# THE $K_{e3}$ DECAY

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The possibility is considered of determining the interaction constants and the properties of the neutrino from studies of the polarizations and spectra of the particles emitted in $K_{e3}$ decays.

At the present time it is not clear whether or not the neutrino is a two-component particle. The study of $K_{e3}$ decays can provide information about the neutrino and the interaction constants.

All of the experiments on measurements of the spectra and polarizations in $\mu^{\pm}$ decays must be divided into two broad groups: those requiring a knowledge of the energies of neutral $\pi$ mesons, and those not requiring such knowledge. As regards $K^0$ decays, in this case a study of the spectra of the particles can be carried out only if the direction of the $K^0$ particle's momentum is known, since otherwise one cannot transform to the center-of-mass system. If, however, a powerful directed beam of $K^0$ mesons has been produced, the detection of the charged particles does not present the same difficulties as that of $\pi^0$ mesons; therefore we may assume that in this case all the momenta will be known.

The calculation of the polarization on the hypothesis that the neutrino is a longitudinal particle has been carried out by Okun'[1] and Werle.[2] Werle[3] has carried through the calculation of the polarization of the $\mu$ mesons for the pure S, V, and T interactions. Charap[4] has calculated the probabilities of the different polarizations and the spectra of the $\mu$ mesons from $K_{\mu3}$ decays for the cases of the two-component neutrino and the twin-component neutrino. But Charap did not consider the case of arbitrary constants, and did not discuss the possibility of determining the constants from experiments.

In the present paper the form of the interaction given in Refs. 5 to 7 is used, but on the assumption that the neutrino is not necessarily a two-component particle. The matrix element has the form:

$$\langle\overline{\psi}_e\{(g_S+i\gamma_5g'_S)+(g_V+i\gamma_5g'_V)\gamma_4 + \frac{1}{2M}(g_T+i\gamma_5g'_T)(\gamma_4\hat{p}_\pi-\hat{p}_\pi\gamma_4)\}\psi_\nu\rangle(2M^{1/2}E_\pi^{1/2})^{-1}. \quad (1)$$

The most convenient way of writing the expression for the probability of emission of electrons and $\pi$ mesons has been given in a paper by Okun'.[7] If one includes all 12 constants corresponding to the general form of the interaction with parity nonconservation, and eliminates the momentum of the $\pi$ meson, Eq. (2) of Ref. 7 takes the following form

$$W_1(E_\pi, E_e)\,dE_\pi\,dE_e = \{(|g_S|^2+|g'_S|^2)[\Delta-2E_\pi M] + (|g_V|^2+|g'_V|^2)[-(M-2E_e)^2-m_\pi^2+2E_\pi(M-2E_e)] + (|g_T|^2+|g'_T|^2)[\Delta-2E_\pi M][M-E_\pi-2E_e]^2M^{-2} + 2\mathrm{Re}(g_Tg_S^*+g'_Tg'^*_S)[\Delta-2E_\pi M][M-E_\pi-2E_e]M^{-1}\} \times(32\pi^3M^3)^{-1}\,dE_\pi\,dE_e. \quad (2)$$

Here $\Delta = M^2 + m_\pi^2$, M and $m_\pi$ are the masses of the K particle and $\pi$ meson, $E_\pi$ and $E_e$ are the energies of the $\pi$ meson and electron in the center-of-mass system, and $p_\pi$ and $p_e$ are their momenta.

If in Eq. (2) one introduces the variable $\epsilon = 2E_e/(M-E_\pi)$, the result is the analogue of Eq. (3) of the paper of Okun':

$$W_1(\varepsilon) = \Phi_{S'S'}^{SS} + \Phi_{V'V'}^{VV}[\varepsilon_0^2-(1-\varepsilon)^2] + \Phi_{T'T'}^{TT}(1-\varepsilon)^2 + \Phi_{T'S'}^{TS}(1-\varepsilon). \quad (3)$$

As in Ref. 7

$$\varepsilon_0 = p_\pi/(M-E_\pi), \quad 1-\varepsilon_0 \leqslant \varepsilon \leqslant 1+\varepsilon_0, \quad m_e = 0.$$

This formula differs from Eq. (3) of reference 7 only by the additional indices on the unknown functions $\Phi$ of the $\pi$-meson energy.

If we assume nothing about the nature of the dependence of $\Phi$ on the $\pi$-meson energy, then we can get from experiments measuring the differential spectrum only three equations for the determination of four combinations of constants. The analysis of the expression (3) has been carried out in Ref. 7, but in one respect this was not done with sufficient completeness: it is stated there that the presence of a maximum (or a minimum) at $\epsilon = 1$ would be evidence of the existence of a V (or T) type of interaction. Actually the condition $\epsilon = 1$ is superfluous, so that the presence of a maximum (or minimum) at any value of $\epsilon$ would indicate the existence of a V (or T) interaction.

If we assume that the constants do not depend on the $\pi$-meson energy, then we can determine all four combinations of constants. Otherwise we can only express three of the combinations in terms of the fourth. There is an exception, however — if the vector interaction is absent. This fact could be ascertained, as noted in Ref. 7, from the shape of the $\pi$-meson spectrum near the maximum $\pi$-meson energy $E_{\pi\,max} = \Delta/2M$.

