In conclusion, the authors wish to thank Prof. I. M. Frank and I. Ia. Barit for help in this work, G. M. Vagradov for useful discussion, and also the group of laboratory workers who took part in the scanning of the plates.

5 Fernbach, Green and Watson, Phys. Rev. 84, 1094 (1951).

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FISSION OF SILVER BY HIGH-ENERGY PROTONS

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Certain parameters for fission of silver induced by protons with energies ranging from 300 to 660 Mev are investigated. An analysis of the experimental data reveals that (1) fission of silver by high-energy protons is characterized by fragments which are predominantly of equal mass; (2) the fission cross section shows no essential change in the region of incident-proton energy which was investigated \((\sigma_f = (3.2 \pm 1) \times 10^{-28} \text{ cm}^2)\); (3) a large number of charged particles are emitted in fission, indicating a high initial excitation for the fissioning nucleus.

A complete yield curve for the residual nuclei as a function of charge is given along with the differential yield curves for various nuclear interactions: spallation, fission and cascade-evaporation.

EXPERIMENTAL DATA

At the present time there is available a large body of data concerned with fission in the heavy nuclei \((\text{U, Th, Bi})\) at the end of the periodic table. There is also some information concerning fission of lighter nuclei such as \(\text{W and Ta}\). The situation is entirely different with respect to fission of elements at the middle of the periodic table. Aside from the fact that fission does occur in these elements' little else is known.

In the present work, using nuclear emulsions, an attempt has been made to obtain certain preliminary information on fission of silver by high-energy protons. A fine-grain nuclear emulsion ("P9-sensitive"), sensitive to 45-Mev protons, was irradiated in the intense proton beam from the synchrocyclotron of the Joint Institute for Nuclear Research. The entire area through which the proton beam passed was then scanned with a microscope (all the plates were irradiated with the beam perpendicular to the surface of the emulsion; the same collimator (diameter \(d = 2 \text{ cm}\)) was used in all experiments).

In carrying out the analysis, only those events were selected in which, in addition to the other nuclear disintegration products, \((\alpha, p)\) there were two tracks of highly ionizing fragment-type particles. In order to distinguish the fission events from spallation in heavy elements of the emulsion, we arbitrarily assigned to fission only those events in which the range ratio for the two
fragments was less than two \((1_f/1_h \leq 2)\).* In spallation the range ratio between the multiply charged particle and the recoil nucleus is considerably greater than two. The emulsion contains two kinds of heavy nuclei \((\text{Ag} \text{ and } \text{Br})\) in addition to light nuclei \((\text{C}, \text{N}, \text{O}, \text{H})\). Since it is difficult to distinguish between effects due to these heavy nuclei, in what follows we shall assume that the fission parameters* for Ag and Br are the same in high-energy, proton-induced fission. When this assumption is made, silver-fission cross section may be somewhat low, but in preliminary data of this kind this fact is not of great importance.

The result of an analysis of fission events in Ag and Br are given in Table I.

As is indicated by Table I the fission cross section for silver remains essentially constant \(\sigma_f = (3.2 \pm 1) \times 10^{-28}\) cm\(^2\) over a wide range of incident-proton energies \((E_p = 300 \text{ to } 660\) MeV\). Furthermore, fission is generally characterized by the emission of a large number of charged particles; this number is essentially constant in the indicated incident-proton energy range. The average total fission-fragment range is considerably greater than that found in fission of heavier nuclei such as U, Th, Bi and W.

In order to investigate the symmetry characteristics of the fission process an investigation was made of the range ratio for the fission fragments. The results of this analysis are given in Table II.

**TABLE I**

<table>
<thead>
<tr>
<th>(l_\text{cm} )</th>
<th>(E_p)</th>
<th>(N)</th>
<th>(N_p)</th>
<th>(N_f)</th>
<th>(N_{\text{AgBr}})</th>
<th>(\sigma_f(\text{AgBr}))</th>
<th>(\Delta \sigma_f)</th>
<th>(n_{\alpha,p})</th>
<th>(L_{\text{sr}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>660</td>
<td>104</td>
<td>(2.14\times10^8)</td>
<td>15</td>
<td>(2.1\times10^9)</td>
<td>(5.2\times10^{-28})</td>
<td>1-10^{-28}</td>
<td>6.5</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
<td>111</td>
<td>(2.22\times10^8)</td>
<td>12</td>
<td>(2.1\times10^9)</td>
<td>(2.7\times10^{-28})</td>
<td>1-10^{-28}</td>
<td>5.7</td>
<td>16</td>
</tr>
<tr>
<td>20</td>
<td>300</td>
<td>118</td>
<td>(2.19\times10^8)</td>
<td>13</td>
<td>(2.1\times10^8)</td>
<td>(2.7\times10^{-28})</td>
<td>1-10^{-28}</td>
<td>6.3</td>
<td>15</td>
</tr>
</tbody>
</table>

*The subscript \(l\) refers to the light fragment and the heavy fragment respectively.

