One cannot exclude the existence of more complicated mechanisms leading to an added longitudinal potential difference in a magnetic field exist. These added effects are, however, apparently substantially less than the effects noted above.

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273

THE DIFFERENTIAL FORM OF THE KINETIC EQUATION OF A PLASMA FOR THE CASE OF COULOMB COLLISIONS

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As is known,[1] the kinetic equation for the particle distribution function $f_\alpha(t, \mathbf{r}, \mathbf{v})$ of a completely ionized plasma ($\alpha = e$ and $\alpha = i$ denote electrons and ions, respectively) can be written for the case of Coulomb collisions in the form

$$\frac{\partial f_\alpha}{\partial t} + (\mathbf{v}\,\mathrm{grad}_r) f_\alpha + \frac{1}{m_\alpha}(\mathbf{F}_\alpha \nabla_v f_\alpha) = -\sum_\beta (\nabla_v \mathbf{j}_{\alpha\beta}), \quad (1)$$

$$(\mathbf{j}_{\alpha\beta})_i = \frac{2\pi\lambda e_\alpha^2 e_\beta^2}{m_\alpha}\int d\mathbf{v}' U_{ik}\left(\frac{f_\alpha}{m_\beta}\frac{\partial f'_\beta}{\partial v'_k} - \frac{f'_\beta}{m_\alpha}\frac{\partial f_\alpha}{\partial v_k}\right), \quad (2)$$

$$U_{ik} = \partial^2 |V| / \partial v_i \partial v_k = (V^2\delta_{ik} - V_i V_k)/V^3;$$

$$V_i = v_i - v'_i.$$

The present note shows that the particular structure of the integrals in Eq. (2) for the current allows one to introduce new variables in the form of new unknown "potential" functions

$$\Phi_\alpha(t, \mathbf{r}, \mathbf{v}) = \int d\mathbf{v}' f_\alpha(\mathbf{v}') |\mathbf{v} - \mathbf{v}'|, \quad (3)$$

which will transform the integro-differential equations (1) into pure differential form.

Let us consider the first integral in the expression for $\mathbf{j}_{\alpha\beta}$. If we make use of the expression for U_{ik}, we obtain

$$\int d\mathbf{v}' U_{ik}\frac{\partial f'_\beta}{\partial v'_k} = 2\frac{\partial \varphi_\beta}{\partial v_i}, \qquad \varphi_\beta(t, \mathbf{r}, \mathbf{v}) = \int \frac{f_\beta(\mathbf{v}')\,d\mathbf{v}'}{|\mathbf{v} - \mathbf{v}'|}. \quad (4)$$

Further, noting the identity

$$V^{-3}\{V^2 \nabla_v f_\alpha - \mathbf{V}(\mathbf{V}\nabla_v f_\alpha)\} = (\nabla_v f_\alpha \nabla_v)\nabla_v |\mathbf{v} - \mathbf{v}'|, \quad (5)$$

we can transform the second integral in (2) according to

$$\int d\mathbf{v}' f'_\beta U_{ik}\frac{\partial f_\alpha}{\partial v_k} = \frac{\partial f_\alpha}{\partial v_k}\cdot\frac{\partial^2 \Phi_\beta}{\partial v_k \partial v_i}. \quad (6)$$

The other quantities entering into (1) are also easy to express in terms of the Φ_β. In particular,

$$\varphi_\beta = \frac{1}{2}\nabla_v^2 \Phi_\beta, \quad f_\beta = -\frac{1}{4\pi}\Delta_v \varphi_\beta = -\frac{1}{8\pi}\nabla_v^4 \Phi_\beta. \quad (7)$$

Inserting (4), (6), and (7) into (1), we obtain the differential equation

$$\frac{\partial}{\partial t}(\nabla_v^4 \Phi_\alpha) + (\mathbf{v}\,\mathrm{grad}_r)(\nabla_v^4 \Phi_\alpha) + \frac{1}{m_\alpha}(\mathbf{F}_\alpha \nabla_v^5 \Phi_\alpha)$$
$$= -\sum_\beta \frac{2\pi\lambda e_\alpha^2 e_\beta^2}{m_\alpha}\left(\nabla_v\left[\frac{1}{m_\beta}(\nabla_v^4 \Phi_\alpha)\nabla_v^3 \Phi_\beta - \frac{1}{m_\alpha}(\nabla_v^5 \Phi_\alpha \nabla_v)\nabla_v \Phi_\beta\right]\right). \quad (8)$$

for the "potential" functions. In the special case of a "moving" Maxwell distribution given by

$$f_\alpha^{(0)}(\mathbf{v}) = n_\alpha (m_\alpha / 2\pi T_\alpha)^{3/2} \exp\{-s_\alpha^2\}, \quad (9)$$

$$\mathbf{s}_\alpha = \sqrt{\frac{m_\alpha}{2T_\alpha}}(\mathbf{v} - \mathbf{v}_\alpha^0)$$

(where $n_\alpha(t, \mathbf{r})$ is the density, $\mathbf{v}_\alpha^0(t, \mathbf{r})$ is the mean velocity, and $T_\alpha(t, \mathbf{r})$ is the temperature) we have

$$\Phi_\alpha^{(0)}(\mathbf{v}) = n_\alpha\left(\frac{2T_\alpha}{\pi m_\alpha}\right)^{1/2} M(s_\alpha),$$

$$M(s) = e^{-s^2} + (1 + 2s^2)\int_0^1 e^{-s^2x^2}dx. \qquad (10)$$

It is easy to verify that $\Phi_\alpha^0(\mathbf{v})$ causes the self-current $\mathbf{j}_{\alpha\alpha}$ to vanish, which is as it should be.

