The Cerenkov radiation produced by a charged particle which moves through a medium with
gyrotropic magnetic permeability is considered. Losses due to spin-wave excitation by the
particle are computed.

1. The electromagnetic radiation produced by the
motion of a charged particle in a ferrite (ferro-
dielectric) is distinguished by a number of features
which derive from the dispersion properties of the
ferrite. In the gyrotropic magnetic permeability tensor \( \mu \), which characterizes the ferrite
\[
\mu = \begin{pmatrix}
\mu_1 & i\mu_2 & 0 \\
-i\mu_2 & \mu_3 & 0 \\
0 & 0 & \mu_4
\end{pmatrix},
\]
the components \( \mu_1 \) and \( \mu_3 \) approach unity as the
frequency increases, whereas \( \mu_2 \) approaches
zero at relatively low frequencies.\(^1\) For this rea-
son the above-mentioned features can be observed
only in the low-frequency region (\( \omega \sim 10^{10} \) to \( 10^{11} \)
sec\(^{-1} \)). Under these conditions the gyrotropic ef-
fects and the anisotropic effects\(^2\) can lead to Ce-
renkov radiation at low particle velocities.

The tensor \( \mu \) does not give a complete descrip-
tion of the dispersion properties of a ferrite me-
dium, because it does not take account of the pos-
sible propagation of spin waves. Spin-wave exci-
tation can occur at almost any particle velocity.

The energy losses associated with this excitation
are of the same nature as polarization losses but
are considerably smaller in magnitude.

It is the purpose of this note to investigate the
effect of the gyrotropic properties of the magnetic
permeability tensor on energy loss in the motion
of a charged particle through a ferrite. For sim-
plcity we assume that the motion takes place along
the gyrotropic axis (z axis).

The radiation of a charged particle in a gyro-
tropic medium has been considered by Kolomenskii
and Sitenko\(^3\) who treated the gyrotropic elec-
trical effects rather than the magnetic effects. Magnetic
effects in the Cerenkov radiation have been exam-
inied in references 4 and 5. In reference 4 the mag-
netic permeability was assumed isotropic; in ref-
ence 5 it was characterized by an anisotropic
symmetric tensor.

2. We shall assume that the ferrite is electric-
ally isotropic. In point of fact the gyrotropic elec-
trical effects generally found in ferrites should be
manifest at frequencies associated with the elec-
tronic dispersion of the dielectric permittivity \( \varepsilon \).
As a rule, the electronic dispersion in \( \varepsilon \) and the
gyrotropic effect in \( \mu \) occur in different frequency
regions so that in the present work we shall con-
sider the gyro tropic magnetic effect only. These
results, however, do not apply in the optical re-
region. On the other hand, \( \mu_{ik} \rightarrow \delta_{ik} \) in the optical
region and the results obtained by Kolomenskii and
Sitenko\(^5\) can be used to calculate losses at these
frequencies.

We calculate the intensity of the Cerenkov radiation in the usual way (cf. reference 1):

\[
dW/dV = -\varepsilon^3 \int \mu_1 \left( 1 - \frac{1}{\beta \varepsilon \mu_1} - \frac{\mu_3^2}{\mu_1^2} \right) d\sigma d\omega - \varepsilon^3 \int \mu_1 \left( 1 - \frac{1}{\beta \varepsilon \mu_1} - \frac{\mu_3^2}{\mu_1^2} \right) d\sigma d\omega,
\]

\[
+ \frac{1}{\beta \varepsilon |\mu_1|} \left( (\beta^2 \varepsilon \mu_1 - 1) (\mu_1 - \mu_3)^2 + 2 \beta^2 \varepsilon \mu_1 \mu_3 (1 + \mu_4) \mu_3 - \mu_1 \mu_3 (\beta^2 \varepsilon \mu_1 - 1) + \beta^4 \varepsilon^2 \mu_3^4 \right) d\sigma d\omega,
\]