*By fission parameters we mean the following: fission cross section, mean number of charged particles per fission, distribution over number of charged particles, fragment range ratio \((l_f/l_h)\) etc.

**TABLE II**

<table>
<thead>
<tr>
<th>Range ratio (l_f/l_h)</th>
<th>Percentage of cases with a given range ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1-1.15)</td>
<td>(42)</td>
</tr>
<tr>
<td>(1.15-1.30)</td>
<td>(14)</td>
</tr>
<tr>
<td>(1.30-1.45)</td>
<td>(11)</td>
</tr>
<tr>
<td>(1.45-1.60)</td>
<td>(9)</td>
</tr>
<tr>
<td>(1.60-1.75)</td>
<td>(11)</td>
</tr>
<tr>
<td>(1.75-1.90)</td>
<td>(6.6)</td>
</tr>
<tr>
<td>(1.90-2.05)</td>
<td>(6.5)</td>
</tr>
</tbody>
</table>

Examination of the table shows that the most probable fission process is the one in which two fragments of approximately equal range are produced; thus, in fission of nuclei at the middle of the periodic table, just as in fast proton-induced fission of heavy nuclei, the most probable process is symmetric fission.

An approximate charge distribution for the fission fragments can be obtained by assuming that the fragment range ratio is related to the fragment charge ratio as follows:

\[
\frac{l_f}{l_h} \approx \left( \frac{Z_h}{Z_f} \right)^{0.6}. \tag{1}
\]

The relation given in (1) may be derived on the basis of the following considerations:

1. According to the Bohr formula,\(^2\) for equal velocities \(v_\alpha = v_f\) the range ratio for a fragment and an \(\alpha\) particle is given by:

\[
\frac{R_f}{R_\alpha} \approx \frac{(\pi \sigma)}{A_f} \frac{Z_f^2}{Z_\alpha^2}; \tag{2}
\]

2. The dependence of \(\alpha\)-particle range on velocity (for velocities \(v_\alpha \approx 10^8\) cm/sec) is given by the Geiger relation

\[
R_\alpha \approx k_\alpha v_\alpha^2. \tag{3}
\]

Thus, if the velocity of the \(\alpha\) particle is equal
to the velocity of the fragment, the fragment range is

$$R_f = \gamma p_f / Z_f^\alpha,$$  \hspace{1cm} (4)

where \( \gamma = \frac{E_f}{m_f c^2} \) and \( p_f = m_f v_f \) is the fragment momentum.

At fission the momenta of the fission fragments are equal \( (p_f)_h = (p_f)_l \); thus, solving Eq. (4) for the ranges of the light fragment and the heavy fragment and taking the range ratio we obtain the expression given in (1)

$$\frac{(R_f)_l}{(R_f)_h} = (\frac{Z_l}{Z_h})^\alpha / (\frac{Z_f}{Z_f})^\alpha.$$  \hspace{1cm} (5)

The validity of (1) can be demonstrated in the case of uranium fission by slow neutrons, where the range ratio and the charge ratio (for the most probable fission case) are well known.

Using (1), we obtain the following expressions for the charge of the light fragment and the heavy fragment:

$$Z_l = \frac{Z_l}{1 + (\frac{l}{h})}; \quad Z_h = Z - Z_l;$$

$$Z_l = Z_0 - n_1 (H_1) - 2n_2 (He_2),$$

where \( Z_f \) is the total charge of the light and heavy fragments, \( Z_0 \) is the charge of the original nucleus (Ag or Br), \( n_1 \) is the number of protons associated with the fission event, and \( n_2 \) is the number of \( \alpha \) particles associated with the fission event.

The further analysis of the fragment charge distribution is based on the assumption that \( Z_0 = Z_{Ag, Br} \), i.e., all fissions are assumed to occur in silver. To obtain the fragment charge distribution for silver fission, we thus determine in each case the number of protons, \( \alpha \) particles, and the range ratio \( l/l_h \) and then, using (1) and (5), the charges \( Z_l \) and \( Z_h \).

The results of this analysis are given in Table III.

