[10] Ila. B. Zel'dovich, Dokl. Akad. Nauk SSSR **89**, 33 (1953).

[11] Tanikawa, Phys. Rev. **10**, 1615 (1958).

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NUCLEONIC INTERACTION WHICH PRODUCES A SUPERFLUID STATE OF THE NUCLEUS

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WE shall consider the possible appearance of superfluid states among medium and heavy nuclei, i.e., states which are energy wise more advantageous than the state of completely degenerate Fermi gas (normal state). In order to do this we shall apply the variational principle of Bogoliubov[1] as it arises from generalizing the method of Fock,[2] and mathematical techniques developed in the theory of superconductivity.[3]

Basing ourselves on the shell model of the nucleus, we shall examine the weak interaction[4] existing among protons (or neutrons) of the same shell with equal and opposite values of the projection of their angular momenta upon the axis of symmetry of the nucleus. Assuming that the shell is characterized by a set of quantum numbers s, the model Hamiltonian may be written in the following form:

$$H = \sum_{s,m} \{[E(s,m) - E_F]\, a^+_{m+}(s)\, a_{m+}(s) + [E(s,-m) - E_F]\, a^+_{-m-}(s)\, a_{-m-}(s)\} + \frac{1}{N} \sum_{\substack{s,m,m' \\ m \neq m'}} J_0(s\,|\,m, m')\, a^+_{m+}(s)\, a^+_{-m-}(s)\, a_{-m'-}(s)\, a_{m'+}(s), \tag{1}$$

where N is the number of levels and the other quantities are described in reference 4. Let us make the following canonical transformations.

$$a_{m+}(s) = u_m(s)\,\alpha_{m0}(s) + v_m(s)\,\alpha^+_{m1}(s), \qquad a_{-m-}(s) = u_m(s)\,\alpha_{m1}(s) - v_m(s)\,\alpha^+_{m0}(s), \tag{2}$$

$$\eta_m(s) = u_m(s)^2 + v_m(s)^2 - 1 = 0. \tag{3}$$

We shall find the average value of $\overline{H} = \langle C_0^* H C_0 \rangle$ in the new vacuum $\alpha_{m_1} C_0 = \alpha_{m_0} C_0 = 0$. We then find the functions $u_m(s)$, $v_m(s)$ from the requirement that $\overline{H}$ have a minimum value with the additional condition (3). As a result we obtain the following equation for the new unknown function $C_m(s)$:

$$C_m(s) = -\frac{1}{2N} \frac{\sum_{m'} J_0(s\,|\,m, m')\, C_{m'}(s)}{\sqrt{\{\overline{E}(s,m) - E_F\}^2 + C_{m'}(s)^2}}, \tag{4}$$

thus

$$u_m(s)^2 = \frac{1}{2}\left[1 + \frac{\overline{E}(s,m) - E_F}{\varepsilon_m(s)}\right], \quad v_m(s)^2 = \frac{1}{2}\left[1 - \frac{\overline{E}(s,m) - E_F}{\varepsilon_m(s)}\right], \quad \varepsilon_m(s) = \sqrt{\{\overline{E}(s,m) - E_F\}^2 + C_m(s)^2}, \quad \overline{E}(s,m) = \frac{1}{2}\{E(s,m) + E(s,-m)\}. \tag{5}$$

When the energy is close to the Fermi surface energy we obtain the following approximate solution to Eq. (4) for small values of J

$$C_m(s_0) = \omega \frac{J_0(s_0\,|\,m, m_0)}{J_0(s_0\,|\,m_0, m_0)} \exp\left\{\frac{(m_2 - m_1)\, d\overline{E}(s_0, m_0)/dm_0}{J_0(s_0\,|\,m_0, m_0)}\right\},$$

$$\ln\frac{\omega}{\mu} = -\frac{1}{2}\int_{m_1}^{m_2} dm' \frac{d\overline{E}(s_0, m_0)}{dm_0} \ln\frac{|\overline{E}(s_0, m' - E_F|}{\mu} \times \frac{d}{dm'}\left[\frac{J_0(s_0\,|\,m_0, m')}{J_0(s_0\,|\,m_0, m_0)} \frac{1}{d\overline{E}(s, m')/dm'}\right], \tag{6}$$

where $\overline{E}(s_0, m_0) = E_F$. The functions $u_m(s)$ and $v_m(s)$ of the superfluid state are obtained from Eq. (5) and (6), while for the normal state $u_m(s) = 1 - \theta_F(s, m)$, $v_m(s) = \theta_F(s, m)$ where $\theta_F(s, m) = 1$ if $E(s, m) < E_F$, and $\theta_F(s, m) = 0$ if $\overline{E}(s, m) > \overline{E}_F$.