\[
+ \frac{\mu_3 (\beta^2 \varepsilon \mu_1 - 1) (\mu_1 - \mu_3) - \beta^2 \varepsilon \mu_1 (1 + \beta^2 \varepsilon \mu_1))}{\beta^2 \varepsilon |\mu_1|} \left( (\beta^2 \varepsilon \mu_1 - 1) (\mu_1 - \mu_3)^2 + 2 \beta^2 \varepsilon \mu_1 \mu_3 (1 + \beta^2 \varepsilon \mu_1) \mu_3 - \mu_1 \mu_3 (\beta^2 \varepsilon \mu_1 - 1) + \beta^4 \varepsilon^2 \mu_3^4 \right) d\sigma d\omega,
\]

where the region of integration is given in the table for \( \mu_3 > 0 \).
The conditions given in the table do not take account of the fact that macroscopically we are not actually considering short waves. This limitation must be taken into account close to points at which the integrand diverges. By introducing a finite attenuation we avoid the divergence and obtain a determinate result.

When \( \mu_2 = 0 \) the integral vanishes in region II and we obtain the result given by Sitenko:

\[
\frac{dW}{dz} = -\frac{e^2}{c^2} \int_{\rho \kappa z_2 > 1} \mu_1 \left(1 - \frac{1}{\beta \delta \mu_2} \right) \omega \, d\omega. \tag{3}
\]

3. In considering Cerenkov radiation in the radio-frequency region (wavelengths greater than 0.1 to 1 mm) we use a simplified model in which the ferrite is characterized by a spontaneous magnetization \( M \). Then, following reference 7, the quantities \( \mu_1 \) and \( \mu_2 \) are given by

\[
\mu_1 = \frac{\alpha^2 - \omega^2 + 2i\alpha \omega H_e + 2\pi M}{\alpha^2 - \omega^2 + 2i\alpha \omega H_e / M} \quad \text{and} \quad \mu_2 = \frac{\omega^2 - \omega_0^2}{\omega^2 - \omega^2 / M} \quad \mu_2 = 1; \tag{4}
\]

where \( \omega_0 = gH_e, \quad \omega_e = gV \sqrt{H_e B} \), \( B = H_e + 4\pi M \), \( H_e = H + \beta M \)

where \( H \) is the magnetic field applied along the axis of easiest magnetization, \( \beta \) is the anisotropy constant and \( \lambda \) is the relaxation frequency in the Landau-Lifshitz equation.

Substitution of Eq. (4) in Eq. (2) (with \( \lambda \ll \omega_0 \)) yields

\[
\frac{dW}{dz} = -\frac{e^2}{c^2} \int \left(1 - \frac{1}{\beta \delta} \right) + \omega_0^2 \left(\frac{\mu_0 - 1}{\omega_0 - \omega} \right) + \omega_0 \left(\frac{\beta \delta \mu_0 - 1}{\omega_0 - \omega} \right) \omega \, d\omega \tag{5}
\]

where \( \mu_0 \) is the value of \( \mu_1(\omega) \) at \( \omega = 0 \).

The limits of the regions of integration which are determined by the radiation conditions (cf. table), can be obtained by using the explicit forms of \( \mu_1(\omega) \) and \( \mu_2(\omega) \). A characteristic feature of Cerenkov radiation in a ferrite is the existence of a low-frequency spectrum at small particle velocities. In terms of the velocity of the charged particle, \( \beta = v/c \), the limits of the low-frequency radiation region are given by

- for \( 0 < \beta \mu_0 \ll 1 \):
  - \( \Omega < \omega < \omega_0 \)
- for \( 1 < \beta \mu_0 < \mu_0^* \):
  - \( 0 < \omega < \omega_0 \)
- for \( \mu_0^* \ll \beta \mu_0 \ll 1 \):
  - \( 0 < \omega < \Omega \).

