{dn}{d\lambda},$$

where

$$\sin \alpha = \left( n_0^2 + 4 (n_0^2 \beta_0^2 - 1) \right)^{-1/2},$$

where $\alpha$ is the vertex angle of the prism, $n_0$ is the refractive index for the wavelength $\lambda_0$ satisfying the condition $d\psi/d\lambda = 0$, and $\beta_0$ is the approximate mean velocity. Equation (3) can be used when the radiator and prism are made of the same material.

Proton velocities were determined by precise measurement of the emission angle of Vavilov-Cerenkov radiation from protons passing through Plexiglas. The choice of radiator material and thickness was dictated by the experimental conditions. Plexiglas was chosen as a substance with low atomic number, low density and relatively low dispersion, thus reducing multiple scattering and slowing down.

Preliminary energy measurements were performed...
formed with a proton beam of low density \((10^8 \text{ protons} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1})\) and, consequently, a relatively thick radiator. Radiation was registered by a camera with a 1:1.5 Jupiter-3 objective on “Negative-A” cine film of 50-unit GOST (All-Union State Standard) sensitivity. With a radiator thickness of 2.9 g/cm^2 in the beam direction exposures lasted 3 minutes.

The present measurements were obtained after the system of proton beam extraction was changed, increasing the intensity more than 100 times. The exposures used with a beam of \(4 \times 10^7 \text{ protons} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1}\) ranges from 3 to 5 seconds.

**MEASUREMENT OF REFRACTIVE INDEX**

An IRF-3 refractometer was used to measure the refractive index of a few samples taken from the same piece of material as the radiator. The absolute refractive index for \(\lambda = 5461 \text{ Å}\) was 1.4926 ± 5 \times 10^{-4}. This value of \(n\) was confirmed after the experiment by measurements performed on samples cut from the radiator in the region traversed by the proton beam.

**EXPERIMENTAL RESULTS**

The emission angle of Vavilov-Cerenkov radiation in Plexiglas was found to be \(\theta = (34° + 0.5') ± 3'\) for \(\lambda = 5461 \text{ Å}\). The proton energy was calculated from

\[
E = E_0 \left(1 - \frac{n \cos \theta}{(n \cos \theta - 1)^{1/5}}\right)^{1/3},
\]

where \(E_0\) is the proton rest mass, 938.2 Mev, and \(n\) is the refractive index of Plexiglas for \(\lambda = 5461 \text{ Å}\).

According to the measured values of \(\theta\) and \(n\) the proton energy at the center of the radiator was 654.9 Mev. When account is taken of slowing down up to the middle of the radiator the mean proton energy becomes 658.4 Mev \((\Delta E = 3.5 \text{ Mev for } t = 1.45 \text{ g/cm}^2\) and \(-dE/dx = 2.40 \text{ Mev/g/cm}^2\)). This was the average of three energy measurements: \(E_1 = 658.0 \text{ Mev}, E_2 = 657.6 \text{ Mev},\) and \(E_3 = 659.6 \text{ Mev}\).

The error in the energy values includes errors in measuring \(\theta\) and \(n\). \(\Delta E_1 / \Delta \theta\) and \(\Delta E_2 / \Delta n\) were determined by means of the relations

\[
\Delta E = E_0 \left(1 - \frac{n \cos \theta}{(n \cos \theta - 1)^{1/5}}\right)^{1/3}.
\]

In our case \(\Delta E_1 = \Delta \theta \times 0.59 \text{ Mev when } \Delta \theta\) is given in minutes, and \(\Delta E_2 = 2 \times 10^3 \Delta n \text{ Mev. For } \Delta \theta = ± 3.0, \Delta n = ± 5 \times 10^{-4}, n = 1.493\) and \(\beta \approx 0.81\) we obtain \(\Delta E_1 = ± 1.8 \text{ Mev}\) and \(\Delta E_2 = ± 1 \text{ Mev.}\)

The total energy error, which is the root mean square of the given errors, is \(\Delta E = ± 2.1 \text{ Mev.}\)

**MEASUREMENT OF TOTAL RANGE IN COPPER**

While measuring the mean proton energy we also measured the total range in copper (Fig. 1b). A collimated proton beam of 20 mm diameter traversed ionization chambers \(M_1\) and \(M_2\), which were separated by copper blocks. The ionization ratio \(J_2/J_1\) in the chambers was measured as a function of copper thickness.