We shall now compute the difference ΔE^I between the ground and the first excited superfluid states in such a way as to avoid departing from even-even nuclei to odd ones, and we find

$$\Delta E^I_m(s) = \langle C^*_s \alpha_{m0}(s)\, \alpha_{m1}(s)\, H \alpha^+_{m1}(s)\, \alpha^+_{m0}(s)\, C_s \rangle - \langle C^*_s H C_s \rangle = 2\varepsilon_m(s), \tag{7}$$

and for $s = s_0$, $m = m_0$

$$\Delta E^I_{m_0}(s_0) \approx 2\omega \exp\left\{\frac{(m_2 - m_1)\, d\overline{E}(s_0 m_0)/dm_0}{J_0(s_0/m_0, m_0)}\right\}. \tag{7'}$$

It can be seen from this that the first excited state is separated from the ground state by a gap (7'). Note that there is no energy splitting when the normal state is perturbed.

We can find the difference ΔE between the superfluid and normal states in the form

$$\Delta E_m(s) = \langle C^*_{(s)} HC_{(s)} \rangle - \langle C^*_{(n)} HC_{(n)} \rangle$$
$$= -\frac{1}{2}\varepsilon_m(s) - \frac{\{\bar{E}(s, m) - E_F\}^2}{2\varepsilon_m(s)} + \{\bar{E}(s, m) - E_F\}\{1 - 2\theta_F(s, m)\}, \quad (8)$$

and for $s = s_0$, $m = m_0$

$$\Delta E_{m_0}(s_0) = -C_{m_0}(s_0)/2. \quad (8')$$

From this it may be seen that the superfluid state turns out to be more advantageous energy wise than the normal state and is separated from it.

Thus the interaction among protons of the same shell having equal and opposite z components of angular momentum gives rise to a superfluid state of the atomic nucleus. The presence of an energy split between the first excited and ground superfluid states confirm the considerations of Bohr, Mottelson, and Pines on the possibility of explaining in this way the energy split in heavy even-even nuclei.

In conclusion the author wished to express his profound gratitude to academician N. N. Bogoliubov for his constant interest in this work and for very valuable remarks.

1 N. N. Bogoliubov, Dokl. Akad. Nauk SSSR **119**, 244 (1958), Soviet Phys. "Doklady" **3**, 292 (1958).

2 V. Fock, Z. Physik **61**, 126 (1930).

3 Bogoliubov, Tolmachev, and Shirkov, Новый метод в теории сверхпроводимости (New Approach to the Theory of Superconductivity), Acad. Sci. Press, 1958.

4 V. G. Solov'ev, Dokl. Akad. Nauk SSSR (in press).

5 Bohr, Mottelson, and Pines (in press).

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PRODUCTION OF PIONS IN CONDENSED MEDIA BY COSMIC RAYS IN THE STRATOSPHERE

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EXPERIMENTS designed to study the dependence of the cross section for meson production by low energy particles on the atomic weight A have shown that this dependence is not stronger than the $A^{2/3}$ law for elements up to aluminum, and somewhat weaker than $A^{2/3}$ for heavy nuclei.[1,2] For primary protons with an energy of 3 Bev the cross section for meson production increases as A for elements up to aluminum and somewhat faster than $A^{2/3}$ for heavier elements.[3]

We report here data on production of very slow π mesons in condensed media by cosmic rays in the stratosphere. Unbounded photoemulsions of 10 cm diameter and 400 μ thickness were used to detect the slow mesons; 12×12 cm aluminum and lead plates of varying thicknesses were used as targets. The photoemulsions were pressed between two plates of aluminum or lead, lifted into the stratosphere in balloon probes and irradiated by cosmic

TABLE I

Substances surrounding the emulsion	Thickness in g/cm²	Number of mesons per cm² including prongless stars		Total number of π-mesons in a g/cm² of the substance and in a cm² of the emulsion after geometric correction	Upper limit of meson energy E_0 in Mev, estimated from the thickness of the target substance
		π^+	π^-		
Packing material		0.17±0.06 *	0.53±0.12		
Aluminum	1.62	0.58±0.09	1.28±0.15	0.75±0.13	13.5
	2.7	0.84±0.12	1.68±0.21	0.73±0.11	18
	5.4	1.79±0.17	3.19±0.25	0.94±0.07	27.5
Lead	2.27	0.34±0.07	1.90±0.19	0.68±0.1	12
	4.54	0.92±0.13	3.96±0.32	0.94±0.08	18
	6.80	1.51±0.16	4.30±0.32	0.80±0.06	23
	11.34	2.04±0.20	4.84±0.37	0.59±0.05	31

*The errors shown are purely statistical.