Here

\[
n = \sqrt{\frac{\mu_0^* \omega_0}{\omega_0}} = \sqrt{\frac{\beta \mu_0 - 1}{\beta \mu_0^* - 1}}.
\]

The integral in Eq. (5) must be computed before we can obtain the total intensity of the low-frequency radiation. Computing this integral we obtain the following results:

for \( 0 < \beta \mu_0 < 1 \):

\[
\frac{dW}{dz} = -\frac{e^2}{c^2} \omega_0 \frac{\mu_0 - 1}{\omega_0 - 1} \ln \frac{\omega_0}{\omega_0 - 1} - 1 - \frac{\mu_0 - 1}{2 \omega_0 (1 - \mu_0)} \quad \tag{6}
\]

for \( 1 < \beta \mu_0 < \mu_0^* \):

\[
\frac{dW}{dz} = -\frac{e^2}{c^2} \omega_0 \frac{\mu_0 - 1}{\omega_0 - 1} \ln \frac{\omega_0}{\omega_0 - 1} \frac{(\beta \delta \mu_0 - 1)}{\beta \delta \mu_0 (\mu_0 - 1)} - \frac{e^2}{4 \omega_0^2} \omega_0^2 \left[ (\beta \delta \mu_0 - 1) + 2 \mu_0 (\beta \delta \mu_0 - 1)^2 + (\beta \delta \mu_0 - 1)^2 \right] \frac{1}{\beta \delta \mu_0 (1 - \mu_0)} \quad \tag{7}
\]

for \( \mu_0^* < \beta \mu_0 < \mu_0^{1/2} \):

\[
\frac{dW}{dz} = -\frac{e^2}{c^2} \omega_0 \frac{\mu_0 - 1}{\omega_0 - 1} \ln \frac{\omega_0}{\omega_0 - 1} \frac{(\beta \delta \mu_0 - 1)}{\beta \delta \mu_0 (\mu_0 - 1)} - \frac{e^2}{4 \omega_0^2} \omega_0^2 \left[ (\beta \delta \mu_0 - 1) + 2 \mu_0 (\beta \delta \mu_0 - 1)^2 + (\beta \delta \mu_0 - 1)^2 \right] \frac{1}{\beta \delta \mu_0 (1 - \mu_0)} \quad \tag{8}
\]

We have not considered the Cerenkov-radiation intensity for particle velocities which satisfy the condition \( \beta \mu_0 > \mu_0^{1/2} \) because in this case there is no separate low-frequency region (isolated in terms of frequency) and it is necessary to introduce the actual frequency dependence of \( \epsilon \) to obtain the integrated intensity.

4. In deriving Eqs. (5) to (8) we used Eq. (4) which was derived assuming a simplified model for the ferrite. Actually, a ferrite must be considered as a system of magnetized sub-lattices. Analyzing the motion of the magnetic moments of each of these sub-lattices in a high-frequency magnetic field, we find the following expressions for \( \mu_1 \) and \( \mu_2 \):

\[
\mu_1(\omega) = \frac{\omega^2 - \omega^2 / M}{\omega^2 - \omega^2 / M} \mu_0 + \frac{\mu_0 - 1}{\omega_0 - \omega} \omega \, d\omega \quad \text{and} \quad \mu_2(\omega) = \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2 / M} \mu_0 + \frac{\mu_0 - 1}{\omega_0 - \omega} \omega \, d\omega.
\]

The limits of the low-frequency radiation region are given by

- for \( 0 < \beta \mu_0 \ll 1 \):
  - \( \Omega < \omega < \omega_0 \)
- for \( 1 < \beta \mu_0 < \mu_0^* \):
  - \( 0 < \omega < \omega_0 \)
- for \( \mu_0^* \ll \beta \mu_0 \ll 1 \):
  - \( 0 < \omega < \Omega \).
\[
\begin{align*}
\mu_1 &= (\omega^2 - \omega_0^2_i)(\omega^2 - \omega_0^2_f)/(\omega^2 - \omega_0^2_j)(\omega^2 - \omega_0^2_k), \\
\mu_2 &= -4\pi\mu_i(g_1M_1 + g_2M_2)(\omega^2 - \omega_0^2_i)/(\omega^2 - \omega_0^2_f)(\omega^2 - \omega_0^2_j), \\
\mu_3 &= 1; \\
\omega_1 &= -g_i(g_1M_1 + g_2M_2)(\omega^2 - \omega_0^2_i)/(\omega^2 - \omega_0^2_f)(\omega^2 - \omega_0^2_j), \\
\omega_2 &= -\pi(g_1M_1 + g_2M_2), \\
\omega_3^a &= \omega_0^2 + 4\pi(g_1 - g_2)^2|\alpha M_1 M_2|, \\
\omega_3^b &= \omega_0^2 + (g_1 - g_2)^2|\alpha M_1 M_2| (\omega_1 + \omega_2)/(\omega_1 + \omega_3), \\
\end{align*}
\]