Ionization chamber \(M_1\) was filled with helium to 0.5 atmos and \(M_2\) was filled with argon to 1 atmos. Measurements at the end of the absorption curve were performed in steps of 1.5 g/cm^2. A portion of a Bragg curve is shown in Fig. 2. Comparison of the experimental curve 1 for \(J_2/J_1\) with the theoretical curve 2 of Mather and Segre yields the energy spread of the proton beam.

The calculation of the theoretical proton range spread due to energy loss fluctuations took a relativistic factor into account:

\[
(\Delta R)^2 = 4 \pi \alpha^2 e^4 n Z^2 \int \frac{dE}{E} \left(1 - \frac{E'}{E}\right)^{1/2} \left(1 - \frac{\beta^2}{1 + \beta^2} \frac{E'}{E}\right) dE',
\]

where \(\beta = \beta (E')\) is the particle velocity. For a copper absorber and proton energy \(E = 658 \text{ Mev}\) we have \(\Delta R = 2.65 \text{ g/cm}^2\).

Besides the indicated spread we took into account range fluctuations due to multiple scattering fluctuations:

\[
\frac{\Delta R_{\text{sf}}^2}{R^2} = \frac{2 \alpha^2 (E/\mu)^3 \sum_{n=0}^{\infty} \frac{n+1}{(n+2)^2} \left(\frac{E_0 - \mu}{E_0 + \mu}\right)^{1/2}}{4 \pi \alpha^2 (E/\mu)^3},
\]

where \(\mu = m L / (2 \pi L_r)\),

\(L_1 (E_1)\) and \(L_r\) being given below [see Eq. (9)].

A calculation using (7) and (8) gives

\(\Delta R = 2 (\Delta S^2 - S^2)/S^2 = 0.83 \text{ g/cm}^2\).
The effective range \( t \) is given by
\[
\frac{s-t}{t} \approx \frac{Zs \epsilon^2 \rho_0}{e m (E_0 - \nu)} \frac{L_{s}}{L_{E_0}} \ln \left( \frac{(E_0 + \nu)^2}{4E_0\nu} \right),
\]
where \( Z \) is the charge of absorber atoms, \( E_0 \) is the total particle energy, \( m \) is the electron rest mass in Mev, \( E_8 = m (4\pi \chi_0)^{1/2} = 21 \) Mev, \( \mu \) is the incident particle rest mass, \( L_\nu \) is the "radiation logarithm" and \( L_{E_0} \) is the "ionization logarithm" for an intermediate energy \( E_1 \). \( E_1 \) is given by
\[
E_1 = \frac{1}{2} \left[ E_0 - \mu - 2\mu \ln \left( \frac{E_0 + \nu}{2\mu} \right) + K\mu^2 \right] \times \left[ \ln \left( \frac{(E_0 + \nu)^2}{4E_0\nu} \right) \right]^{-1},
\]
where
\[
K = \frac{e}{\hbar} \left( 1 - b e^{-t} \right)^{-1} \left( 1 - be^{-t} \right)^{1/2} e^{-et} \int dt,
\]
\( b = (E_0 - \nu)/(E_0 + \nu) \).

The values of \( E_1 \) (in \( \mu \)) for \( E_0 \) from 1.2 to 5\( \mu \) are as follows:
\[
\begin{align*}
E_1 &= 1.20 \quad 1.30 \quad 1.40 \quad 1.50 \quad 1.60 \quad 1.70 \quad 1.80 \quad 1.90 \\
E_1 &= 1.11 \quad 1.16 \quad 1.20 \quad 1.26 \quad 1.31 \quad 1.40 \quad 1.45 \quad 1.48 \\
E_1 &= 2.0 \quad 2.2 \quad 2.5 \quad 3.0 \quad 3.5 \quad 4.0 \quad 4.5 \quad 5.0
\end{align*}
\]

Figure 3 gives the values of \((s-t)/t\) for C, Al, Be, Cu, and Pb. The ordinates give \((s-t)/t\) as a percentage for a particle with the rest mass \( \mu = 100 \) Mev; for other masses we use the relation
\[
\frac{s-t}{t} = 100 \left( \frac{s-t}{t} \right)_{\mu=100}
\]

The abscissas give the initial total energy in units of \( \mu \). The accuracy of \((s-t)/t\) based on (9), (10), and (11) is \( \sim 1\% \).

