RADIO EMISSION FROM AN ATOMIC EXPLOSION

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It is shown that in an atomic explosion, which is accompanied by asymmetric emission of γ quanta, radio waves are emitted due to the presence of a current in the ionized air. The duration of the oscillation in each half-wave is of the order of $10\,\mu$ sec. For a given asymmetry in emission of the γ rays, the amplitude of the oscillations depends weakly on the total number of quanta.

It is known that atomic bomb blasts are accompanied by characteristic radio signals which are recorded at distances of thousands and tens of thousands of kilometers. In propagating over such large distances the radio pulse spectrum becomes much more complex than the spectrum of the initially emitted pulse.[1] Only the latter spectrum will be studied in the present paper. This is the spectrum which one can record at a distance of say one hundred kilometers from the point of the explosion. In other words, we shall try to determine the effective radiator.

G. M. Gandel'man and L. P. Feoktistov (private communication) have described how an electric field develops in the air under the action of the γ quanta. The quanta give rise to Compton electrons which move preferentially forward, i.e., away from the point of the blast. But since the flux of quanta decreases approximately exponentially with distance from the blast origin, the numbers of negative charges which are transferred to points at different distances from the origin are different. This nonuniformity of the charge density results in the appearance of an electric field.

In moving along their paths, the Compton electrons produce a large number of secondary electrons, thus making the air conducting and producing a current which tends to annul the field. If the γ quanta are emitted asymmetrically, the current radiates an electromagnetic pulse. But the duration of the pulse due to the electronic current can be of the order of one or a few microseconds, which can give radiowaves in the meter wave length band. The longer wave length oscillations are of great interest.

The cause of these oscillations has been given by O. I. Leipunskii (private communication). The electrons attach themselves not to positive ions but mainly to neutral O_2 molecules. The air acquires an ionic conductivity and a current in it lasts much longer than it does during the stage where the conduction is electronic. At the moment when ionic conduction is established there is still an electric field in the air. O. I. Leipunskii showed that the magnitude of this field is practically independent of the initial ionization at the given space point: if the ionization is greater, the initial space charge density is greater, but then so is the electronic conductivity of the air which causes the dissipation of this space charge. This same result is obtained in more rigorous form in the present paper.

But if the field is independent of the initial ionization, the initial asymmetry in emission of the quanta has no effect on it. Thus the field is purely radial and symmetric. An asymmetry arises because of the ionic conduction of the air which is, of course, greater on the side toward which more quanta were emitted. As the later computations show, the asymmetric ionic current gives a pulse of reasonable order of magnitude and the expected duration.

1. THE INITIAL FIELD

Let the number of γ quanta emerging from the blast origin per unit solid angle per unit time in a given direction be $\dot{N}_0/4\pi$. (The total number of emitted quanta will then be $\overline{N}_0$, where the average is taken over all solid angle.) We denote the mean free path of the quanta by λ, and the number of secondary electrons per Compton electron by ν. Then the number of free electrons which appear at distance r from the blast origin per unit time per unit volume is

$$J = \dot{N}_0 \nu e^{-r/\lambda} / 4\pi\lambda r^2. \quad (1)$$

We shall not treat effects related to multiple scattering of quanta since they are of secondary importance.

If γ is the probability per unit time of attachment of an electron to a molecule and n_e is the volume concentration of secondary electrons, then n_e satisfies the differential equation

$$\partial n_e / \partial t + \gamma n_e = J, \tag{2}$$

which has the integral

$$n_e = e^{-\gamma t} \int_0^t J(t') \exp(\gamma t')\, dt'. \tag{3}$$

The space charge density ρ is determined by the law of conservation of charge, which in the present case gives the equation

$$\partial \rho / \partial t = -\operatorname{div} \sigma_e \mathbf{E} + \dot{\rho}_0, \tag{4}$$

where $\mathbf{E}$ is the electric field, σ_e is the electronic conductivity of the air, which is considerably greater than its ionic conductivity so long as there are free electrons present, and $\dot{\rho}_0$ is the rate of increase of the space charge density resulting from displacement of the Compton electrons through the air.

