

tors in the directions of the relative momenta of the initial and final states. From (4) and (5) we have

$$\sigma = \sigma_0(1 + \mathbf{P}_0 \cdot \mathbf{P}); \quad \mathbf{P} = \frac{ab_0^* + a^*b_0}{aa^* + bb^*} [\mathbf{k} \times \mathbf{k}'], \quad (6)$$

where  $\sigma_0$  is the cross section of the unpolarized protons.

2. The intrinsic parity changes, i.e.,  $I_\pi I_P = -I_Y I_K$ . The matrix (2) is pseudoscalar with  $a = 0$  and  $\mathbf{b} = b_1 \mathbf{k} + b_2 \mathbf{k}'$ . In this case we have

$$\sigma = \sigma_0(1 - \mathbf{P}_0 \cdot \mathbf{P}); \quad \mathbf{P} = i \frac{b_1 b_2^* - b_2 b_1^*}{b \cdot b^*} [\mathbf{k} \times \mathbf{k}']. \quad (7)$$

Finally we obtain

$$\sigma(\theta, \varphi) = \sigma_0(1 \pm P_0 P \sin(\delta - \varphi)), \quad (8)$$

where  $\delta$  is the azimuth of the initial polarization vector ( $\mathbf{P}_0$  is in the plane perpendicular to  $\mathbf{k}$ ). Hence the asymmetry is equal to

$$e(\theta) = \pm P_0 P \sin \delta. \quad (9)$$

Thus a measurement of the reaction (1) with a polarized target would permit a determination of the polarization  $P$  of the hyperon in the reaction with an unpolarized target. If the parity ( $KY$ ) relative to ( $\pi p$ ) is known, then this experiment would also allow one to determine the sign of the polarization. On the other hand, if the sign of the polarization is determined from the hyperon decay, then the proposed experiment would afford a possibility to determine the relative parity ( $KY$ ).

<sup>1</sup>F. S. Crawford et al., Phys. Rev. **108**, 1102 (1957).

<sup>2</sup>T. D. Lee et al., Phys. Rev. **106**, 1367 (1957).  
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### SOME REMARKS ON THE STRUCTURE OF SHOCK WAVES

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1. The behavior of entropy in the transition zone of a not very strong shock wave is described by the general heat-transfer equation. Ia. B. Zel'-

dovich has shown that in a gas with a high thermal conductivity and negligible viscosity the entropy goes through a maximum and in that case the speed of sound in the system of coordinates associated with the discontinuity is equal to the local speed of sound.<sup>1</sup> We shall show that this coincidence also occurs in other cases. For a maximum to exist we must have a non-hydrodynamic energy flux associated above all with the thermal conductivity, since the viscosity alone can only increase the entropy. If we differentiate with respect to  $V$  (specific volume) the condition of the conservation of momentum flux at the point  $dS = 0$  we obtain:

$$\begin{aligned} \left(\frac{dp}{dV}\right)_S &= -m^2 + \left(\frac{4}{3}\mu + \zeta\right) \frac{d}{dV} \left(\frac{du}{dx}\right) \\ &= -m^2 + \left(\frac{4}{3}\mu + \zeta\right) \frac{d^2u}{dx^2} \frac{dV}{dx}. \end{aligned} \quad (1)$$

The last term is equal to zero since at the point  $S = S_{\max}$  the velocity has a point of inflection, which can be shown to exist both in the case of weak waves,<sup>2</sup> and also in the case of waves of arbitrary intensity. At the same time  $(dp/dV)_S = -m^2 = -\rho^2 u^2$ , from which it follows that  $\pm u = c_0$ .

If we now turn to magnetohydrodynamics, then finite conductivity (Joule heat) alone can only increase the entropy. It may go through a maximum if we take thermal conductivity into account. Then, by investigating the conditions for the conservation of momentum flux and of the magnetic field perpendicular to the flow of gas,<sup>3</sup> it can be shown that a maximum exists when  $\pm u = c_m = (c_0^2 + H^2/4\pi\rho)^{1/2}$ , if the field has a point of inflection at this point.

If the influence of thermal conductivity or of any kind of diffusion is predominant while the viscosity and the Joule heat can be neglected, then the coincidence noted above can be easily established even in the relativistic case. It is of interest to note that under these circumstances, in the case of detonation, the process passes through the Jouguet point twice: the first time when the medium undergoes shock compression, but subsequently the removal of heat due to thermal conductivity lowers the entropy, and the process arrives at a point on the Hugoniot adiabat, and from there the initiated detonation again brings the process to the Jouguet point.