For 658-Mev protons \((E_0 = 1.7 \mu)\) the correction for multiple scattering of protons in copper is 0.46\%; the mean range is then 257.6 g/cm\(^2\).

Accuracy of the range value is determined by the accuracy in measuring the absorber thickness and by the uncertainty resulting from variation of mean proton energy during the course of the experiment. Several checks of the range during the experiment showed that the deviation did not exceed \( \pm 0.2 \) g/cm\(^2\), while the total error including that involved in measuring absorber thickness amounted to \( \pm 0.3 \) g/cm\(^2\).

**DETERMINATION OF IONIZATION POTENTIAL FOR COPPER**

Precise measurements of proton energy and range permit determination of the ionization potential for copper by means of the relation
\[
R = \int \left( \frac{dE}{dx} \right)^{-1} dE,
\]
where \( R \) is the total range (in grams per cm\(^2\)) of protons with initial energy \( E_0 \) (in Mev) in a
The ionization potential of stopping atoms, \( \Sigma C_i \), represents corrections for particle velocities comparable to electron velocities in K, L, ... shells, \( \delta \) is the Fermi density correction, and \( T \) is the maximum energy transferred by an incident particle to an atomic electron, which is given by:

\[
T = \frac{E^2 - \mu^2 c^4}{\mu c^2} \left( \frac{\mu}{2m + \mu} + E \right) / \mu c^2,
\]

(14)

where \( E \) is the energy of the incident particle, \( \mu \) is the reduced mass of the particle and rest mass, \( \beta = v/c \) is the particle velocity, \( I \) is the mean ionization potential of stopping atoms, \( \Sigma C_i \) represents corrections for particle velocities comparable to electron velocities in K, L, ... shells, \( \delta \) is the Fermi density correction, and \( T \) is the maximum energy transferred by an incident particle to an atomic electron, which is given by:

\[
-\frac{dE}{dx} = \frac{2\pi e^4}{m e^2 c^2} \ln \left( \frac{2m\beta}{\beta - 1} \right) - \frac{2\pi}{Z} \sum_{i=1}^{\infty} \frac{C_i - \delta}{\beta^i}.
\]

(13)

where \( e \) and \( m \) are the electron charge and mass, \( x \) is the charge of the incident particle, \( n \) is the number of electrons per cm\(^2\) of stopping material with density \( \rho \), \( \beta = v/c \) is the particle velocity, \( I \) is the mean ionization potential of stopping atoms, \( \Sigma C_i \) represents corrections for particle velocities comparable to electron velocities in K, L, ... shells, \( \delta \) is the Fermi density correction, and \( T \) is the maximum energy transferred by an incident particle to an atomic electron, which is given by:

\[
T = \frac{(E^2 - \mu^2 c^4)/\mu c^2}{\mu/2m + \mu + E/uc^2}.
\]

The most accurate calculation of the density effect in different substances is that of Sternheimer. The empirical expression for \( \delta \) is:

\[
\delta = 4.606 X + C + (X_i - X)\delta_i
\]

(15)

where \( X = \log (\rho c/\mu) \), and \( C \) and \( \mu \) are the particle momentum and rest mass, \( X_i \) is the value of \( X \) above which \( \delta \) is linearly dependent on \( X \), \( X_0 \) is the value of \( X \) below which \( \delta = 0 \) and is determined from the velocity \( \beta_0 = \left(1 + \sum \nu_i^2/\mu_i^2\right)^{1/2} \) (where \( \nu_i \) is the oscillator strength of a given transition, which is equal to the number of electrons on the given level divided by the total number of electrons in the atom), and \( \nu_i = E_i/h \) is the i-th transition frequency. For copper we have \( C = 4.13, 10\alpha = 0.99, m = 3.40, X_i = 3, \) and \( X_0 = 0.00. * \) The value of \( \delta \) is \( \sim 0.12\% \) for 400-Mev protons and \( \sim 0.25\% \) for 600-Mev protons. In our case the density effect reduces \( I_{Cu} \) by 2 ev.