We use l to denote the average radial component of the free path of the Compton electrons. Obviously, $l \ll \lambda$. $\dot{\rho}_0$ is given by the difference between the numbers of electrons leaving a given point and coming toward it, or

$$\dot{\rho}_0 = e\,[J(r) - J(r + l)] / \nu.$$

For points not too close to the origin, this gives

$$\dot{\rho}_0 = Jle / \lambda\nu. \tag{5}$$

The Poisson equation gives

$$\operatorname{div} \mathbf{E} = 4\pi\rho. \tag{6}$$

Substituting in Eq. (4), we find

$$\operatorname{div}\left(\frac{1}{4\pi}\frac{\partial \mathbf{E}}{\partial t} + \sigma_e \mathbf{E}\right) = \dot{\rho}_0. \tag{7}$$

We note that the diffusion current has been omitted since it is much smaller than the conduction current in the cases we are considering.

As already stated, σ_e and $\dot{\rho}_0$ depend on the angle. Nevertheless we shall show that at the instant of time in which we are interested the field has only a radial component. Thus the divergence operator on the right side of (6) actually contains only derivatives with respect to r:

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \left(\frac{1}{4\pi}\frac{\partial E_r}{\partial t} + \sigma_e E_r\right) = \dot{\rho}_0. \tag{8}$$

We impose the boundary condition $E_r(\infty, t) = 0$ on E_r. Integrating Eq. (8) subject to this condition, and using Eqs. (5) and (1), we get

$$\partial E_r / \partial t + 4\pi\sigma_e E_r = -(\dot{N}_0 el / \lambda r^2) \exp(-r/\lambda). \tag{9}$$

Then

$$E_r = -(el / r^2\lambda)\, e^{-r/\lambda} \int_0^t \exp\left\{-4\pi \int_{t'}^t \sigma_e(t'')\, dt''\right\} \dot{N}_0(t')\, dt'. \tag{10}$$

As already stated, σ_e is the electronic conductivity of the air. Denoting the electron mobility by the symbol ω_e, we get [cf. Eq. (3)]:

$$\sigma_e = e\omega_e n_e = \frac{e\omega_e \nu}{4\pi r^2 \lambda} e^{-r/\lambda} \int_0^t \dot{N}_0(t''') \exp\{\gamma(t''' - t)\}\, dt'''. \tag{11}$$

A double integral of σ_e with respect to the time appears in the expression for E_r. This can be reduced to a single integral, giving

$$4\pi \int_{t'}^t \sigma_e dt'' = \frac{e\omega_e \nu}{r^2 \lambda} e^{-r/\lambda} \frac{1}{\gamma} \left[\int_0^t \dot{N}_0(t'')(1 - \exp\{\gamma(t'' - t)\})\, dt'' - \int_0^{t'} \dot{N}_0(t'')(1 - \exp\{\gamma(t'' - t')\})\, dt''\right]. \tag{12}$$

To find E_r we must assign the time dependence of $\dot{N}_0$. We can say the following about the shape of this time dependence. Initially, while the chain reaction is still developing, $\dot{N}_0(t)$ increases very rapidly. After reaching a maximum, $\dot{N}_0(t)$ decreases, but at a rate much slower than its rate of rise.

We may assume the dependence

$$\dot{N}_0(t) = A \exp\{-\beta t\}. \tag{13}$$

As we shall see from the later calculations the field E_r is extremely insensitive to the value of β so long as the inequality

$$\beta < \gamma \tag{14}$$

is satisfied. $1/\gamma$, the time for attachment of an electron to a molecule, is approximately 4×10^{-7} sec (cf. reference 2), while $1/\beta$ is larger than this, which is the basis for the inequality (14).