<table>
<thead>
<tr>
<th>Charge of the fission fragment ( Z_f/Z_h )</th>
<th>Number of fragments with a given charge</th>
<th>Cross section for the formation of a fragment with a given charge ( \times 10^{-24} \text{ cm}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0</td>
<td>1.4</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0.7</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>4.3</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>4.3</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>5.7</td>
</tr>
<tr>
<td>17</td>
<td>9</td>
<td>6.4</td>
</tr>
<tr>
<td>18</td>
<td>9</td>
<td>6.4</td>
</tr>
<tr>
<td>19</td>
<td>13</td>
<td>9.3</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>9.3</td>
</tr>
<tr>
<td>21</td>
<td>9</td>
<td>8.8</td>
</tr>
<tr>
<td>22</td>
<td>8</td>
<td>5.7</td>
</tr>
<tr>
<td>23</td>
<td>6</td>
<td>4.3</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>2.1</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>1.4</td>
</tr>
<tr>
<td>26</td>
<td>0</td>
<td>0.7</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>0.7</td>
</tr>
</tbody>
</table>

It is evident from the values given in the table that the most probable fragment charge in silver fission is \( Z = 19 \) to \( 20 \); the cross section for the formation of these fragments is approximately 0.1 millibarns which, as will be shown below, is approximately two orders of magnitude greater than the cross section for the production of a multiply-charged particle with \( Z = 19 \) to \( 20 \) in spallation.

**YIELD OF RESIDUAL NUCLEI AS A FUNCTION OF CHARGE IN SPALLATION**

In certain interactions between fast nucleons with nuclei, the disintegration products include particles with \( Z \geq 4 \) in addition to light charged particles (\( \alpha, p \)). The nucleon process that gives rise to these multiply charged particles is usually called spallation. The spallation process in nuclear emulsions has been investigated in considerable detail in a dissertation by Lozhkin. Using the data reported in this work (the total cross section for the formation of spallation products in Ag and Br at an incident proton energy \( E_p = 460 \text{ Mev} \) and the relative yield of spallation products with a given charge \( Z_f \)) we can determine the cross section for the formation of various spallation products from silver nuclei, once again assuming that the spallation process is the same in Ag and Br.

In Fig. 1 (curve B) the circles denote the cross section for the formation of spallation products from Ag with charges \( Z_f = 4 \) to \( 14 \) at an incident proton energy \( E_p = 460 \text{ Mev} \). Because this curve is linear over a wide region of \( Z_f \), it is legitimate to extrapolate to the region of higher charge, making it possible to estimate the cross section for the
production of spallation products with charge \( Z = 19 \) to \( 20 \). It is apparent from the curve that the cross section for the formation of spallation products with charge \( Z = 20 \) is of the order of \( 2 \times 10^{-30} \) cm\(^2\), that is 50 times smaller than the cross section for the formation of a fission fragment of the same charge in silver fission.

The yield of residual nuclei as a function of charge in spallation can be obtained from the distribution of stars, using the number of light charged particles which appear in spallation. If we select a spallation product of given charge, \( Z_f \) = const, the charge of the residual nucleus as a function of \( Z \) for the given spallation process is determined by the number of light charged particles which appear:

\[
Z_{\text{res. nuc.}} = Z_{Ag} - Z_f - \Delta Z,
\]

where \( \Delta Z = n_f (H^1_1) + 2n_f (H^1_2) \). If \( \alpha/p = \text{const} \) (in spallation of Ag and Br, \( \alpha/p = \text{const} = 0.5 \)), then \( \Delta Z = 1.33 n_{\alpha,p} \) and consequently

\[
Z_{\text{res. nuc.}} = Z_{Ag} - Z_f - 1.33 n_{\alpha,p}.
\]

Knowing the cross section for a spallation product with a given charge (curve B, Fig. 1) and the probability for the emission of a given number of light charged particles in spallation \( W (n_{\alpha,p})_1 \), we can estimate the cross section for a residual nucleus with a given charge

\[
\sigma_{\text{res. nuc.}} (Z_f) = \sigma_{\text{inelast}} (Z_f) W (n_{\alpha,p}).
\]

Summing the yields of the residual nuclei with given \( Z_f \) resulting from the various processes (\( Z_f = 4 \) to 20) we obtain the total yield of residual nuclei as a function of \( Z \). The results of an analysis of this kind are given by Curve C of Fig. 1.

