

where the commuting function  $S$  is different for the Larmor and Maxwell photons:

$$= (\alpha, \nabla - \partial/c \partial t) D(x, t), \quad (10)$$

$$S_M(x, t) = (\alpha, \nabla - \partial/c \partial t) (\alpha^*, \nabla - \partial/c \partial t) D(x, t). \quad (11)$$

In momentum space  $S_M$  reduces to the form of the Cayley-Klein transformation of the unit wave vector  $\mathbf{k}^0 = \mathbf{k}/k$

$$S_M = (I + \alpha^*, \mathbf{k}^0) / (I - \alpha, \mathbf{k}^0). \quad (12)$$

For the Maxwell field it is possible to utilize Eqs. (1) and (2) with the reduced wave function  $\psi_M = T\psi$ , where

$$T = (3 + M)/4, \quad M = r_1 + r_2 + r_3. \quad (13)$$

The photon theory, like the new neutrino theory, is intrinsically three-dimensional, since both groups  $G_\alpha$  and  $G_{\alpha^*}$  have a diagonal matrix in common:  $\alpha_4 = \alpha_4^* = r$ . Taking this into account, one can write the wave equations in a symmetric four dimensional form ( $-ir\alpha_k \rightarrow \gamma_k$ ,  $r \rightarrow \gamma_4$ ).

The interaction of photons with the gravitational field is described by the equations

$$\gamma_\lambda \partial \varphi / \partial x_\lambda = 0, \quad \text{or} \quad \beta_\lambda \partial \varphi / \partial x_\lambda = 0,$$

where

$$\varphi(x) = (I + g\gamma_\mu \gamma_\nu^* h_{\mu\nu}(x)) \psi(x) \quad (14)$$

( $h_{\mu\nu}(x)$  is the gravitational potential). The application of perturbation theory to this equation is facilitated by the smallness of the coupling constant  $g$ . The usual formulae apply for the traces of products of  $\alpha_i$  and  $\alpha_i^*$ . The traces of products  $\alpha_i \alpha_k^*$  vanish identically.

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### CERTAIN NEW MAGIC NUCLEON NUMBERS

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NUCLEI containing 30 neutrons have, in most cases, some rare distinguishing properties. The nuclide  $^{26}\text{Fe}_{30}$  is the most abundant of all nuclides having  $Z > 10$ . The relative abundance of this iron isotope is 91.7%, while, for example, the isotope  $^{26}\text{Fe}_{28}$  (in spite of its containing 28 neutrons) has an abundance of only 5.8%. The relative abundance of the lightest nickel isotope  $^{28}\text{Ni}_{30}$  is 67.8%, although in the case of analogous isotopes of other elements it usually does not exceed several percent or fractions of a percent. The nuclide  $^{24}\text{Cr}_{30}$  is different in having a low effective capture cross-section for thermal neutrons (0.36 barns), like nuclides containing the usual magic numbers of neutrons. Along with this, the iron and nickel isotopes having  $N = 30$  have very high effective cross-sections for coherent scattering (without change in spin) of thermal neutrons compared with other isotopes of the same elements, and very high total scattering cross-section ( $\sigma_{\text{free}}$ ), multiplied by  $[(A+1)/A]^2$  to reduce it to the case of the nucleus at rest. The corresponding data (in barns) are given in Table I.

TABLE I

Element	N	A	$\sigma_{\text{free}} [(A+1)/A]^2$	$\sigma_{\text{coh}}$
Fe	28	54	2.5	2.20
	30	56	12.8	12.8
	31	57	2.0	0.64
Ni	30	58	24.4	25.9
	32	60	1.0	1.1
	34	62	9.0	9.5

The effective scattering cross-section of  $^{28}\text{Ni}_{30}$  is particularly high. It is possible that the properties of a 30-neutron configuration manifest themselves differently in different nuclides, depending

on the properties of the proton system of each individual nuclide. The properties of nuclides with  $N = 30$ , particularly the isotopes of iron and nickel, lead to the view that 30 is a magic number for neutrons.\*

Very characteristic phenomena are seen to follow the filling of configurations of 42 and 60 protons or neutrons. (See Table II; the dash indicates the absence of a stable nuclide).