We substitute the time dependence (13) in formula (12) and carry out the integration, giving

$$E_r = \frac{el}{\lambda r^2} e^{-r/\lambda} \times \exp\left\{-\frac{e\omega_e \nu \exp(-r/\lambda)}{r^2\lambda\gamma}\left[\frac{A}{\beta} - \frac{A\gamma \exp(-\beta t)}{\beta(\gamma - \beta)} + \frac{A \exp(-\gamma t)}{\gamma - \beta}\right]\right\} \times \int_0^t A e^{-\beta t'}\, dt' \exp \times \left\{\frac{e\omega_e \nu \exp(-r/\lambda)}{r^2\lambda\gamma}\left[\frac{A}{\beta} - \frac{A\gamma \exp(-\beta t')}{\beta(\gamma - \beta)} + \frac{A \exp(-\gamma t)}{\gamma - \beta}\right]\right\}. \tag{15}$$

The exponential in this equation contains the quantity

$$e\omega_e A\nu \exp(-r/\lambda)/\lambda r^2\beta(\gamma-\beta)$$

in its argument. Setting A/β equal to the total number of quanta, which is say 10^{22}, the number ν of secondary electrons per γ quantum $\sim 3\times 10^4$, $r\sim\lambda\sim 3\times 10^4$ cm, $\omega_e \sim 2.5\times 10^6$ cgs units, we find that the order of magnitude of the expression in the exponent is 4×10^3, i.e., it is very large. Consequently the main contribution to the integral (15) comes from large values of t', when the function $\exp(-\beta t')$ is smallest, since it appears with a negative sign in the integral. (This is also the reason why the stage of increase in number of quanta is unimportant). This statement is valid if the inequality (14) is satisfied, i.e., if the exponential $\exp(-\gamma t')$ is smaller than $\exp(-\beta t')$. Then the exponential $\exp(-\gamma t')$ has no effect on the value of the integral and drops out of the final expression for E_r, if t is not too small. The result of the integration is

$$E_r = E = (\gamma-\beta)\,l/\omega_e\nu. \tag{16}$$

A similar result was obtained by O. I. Leipunskii.

Thus the total number of γ quanta and their angular distribution actually are not affected by the magnitude of the residual field, so long as A/β is sufficiently large. But this is practically always the case.

E denotes the radial field. It does not go to zero at the origin. Consequently the charge density has a singularity at the origin:

$$\rho = \frac{1}{4\pi r^2}\frac{\partial}{\partial r}\,r^2E = \frac{E}{2\pi r}.$$

But this singularity does not contribute anything to the total charge, since ρ is multiplied by $r^2 dr$ when we integrate over the volume. The appearance of such a singularity is entirely reasonable physically, since $J(r,t)$ has a singularity at the origin. A numerical calculation gives a value for E of order of magnitude 2 v/cm.

2. ELECTROMAGNETIC OSCILLATIONS

The time of development of the initial field is of order of magnitude $1/\beta$, which may be assumed to be about one microsecond. The period of the electromagnetic oscillations is approximately ten times as large. Thus the whole process can be divided into two stages: the first stage is the development of the field considered in the preceding section, and the second is the damping of the oscillations produced by the field.

In the second stage, the conductivity of the air is ionic. Consequently the damping of the oscillations is much less than it would be for the case of electronic conductivity.

The number of ions decreases continually as a result of recombination. The recombination process must be taken into account along with the ionic conductivity, because the recombination coefficient is related to the ion mobility by the formula (cf. reference 2)

$$b \approx 4\pi e(\omega_+ + \omega_-) = 4\pi e\omega. \tag{17}$$

This formula is in satisfactory agreement with experiment for air at normal density.