From the definition (3) one easily obtains the asymptotic potential function (for $v \gg \langle v\rangle_\alpha$; we drop terms of order v^{-3}), namely

$$\Phi_\alpha(\mathbf{v}) = vn_\alpha\left\{1 - \frac{(\mathbf{v}\mathbf{v}_\alpha^0)}{v^2} + \left[\frac{T_\alpha}{m_\alpha v^2} - \frac{v_\alpha^{02}}{2v^2} - \frac{(\mathbf{v}\mathbf{v}_\alpha^0)^2}{2v^4} - \frac{(\mathbf{v}\Pi_\alpha\mathbf{v})}{2v^4}\right]\right\}, \qquad (11)$$

where we make use of the tensor

$$\Pi_{\alpha ik} = \frac{1}{n_\alpha}\int d\mathbf{v}' f_\alpha'\left(u_{\alpha i}'u_{\alpha k}' - \frac{u_\alpha'^2}{3}\delta_{ik}\right), \quad \mathbf{u}_\alpha' = \mathbf{v}' - \mathbf{v}_\alpha^0.$$

If the distribution differs only slightly from a Maxwell distribution, so that we may write

$$\Phi_\alpha(\mathbf{v}) \approx \Phi_\alpha^0 + \Phi_\beta^{(1)}$$

$$= n_\alpha(2T_\alpha/\pi m_\alpha)^{1/2}[M(s_\alpha) + \chi(s_\alpha)], \quad \chi \ll M, \qquad (12)$$

the linear approximation gives the following expression for the current $\mathbf{j}_{\alpha\alpha}$ due to collisions among particles only of type α:

$$\mathbf{j}_{\alpha\alpha} = (\lambda e_\alpha^4/4m_\alpha^2)[(\nabla_v^5\Phi_\alpha\nabla_v)\nabla_v\Phi_\alpha - (\nabla_v^4\Phi_\alpha)\nabla_v^3\Phi_\alpha]$$

$$\approx (\lambda e_\alpha^4 n_\alpha^2/4\pi m_\alpha^2)(m_\alpha/2T_\alpha)^{1/2}[16e^{-s^2}(\mathbf{s}\nabla_s)\nabla_s\chi + 8e^{-s^2}\nabla_s^3\chi + (\nabla_s^5\chi\nabla_s)\nabla_s M - (\nabla_s^4\chi)\nabla_s^3 M]. \qquad (13)$$

In this case all the equations can be linearized.

Under certain special conditions, it is possible to lower the order of the differential equation. Such a situation may occur, for instance, when the distribution depends only on the absolute value of the velocity. Since the use of Eq. (8) for the "potential" functions may lead to extra solutions, the final result must be verified by inserting it into the initial equation (1).

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PROOF OF THE ABSENCE OF RENORMALIZATION OF THE VECTOR COUPLING CONSTANT IN BETA-DECAY

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GELL-MANN and Feynman have proposed[1] that the vector coupling constant of the β-decay interaction is not subject to renormalizations due to the strong meson-nucleon interaction, if a direct interaction between the π meson and electron-neutrino fields is introduced such as to make the vector part of the meson-nucleon β-interaction Hamiltonian of the form

$$H = G_V[\bar\psi\gamma_\mu\tau^+\psi + 2i(\Phi^+T^+\nabla_\mu\Phi - (\nabla_\mu\Phi^+)T^+\Phi)]J_\mu + \text{Herm. conj.}$$

$$J_\mu = \tfrac{1}{2}\bar\psi_e\gamma_\mu(1+\gamma_5)\psi_\nu, \qquad (1)$$

where $\tau^+ = \frac{1}{2}(\tau_x + i\tau_y)$, $T^+ = \frac{1}{2}(T_x + iT_y)$ are the isotopic spin operators and $\Phi = (\varphi, \varphi^0, \varphi^+)$ is the meson wave function.

This assumption of Gell-Mann and Feynman may be rigorously proved if it is noted that in the presence of the β-interaction (1) the complete nucleon — π meson Lagrangian (in which the meson-nucleon interaction is included but interactions with the electromagnetic field are not) admits the group of infinitesimal transformations

$$\psi = [1 - i(\tau^+\chi + \tau\chi^*)]\psi'; \qquad \Phi = [1 - 2i(T^+\chi + T\chi^*)]\Phi';$$

$$J_\mu = J_\mu' + \partial\chi/\partial x_\mu \qquad (2)$$

where χ is an infinitesimal numerical function. The existence of the group of transformations (2) makes possible the proof of a theorem analogous to the Ward theorem in quantum electrodynamics. To obtain the proof it is only necessary to calculate the nucleon Green's function $G(x, y, J_\mu)$ in the presence of a time and space independent external β-current J_μ, and to define the vertex part as

$$\Gamma_\mu^+(x, y; \xi) = \partial G^{-1}(x, y; J_\mu)/\partial J_\mu|_{J_\mu=0}\delta(\xi).$$

Putting $\chi(x) = J_\mu x_\mu$, one obtains from the definition of the Green's function

$$G(x, y; J_\mu) = \langle 0 \mid T\{\psi(x), \bar\psi(y)\} \mid 0\rangle$$

and from the relations (2):