\( I_{Cu} \) was computed as follows. Equation (12) was integrated numerically to determine \( R_1 \) and \( R_2 \) for two different ionization potentials close to the experimental value, such as \( I_1 = 375 \text{ ev} \) and \( I_2 = 300 \text{ ev} \). We thus determined \( \Delta R/\Delta I = (R_2 - R_1)/(I_2 - I_1) = 0.126 \text{ g/cm}^2 \text{ ev} \), and the ionization potential satisfying initial energy \( E_0 = 658 \text{ Mev} \) and range \( R_0 = 257.6 \text{ g/cm}^2 \) was \( I_{Cu} = 305 \text{ ev} \). It should be noted that the range was calculated assuming velocity independence of the ionization potential \( I_{Cu} \). The computed proton range for \( I_{Cu} = 377 \text{ ev} \) is 8.84 g/cm\(^2\) greater than the experimental range.

The error in determining the ionization potential by the foregoing method is a combination of the errors in determining the initial proton energy and the range. The r.m.s. error in the range is \( \pm 1.2 \text{ g/cm}^2 \), which leads to the uncertainty \( \Delta I_{Cu} = \pm 0.1 \text{ ev} \) in the ionization potential. The value \( I_{Cu} = 305 \pm 10 \text{ ev} \) obtained in the present experiment agrees well within the limits of error with Mather and Segre's value of 310 ev for 340-Mev protons.

### Table II

<table>
<thead>
<tr>
<th>Element</th>
<th>( q )</th>
<th>( I/\text{ev} )</th>
<th>Bloch constant ( L = I(1/2, \text{ev}) )</th>
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<td>H</td>
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<tr>
<td>Be</td>
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<td>C</td>
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<tr>
<td>Cu</td>
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<tr>
<td>Cd</td>
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<td>498±35</td>
<td>9.8</td>
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<tr>
<td>W</td>
<td>0.794</td>
<td>680±50</td>
<td>9.2</td>
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Ionization potentials were calculated assuming that $I_{Cu} = 305$ ev for $E_0 = 635$ Mev. Corrections were included for the binding of K and L electrons as well as for the density effect. The final expression for ionization potentials based on measurements of relative stopping powers is

$$\ln I = 13.788 - \frac{A}{2} - 3.670q - \frac{1}{2}\sum_{K,L} C_i - \frac{3}{2},$$

(16)

where $A$ is the atomic weight.

Relative stopping powers were measured with an accuracy of about $\pm 1\%$; therefore the accuracy of the ionization potentials is $\pm 10\%$, since $\Delta I/I \approx (\Delta q/q) \ln (2mv^2/1)$. The stopping power of hydrogen* was determined from the CH$_2$ – C difference.

**DISCUSSION OF RESULTS**

The paper of Sachs and Richardson$^9$ contains a graph showing values of the ionization potential for copper that were determined by different methods for different proton energies, as a function of the effective energy, which when $I$ is determined for the entire range equals 0.6 of the initial energy $E$ if we assume $I = I_0 - a \ln E$. The data of the present work for 400 Mev are included in the same graph for comparison (Fig. 4). Up to 60 Mev the ionization potential of copper is $\sim 380$ ev, while at higher energies it is reduced to $\sim 310$ ev.

As indicated above, in the present paper I$_{Cu}$ was calculated assuming it to be independent of proton velocity. I$_{Cu}$ was also calculated with the experimental data for low energies taken into account. For proton energies from 0 to 125 Mev we assumed the constant value I$_{Cu} = 377$ ev; from 125 to 660 Mev we assumed 305 ev. The theoretical and experimental ranges could be brought into agreement only by the value 301 ev for I$_{Cu}$ in the 125 – 660 Mev range. No other assumptions were made regarding the energy dependence of I$_{Cu}$; the assumptions already adopted yield 300 ev as the ionization potential of copper for a mean proton energy of about 400 Mev. The observed drop of 70 – 80 ev in this proton energy range is outside the limits of experimental error and requires an explanation.

If we assume that an ionization potential cannot depend on the velocity of an incident particle the decrease of the former as the particle energy rises can either indicate some additional form of loss at high energies which is not included in Bethe’s theory, or that some effect reduces the loss at low energies.