We shall now find the expression for the ionic conductivity of the air as a function of coordinates and time. For the density of ions of one sign we have $dn/dt = -bn^2$, so that

$$n = 1/(bt + 1/n_0). \tag{18}$$

Here n_0 is the initial number of ions per cc. Considering that there is only a slight angular dependence of the emission of quanta, we write

$$n = 1/[bt + (1+\xi\cos\vartheta)/\bar{n}_0].$$

Limiting ourselves to terms of first order in ξ, we get the following expression for the electrical conductivity:

$$\sigma = ne\omega = \frac{e\omega}{bt + 1/\bar{n}_0} - \frac{e\omega\xi\cos\vartheta}{\bar{n}_0(bt+1/\bar{n}_0)^2} \equiv \sigma_0 + \sigma_1\cos\vartheta. \tag{19}$$

In this formula, $\bar{n}_0$ is defined by the equation

$$\bar{n}_0 = (A\nu/4\pi\beta\lambda^3)\,\mu(r/\lambda), \tag{20}$$

where

$$\mu(x) = x^{-2}e^{-x}. \tag{21}$$

Thus at the initial instant the field is radially symmetric and is given by Eq. (16), while the electric field depends on angle. Consequently the current will have a corresponding asymmetry.

The electrical conductivity leads to a falloff of the initial field with time. Keeping only the term with σ_0, we have

$$\partial E_{0r}/\partial t + 4\pi\sigma_0 E_{0r} = 0, \tag{22}$$

where the index 0 on E_{0r} is a reminder that in this approximation the field is assumed to be centrally symmetric. Integrating (22) we find, by making use of (18):

$$E_{0r} = E\exp\left\{-\int_0^t 4\pi\sigma_0 dt'\right\} = E\exp\left\{-\frac{4\pi e\omega}{b}\ln(b + 1/\bar{n}_0)\Big|_0^t\right\}. \tag{23}$$

If we took Eq. (17) literally, we should replace the coefficient of the logarithm by unity. We shall however set

$$4\pi e\omega / b \equiv \alpha, \tag{24}$$

where α can be determined from experimental data. Thus

$$E_{0r} = E\,(bt\bar{n}_0 + 1)^{-\alpha}. \tag{25}$$

The effect of recombination is to give a power law for the damping instead of an exponential law which one would get if the ion concentration were constant in time. The quantity $\sigma_1 \cos\vartheta$, which was assumed to be small, will be multiplied by E_{0r}, which gives an inhomogeneous term in Maxwell's equations and gives rise to an asymmetric current. The assumption that the asymmetry is small was of course made only for computational reasons. If we assumed arbitrarily large asymmetry, we would have to perform numerical integration of the Maxwell equations with respect to three independent variables (radius, angle, and time), which is immeasurably more difficult than the (also numerical) determination of quantities which depend only on the radius and the time.

We shall write the Maxwell equations in the form

$$\operatorname{curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi\sigma}{c}\mathbf{E}, \tag{26}$$

$$\operatorname{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{27}$$

$$\operatorname{div}\mathbf{H} = 0. \tag{28}$$

As usual for conducting media, we need not write the equation for $\operatorname{div}\mathbf{E}$, since it follows from (26).

We look for a solution of the system (26) to (28) in the form

$$E_r = E_{0r} + E_1\cos\vartheta, \quad E_\vartheta = E_2\sin\vartheta, \quad E_\varphi = 0;$$

$$H_r = H_\vartheta = 0, \quad H_\varphi = H\sin\vartheta.$$

When this form is substituted in the Maxwell equations, the angular dependence separates off, and we get a system of equations for E_1, E_2, and H.