2. We shall investigate qualitatively some aspects of the physical picture of the structure of a normal shock wave in magnetohydrodynamics. In this case a new type of dissipation, Joule heat, appears and manifests itself in the formula for the entropy discontinuity in a weak wave.<sup>4</sup>

$$T\Delta S = \frac{1}{12} \left( \frac{\partial^2 V}{\partial p^2} \right)_S (\Delta p)^3 - \frac{1}{16\pi} \left( \frac{\partial V}{\partial p} \right)_S \Delta p (\Delta H)^2 \quad (2)$$

$$= \frac{\gamma+1}{12} c_0^2 \left( \frac{\Delta p}{p} \right)^3 \left( 1 + \frac{3\eta}{\gamma+1} \right),$$

where  $\eta = H^2/4\pi\rho c_0^2$  is the ratio of the Alfvén velocity to the velocity of sound. The Joule dissipation predominates for  $\eta > 1$ . It can also be estimated directly:

$$T\Delta S = \int \frac{j^2}{\rho\sigma} dt = \frac{c^2}{(4\pi)^2\sigma} \int_0^L \left( \frac{dH}{dx} \right)^2 \frac{dx}{m} \approx \frac{\nu_m}{4\pi} \frac{(\Delta H)^2}{mL}, \quad (3)$$

where, for example,  $L = \Delta H (dH/dx)_{\max}$ . On comparing (3) and the last term in (2) we obtain:

$$L \approx - \frac{4\nu_m}{(\partial V/\partial p)_S \Delta p m} = \frac{4\rho^2 c_0^2 \nu_m}{m\Delta p} = \frac{4\rho}{\Delta p} \frac{\nu_m}{c_m}. \quad (4)$$

This result depends somewhat on the method of averaging, but in order of magnitude we always have  $L \approx \nu_m/c_m$ . If we measure the variation of the field in the shock wave by some kind of induction method, we can estimate the conductivity of the medium.

As  $\sigma$  increases ( $\nu \rightarrow 0$ ) the width of the front decreases, but the field gradients increase in such a way that the discontinuity in the entropy depends only on the discontinuity in the field. While the problem is formulated with the conductivity considered infinite everywhere, this is not the case inside the transition zone. In a reference system associated with the discontinuity front, there exists throughout the whole space an electric field given by  $\mathbf{E} = (\mathbf{u}_1 \times \mathbf{H}_1)/c$ . Behind the front both the magnetic field and the velocity vary and currents appear:

$$-dH/dx = 4\pi j/c = (E - uH/c) 4\pi\sigma/c = (H' - H)u/\nu_m, \quad (5)$$

where

$$H' = \frac{c}{u} E = \frac{c}{u} \frac{u_1}{c_1} H_1 = \frac{\rho}{\rho_1} H_1.$$

Thus  $H'$  is the field which exists at the given point under the condition of being "frozen in." The

equation shows that the field gradually approaches its "frozen-in" equilibrium value. The conductivity determines only the method of relaxation.

We give also the coefficient of absorption for sound propagated at right angles to the field in a medium of sufficiently high conductivity,

$$\alpha = (\omega^2/2c_m^3) [^{4/3}\nu + \zeta/\rho + (\gamma-1)\chi/c_p + \nu_m H^2/4\pi\rho c_m^2] = a\omega^2, \quad (6)$$

where  $\omega$  is the frequency,  $\nu$  and  $\xi$  are the kinematic and the volume viscosities of the medium, and  $\chi\rho$  is the coefficient of thermal conductivity. In the case  $H = 0$  formula (6) reduces to the usual formula for the damping coefficient of a sound wave.<sup>2</sup> In classical gas dynamics the corresponding parameter  $a$  determines, for a given  $\Delta p$ , the smearing of the front of a weak shock wave.<sup>2</sup> As the field increases the value of our quantity  $a$  decreases, and this shows that the dimensions of the transition zone decrease, while everything else remains the same. This can also be seen from (4).

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<sup>1</sup>Ia. B. Zel'dovich, Теория ударных волн и введение в газодинамику (Theory of Shock Waves and Introduction to Gas Dynamics) Acad. Sci. Press, 1946, § XII.

<sup>2</sup>L. D. Landau and E. M. Lifshitz, Механика сплошных сред (Mechanics of Continuous Media), GITTL, 1954, §§ 77, 87.

<sup>3</sup>G. S. Golitsyn and K. P. Staniukovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1417 (1957), Soviet Phys. JETP **6**, 1090 (1958).

<sup>4</sup>L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media), Moscow, Gostekhizdat, 1957, § 54.