The variation of $I$ with energy may possibly be associated with the effective density at low energies. Indeed, the calculations of ionization potentials from absolute measurements of $dE/dx$ at low energies have usually employed an expression which neglected the Fermi density effect, although this effect does not disappear at low energies. Sternheimer$^{10}$ has shown that the reduction of ionization loss $\Delta (dE/dx)$ at low energies due to the density effect is given by

$$\frac{\Delta dE}{dx} = \frac{2n \pi (z^2 e^2)^3 \sum_i f_i \ln \left( \frac{l_i}{\nu_i} \right)}{\rho m v^2} = \frac{2n \pi (z^2 e^2)^3 \delta}{\rho m v^2},$$

(17)

where $l_i = (\nu_i^2 + l_i^2)^{1/2}$ with all other notation as in (13). In first approximation, i.e., using transition frequencies $\nu_i$ taken from Sommerfeld’s tables,$^{20}$ we have $\delta (Cu) = 0.26$. $\Delta (dE/dx)$ for protons with $\sim 20$ Mev in copper may be a few percent.

In calculating the reduction of loss at high proton energies we must use the formula$^{19}

$$\frac{\Delta dE}{dx} = \frac{2n \pi (z^2 e^2)^3 \sum_i f_i \ln \left( \frac{\nu_i^2 + l_i^2}{\nu_i^2} \right)}{\rho m v^2} - l^2 (1 - \beta^2),$$

(18)

where $I$ is defined by

$$\beta^2 - 1 = \sum_i f_i / (\nu_i^2 + l_i^2).$$

In this case for 600-Mev protons with the same values of the frequencies $\nu_i$, $\Delta (dE/dx)$ for copper is only 0.1% and increases to 3% at 2 Bev. The reduction of $dE/dx$ by a few percent at low energies is equivalent to reduction of the copper ionization potential by 20 – 30%, i.e., to $\sim 300$ ev. It should be noted that the value of the sum

$$\delta = \sum f_i \ln (l_i / \nu_i)$$

is uncertain because the transition frequencies $\nu_i$ are uncertain. If a more exact calculation eventually shows that the correction factor $\delta$ actually has the given value, there will hardly be any reason for regarding $I$ as velocity dependent. $I$ could then be given a constant

*For hydrogen the accuracy of $q$ is $\sim 5\%$ and that of $I_H$ is $\sim 50\%$. 

FIG. 4. Ionization potential of copper as a function of proton effective energy. $\bullet$ – present data ($E_{eff}$ = proton effective energy).
value at 10$^{-11}$ Z ev and the density effect would be taken into account in calculations of energy losses over the entire energy range. In the event of the contrary result the reduction of $I$ at higher energies must be accounted for in some other manner.

In conclusion we wish to thank Yu. D. Prokoshkin and I. M. Vasilevskii for discussions and for acquainting us with preliminary data on absolute losses for 650-Mev protons.

1 S. Livingston and H. A. Bethe, Revs. Modern Phys. 9, 264 (1937).
20 A. Sommerfeld, Atombau und Spektrallinien I (Russ. Transl.) GITTL, Moscow, 1956, p. 212. [Vieweg, Brunswick, 1955].
## ERRATA TO VOLUME 9

<table>
<thead>
<tr>
<th>Page</th>
<th>Reads</th>
<th>Should read</th>
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</thead>
<tbody>
<tr>
<td>294, Col. 2, line 4 from bottom</td>
<td>$N = N_{\text{exp}} (p, \theta) F (p, \theta)$ which are approximately $13Z \text{ ev}$</td>
<td>$N = N_{\text{exp}} (p, \theta) 1 + F (p, \theta)$ and approximately equal to $13Z \text{ ev}$</td>
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<tr>
<td>462, Col. 1, line 8 from top</td>
<td>$\langle j_1 t' \alpha</td>
<td>R_{J_2}</td>
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<td>646, Col. 1, line 3 from top</td>
<td>$\chi = 2.14 \times 10^{-13}$</td>
<td>$\chi = 1.04 \times 10^{-13}$</td>
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<td>661, Col. 1, line 6 from top</td>
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