We introduce the following dimensionless variables:

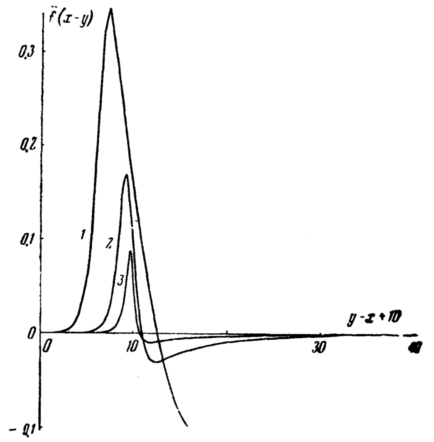
$$x = r/\lambda, \quad y = ct/\lambda, \quad H = \xi E H', \quad E_1 = \xi E E_1',$$

$$E_2 = \xi E E_2', \quad m = (e\omega\nu / \alpha\lambda^2 c)\,A/\beta.$$

Then the final system of equations which is to be integrated numerically has the form*

$$\frac{2H'}{x} = \frac{\partial E_1'}{\partial y} + \frac{m\alpha\mu(x)\,E_1'}{m\mu(x)\,y+1} + \frac{m\alpha\mu(x)}{(m\mu(x)\,y+1)^{2+\alpha}}; \tag{29}$$

*If the asymmetry of the emission of quanta is quadrupole, with the shape $\xi(3\cos^2\vartheta - 1)/2$, there will be a coefficient of 3 before the E_1' in (31).



*Values of parameters: 1) $\alpha = 4$, $m = 200$; 2) $\alpha = 1$, $m = 10$; 3) $\alpha = 1$, $m = 1$.

$$-\frac{1}{x}\frac{\partial}{\partial x}\left(xH'\right) = \frac{\partial E_2'}{\partial y} + \frac{m\alpha\mu(x)\,E_2'}{m\mu(x)\,y+1}, \tag{30}$$

$$\frac{1}{x}\left(\frac{\partial}{\partial x}(xE_2') + E_1'\right) = -\frac{\partial H'}{\partial y}, \tag{31}$$

where $\mu(x)$ is given by (21).

The initial condition on H', E_1' and E_2' is that all three are zero for $t = 0$.

From symmetry considerations, the electric field at $r = 0$ must be along the line from which the angle ϑ is measured. This gives

$$E_1'(0, t) + E_2'(0, t) = 0. \tag{32}$$

The magnetic field at the origin must be equal to zero:

$$H'(0, t) = 0. \tag{33}$$

We now consider the question of radiation of electromagnetic waves. At a large distance from the point of the blast, where the function $\mu(x)$ may be assumed to be equal to zero, we can make the substitution

$$\zeta = \frac{1}{x}\frac{\partial}{\partial x}(x^2 H') \tag{34}$$

and reduce the system (29) to (31) to the wave equation

$$\partial^2\zeta/\partial x^2 - \partial^2\zeta/\partial y^2 = 0, \tag{35}$$

whose solution we write as $\zeta = \ddot{f}(x-y)$. The field is given in terms of the function f by the formulas

$$H' = x^{-1}\ddot{f}(x-y) - x^{-2}\dot{f}(x-y),$$
$$E_2' = x^{-1}\ddot{f}(x-y) - x^{-2}\dot{f}(x-y) + x^{-3}f(x-y). \tag{36}$$

Thus the function f determines the effective radiating dipole. It is determined by integrating the system numerically along the characteristic $x = y$ out to the region where $\mu(x) = 0$. The results of the integration are given in the figure for three cases. The ordinate is the function $\ddot{f}(x-y)$ and the abscissa is the quantity $y - x + 10$. As we see, the signal amplitude depends approximately logarithmically on the value of m, i.e., on the total number of quanta. The signal duration has approximately the same dependence. The latter result seems very natural, since the wave length is comparable with the dimensions of the ionized region, which depend logarithmically on the number of quanta.

In conclusion, I express my gratitude to O. I. Leipunskii who pointed out the role of the ionic conductivity of the air in the production of the radio signal and stimulated the present work by numerous discussions. The analysis of the equations and the necessary computations for the present problem were done by A. A. Miliutin, S. L. Kamenomostskaia, and V. I. Kozhevnikov, to whom I express my deep indebtedness. I thank A. A. Dorodnitsyn and M. V. Keldysh for providing the computing machines.

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