where \( H_{M_j} = \beta_j M_j + \mathbf{H} \) and \( M_j, \beta_j \) and \( g_j \) are respectively the magnetic moment per unit volume, the anisotropy constant and the gyromagnetic ratio for the \( j \)-th sub-lattice. The dimensionless parameter \( \alpha \) characterizes the exchange reaction between the sub-lattices. In Eq. (9) it is assumed that the axes of easiest magnetization of the sub-lattices are parallel to each other and that the magnetic field \( H \) is along this direction.

It is apparent from Eq. (9) that "magnetic" Cerenkov radiation should be observed in the region of the high-frequency resonance \( (\omega \sim \omega_2) \) as well as in the region of the usual ferromagnetic resonance \( (\omega \sim \omega_1) \). This result is of interest because \( \omega_2 \) is a sensitive function of temperature; thus it is possible to observe Cerenkov radiation in different regions of the spectrum (from radio frequencies to the infrared) at low particle velocities.

5. Up to this point we have not considered the possibility that spin waves can be excited by the particle. In other words, we have not taken account of the spatial dispersion in the tensor \( \mu \); this dispersion is a result of the exchange interaction between the spins.

Inasmuch as the coupling between the electromagnetic waves and the spin waves is weak,4 we can compute the intensity of the spin-wave excitation by successive approximations, assuming zero electric field in the first approximation.

The following expressions, which give the loss due to spin-wave excitation, can be obtained by some rather laborious computations:

\[
\begin{align*}
\frac{dW}{dz} &= -\pi\varepsilon^2\varepsilon'^2\varepsilon g M \frac{\Theta_c}{k} \left( \frac{\Theta_c}{\mu M} \right)^{2/3} \left( \frac{v}{v_S} \right)^{4/3} \\
&\times \left[ \ln \left( \frac{\Theta_c}{\mu M} \frac{v_S}{v} \right)^{2/3} - 1 \right], \quad \mu M \gg v_S; \\
\frac{dW}{dz} &= -\pi\varepsilon^2\varepsilon'^2\varepsilon g M \frac{\Theta_c}{k} \left( \frac{\Theta_c}{\mu M} \right)^{2/3} \left( \frac{v}{v_S} \right)^{4/3} \left( \frac{v}{v_S} \right)^{1/3}, \quad v_S \gg v \gg \mu M \Theta_c; \\
\frac{dW}{dz} &= -\pi\varepsilon^2\varepsilon'^2\varepsilon g M \frac{\Theta_c}{k} \left( \frac{\Theta_c}{\mu M} \right)^{2/3} \left( \frac{v}{v_S} \right)^{4/3} \left( \frac{v}{v_S} \right)^{1/3}, \quad v_S \gg v, \end{align*}
\]

where \( \Theta_c \) is on the order of the Curie temperature and \( v_S = \Theta_c g/\hbar \) is the maximum \( (k = 1/a) \)

\*The high-frequency resonance in ferrites will be considered in detail in a separate paper.

It is interesting to note that conservation of energy and conservation of momentum (the radiation conditions) permit excitation of spin waves at almost any particle velocity.

Comparing Eqs. (10) to (12) with the polarization losses in a dielectric, one is easily convinced that the excitation of spin waves by the charged particle is responsible for only a small part of the total energy loss. The polarization losses are of order \( dW/dz \approx (\varepsilon^2/\varepsilon') \Theta^2 \) where \( \Theta^2 = 4\pi N e^2/m \left( \omega_p \sim 10^{15} \text{ sec}^{-1} \right) \). Hence the ratio of spin-wave loss to polarization loss is of order \( (\Theta_c/\Theta) \times (\Theta_c/\Theta_p) \ll 1 \).

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1 L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media) GITTL, M. 1957.

Translated by H. Lashinsky