**YIELD OF RESIDUAL NUCLEI AS A FUNCTION OF CHARGE IN THE CASCADE-EVAPORATION PROCESS**

In order to obtain the complete picture of the yield of residual nuclei as a function of \( Z \) in the interaction of fast protons (\( E_p = 460 \) Mev) with silver nuclei, it is necessary to take account of the residual nuclei which result from the cascade-evaporation process. To evaluate this part of the yield curve we used the data reported by Ostroumov\(^4\) on the interaction of protons (\( E_p = 460 \) Mev) with heavy emulsion nuclei. Knowing the total cross section for the inelastic interaction of fast protons with silver nuclei \( \sigma_{\text{inelast.}} \approx 10^{-24} \) cm\(^2\) and the relative yield of stars with a given number of prongs \( W (n_{\alpha,p})_1 \), we can determine the yield of residual nuclei as a function of \( Z \) from the following relation:

\[
\sigma_{\text{res. nuc.}} (Z_f) = \sigma_{\text{inelast}} (Z_f) W (n_{\alpha,p});
\]

where

\[
Z_f = Z_{Ag} - \Delta Z_{sp}; \quad \Delta Z_{sp} = \left[ \frac{n_{sp}}{1 + (\alpha/p)} \right] (2\alpha/p + 1)\).
\]

An analysis of the yield of residual nuclei as a function of \( Z \) in cascade-evaporation gives the values shown in Fig. 1 (curve D). Summing all the yields for a given \( Z \) we determine the total yield resulting from all three processes: cascade-evaporation, spallation and fission. The total curve showing yield of residual nuclei as function of \( Z \) is given in Fig. 2.

**DISCUSSION OF RESULTS**

In analyzing the total yield for various nuclei produced in the interaction of fast protons with silver nuclei (Figs. 1 and 2) it is convenient to divide the curve into four sections I, II, III, and IV.

1. Region I: \( Z \geq 32 \). In this region the residual nuclei are formed chiefly in the cascade-evaporation process and are not connected with spallation or fission.

2. Region II: \( 25 \geq Z \geq 32 \). The residual nuclei in this region are due chiefly to spallation.

3. Region III: \( 15 \leq Z \leq 25 \). In this region of \( Z \) the nuclei are formed predominantly as a result of fission.

4. Region IV: \( 4 \leq Z \leq 15 \). The multiply charged ions in this region are predominantly spallation products.

Obviously, the boundaries indicated above are arbitrary, since these regions overlap.
The yield curve obtained in the present work, using an emulsion method, is similar to the yield curve obtained by a radiochemical method¹ (curve 2, Fig. 2) in the region \( Z > 26 \). The numerical values for the yields in this region do not differ from the present values by more than a factor of 2 or 2.5. If curves are normalized at the highest yield (black dots in Fig. 2) the agreement in this region of \( Z \) is even more striking.

For silver irradiated by 480-Mev protons the radiochemical work, after summation of all the yields, gives a total inelastic cross section \( \sigma = 0.43 \times 10^{-24} \text{ cm}^2 \) whereas direct measurements⁵ of the inelastic cross section for 660-Mev protons with nuclei in the silver region (Sn) give \( \sigma = 1.2 \times 10^{-24} \text{ cm}^2 \). Thus, the total yield curve in the region \( Z > 26 \) obtained by the radiochemical method is in good agreement with the general shape of the present curve but gives yield values which are too low.

In the region \( Z \leq 15 \) the data obtained in the present case differ basically from the data obtained by the radiochemical method.¹ In this region the radiochemical method does not give a true picture. There is little doubt that the radiochemical results are more reliable in the region \( 25 \leq Z \leq 32 \) than in the region \( Z \leq 15 \). Since our yields in this region of \( Z \) (4 \( \leq Z \leq 15 \)) were obtained by calculation, starting from the experimentally obtained yields³ of multiply charged ions (4 \( \leq Z \leq 15 \)) and the agreement of the present data with the radiochemical data in the region 25 \( \leq Z \leq 32 \) is rather good, it would seem that the present results on the yield of multiply charged ions 4 \( \leq Z \leq 15 \) are as reliable as the data obtained radiochemically in the region 25 \( \leq Z \leq 32 \).

In the fission-fragment region 15 \( \leq Z \leq 25 \) the present data also differ from the data obtained by radiochemical methods. This discrepancy lies essentially in the fact that the present data lead to a yield curve which exhibits a maximum, indicating symmetric fission for silver, whereas the yield curve obtained radiochemically indicates a predominance of asymmetric fission. However, the statistics used in the present data are still rather inadequate so that the numerical values in this work must be considered as tentative.

In conclusion the author wishes to take this opportunity to thank Professor N. A. Perfilov for a number of valuable comments in discussing the present work. The author is also indebted to O. V. Lozhkin, V. I. Ostroumov and V. F. Darofskii for making certain invaluable data available to the author.

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