TABLE II

Element	Z	A	I	Element	N	A	I
Nb	41	93	9/2	Ge	41	73	9/2
Tc	43	—	—	Se	43	77	1/2
Rh	45	103	1/2	Se	45	—	—
Ag	47	107, 109	1/2	Kr	47	83	9/2
Pr	59	141	5/2	Pd	59	105	5/2
Pm	61	—	—	Pd	61	—	—
Eu	63	151	5/2	Cd	63	111	1/2
Tb	65	159	3/2	Cd	65	113	1/2

The filling of the shell terms  $5g_{9/2}$  and  $4d_{5/2}$  begins (in the case of nuclides with 59 and 41 protons or neutrons) in a quite normal manner, but when the number of the same nucleons is one more than 60, the nuclide loses its stability, and it is quite characteristic that this takes place both for 61 protons and for 61 neutrons. Nor does a stable nuclide containing 43 protons exist. Thus, of the four cases considered, three exhibit loss of stability.

As to the nuclide  $\text{Se}^{77}$ , which contains 43 neutrons, its spin is not  $9/2$  like that of the  $^{32}\text{Ge}_{41}$  nuclide, but  $1/2$ . This must be compared with the fact that the unstable nuclides with 43 protons or 61 neutrons are followed by nuclides having an odd number of protons (Rh, Ag) or neutrons (Cd, Sn, and the following) whose spin also drops to one-half.† Neither the level-crossing hypothesis (Ref. 1 and others) nor any other modern theory is capable of explaining why the spin diminishes not immediately after the closing of the subshells that contain 40 protons or 40 and 50 neutrons, but only at  $N = 43$  and only after  $Z$  exceeds 43 or  $N$  exceeds 61.

The cause of absence of  $\beta$ -stable nuclides with  $Z = 43$  and 61 has been the subject of many investigations (Ref. 2 and others). This absence is usually made dependent on the closing of the magic configurations of 50 and 82 neutrons. The influence of the reduced binding energy of the neutrons must naturally be taken into account in both cases. However, such an explanation cannot be considered exhaustive, particularly for the following reasons:

(1) One cannot ignore the fact that the absence of a stable nuclide, or certain other post-magic phenomena, are observed not only past 42 and 60 protons, but also past 60 and 42 neutrons; in many elements with even  $Z$ , from  $^{42}\text{Mo}$  to  $^{56}\text{Ba}$ , there at least two isotopes with odd  $N$ , and only in the case of Pd does the nuclide  $^{46}\text{Pd}_{61}^{107}$  lose stability, unlike its isobar  $^{47}\text{Ag}_{60}^{107}$ . (2) It is known that the only nuclide with  $N = 51$  that remains stable, in spite of the presence of a post-magic neutron configuration, is  $^{40}\text{Zr}_{51}$ , and the stability of this nuclide must be ascribed to the properties of its proton configuration, i.e., to the closing of the  $3p_{1/2}$  term. In exactly the same way, the only stable nuclide with  $N = 83$  is  $^{60}\text{Nd}_{83}$ , and in this case the nuclide probably owes its stability to the magic properties of its proton configuration. We see that, in addition to the usual explanation for the absence of stable isotopes of Tc and Pm, it is necessary to take into account also the peculiarities of the numbers 42 and 60.

Thus, it becomes quite probable that a configuration of 30 neutrons and of 42 and 60 protons or neutrons has certain magic properties. These configurations do not coincide with shell configurations, and it must be emphasized that the nature of their stability may be entirely different than that due to closing of the shell terms.

This probably does not exhaust the list of new magic numbers. However, recognition of these three numbers alone is enough to raise the question of the existence of some other uninvestigated properties of the nucleus, along with those already known at the present time.

\*Such a conclusion is not disproved by the fact that a configuration consisting of 30 protons does not have clearly pronounced magic properties. It must be taken into account that the  $^{30}\text{Zn}$  nuclides lie in that portion of the periodic system (between copper and bromine) which is, in general, characterized by a considerably lower binding energy compared with the general course of the packing-fraction curve.

†A similar reduction in spin, as compared with its theoretical value is absent only for odd  $Z > 61$  (it is known that heavy nuclides retain high values of spin for odd  $Z$  to a considerably greater extent than for odd  $N$ ).

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