If the neutrino is not assumed to be a two-component particle, the corresponding expression given by Okun'[7] for the probability of emission of a $\pi$ meson will have the following appearance:

$$W(E_\pi)\,dE_\pi = \{(|g_S|^2 + |g_S'|^2)(\Delta - 2E_\pi M)\,p_\pi + (|g_V|^2 + |g_V'|^2)\,2p_\pi^3/3 + (|g_T|^2 + |g_T'|^2)(\Delta - 2ME_\pi)\,p_\pi^3/3M^2\}\,dE_\pi/32\pi^3M^3. \quad (4)$$

The study of the polarization of the electrons for fixed momenta of the electron and $\pi$ meson can give additional information about the interaction constants, and thus also about the properties of the neutrino.

It is more convenient to examine the probability distribution of the longitudinal polarizations by using the variables $E_e$ and $E_\pi$, as was done for the spectrum in Ref. 7. The calculations lead to the following expression for the difference of the probabilities of the polarizations parallel and antiparallel to the momentum of the electron:

$$\delta W_2(E_e, E_\pi)\,dE_\pi\,dE_e = (ME_\pi - \Delta/2)\Big\{a_2 + a_8[(M - E_\pi) - 2E_e]\,M^{-1} + a_{15}[(M - E_\pi)^2 - 4E_e(M - E_\pi) + 4E_e^2]\,M^{-2} + a_{10}\Big[-1 + \frac{2E_e(E_\pi - M)}{ME_\pi - \Delta/2} + \frac{2E_e^2}{ME_\pi - \Delta/2}\Big]\Big\}\frac{dE_\pi\,dE_e}{32\pi^3M^3}. \quad (5)$$

Here

$$a_2 = -2\mathrm{Re}(g_S' g_S^*);\quad a_8 = -2\mathrm{Re}(g_S' g_T^* + g_S g_T'^*);$$
$$a_{10} = -2\mathrm{Re}(g_V' g_V^*);\quad a_{15} = -2\mathrm{Re}(g_T' g_T^*).$$

If we write (5) in the more compact form, we have:

$$\delta W_2(\varepsilon) = F_{SS'} + F_{ST'}^{S'T}(1-\varepsilon) + F_{T'T}(\varepsilon - 1)^2 + F_{VV'}[(\varepsilon - 1)^2 - \varepsilon_0^2]. \quad (6)$$

Thus the expression for the longitudinal polarization, Eq. (6), is very similar to the expression (3) for the spectrum.

If $F_{ST'}^{S'T} = 0$, the distribution is symmetrical with respect to the point $\epsilon = 1$. The converse statement is also true: if the distribution is symmetrical around the point $\epsilon = 1$, then $F_{ST'}^{S'T} = 0$.

If the distribution has an extremum, then either $F_{VV'}$ or $F_{TT'}$ is different from zero. We call attention to the fact that if we had started with the two-component model of the neutrino, the presence of a minimum would have meant the emission of a neutrino, and a maximum that of an antineutrino. Therefore if it should turn out, for example, that in the $K_{e3}$ decay there is a maximum in the distribution instead of a minimum, this would mean either that the lepton charge is not conserved, or else that the neutrino is not a two-component particle. The same statement could be made if a minimum were observed in the $K_{e3}$ decay or the decay $K^0 \to e^- + \tilde{\nu} + \pi^+$, or if a maximum were observed in the decay $K^0 \to e^+ + \nu + \pi^-$.

We note that if one of the constants of the vector interaction, $g_V$ or $g_{V'}$, is equal to zero, or if these constants are displaced in phase relative to each other by 90°, the distribution passes through zero when the $\pi$-meson energy takes the value $E_{\pi\,max}$. As was pointed out in Ref. 7, it is improbable that these constants should vanish right at just this point; therefore the absence of one of the vector interaction constants is a condition that can be ascertained experimentally, independently of the determination of the functional dependence of the constants on the $\pi$-meson energy.

Equation (6) contains three powers of $\epsilon$ (0, 1, 2); therefore if $F_{VV'} \neq 0$, one can only express three combinations of constants in terms of the fourth, but if $F_{VV'} = 0$, one can find all four combinations.

If the assumption is made that the constants do not depend on the energy of the $\pi$ meson, then of course all four combinations can be determined.

Let us consider the problem of the transverse polarization in the plane of the decay. By standard calculations one gets the following expression for the difference of the probabilities of the polarizations parallel and antiparallel to the direction $\mathbf{J}_2$ ($\mathbf{J}_2$ lies in the plane of the decay and makes a right angle with the electron momentum and an acute angle with the momentum of the $\pi$ meson):

$$\delta W_3 = -|p_\pi||\sin\theta|\{2\,\mathrm{Re}(g'_V g_T^* - g_V g_T'^*) \times [2E_e^2 - (M-E_\pi)E_e]M^{-1} + 2\,\mathrm{Re}(g'_S g_V^* - g_S g_V'^*)E_e\}\,dE_\pi\, dE_e/32\pi^3 M^3. \quad (7)$$

Writing this in the more compact form, we get

$$\delta W_3 = |\sin\theta|[F^{V'T}_{VT'}(\varepsilon^2-\varepsilon) + F^{S'V}_{SV'}\varepsilon]. \quad (8)$$

Here $\theta$ is the angle between the momenta of the $\pi$ meson and the electron

$$\sin\theta = [1-(E_\pi E_e + \Delta/2 - ME_e - ME_\pi)^2/E_e^2(E_\pi^2 - m_\pi^2)]^{1/2}.$$

It is convenient to investigate not the expression (8), but the expression for $\delta W_3/|\sin\theta| \equiv q$. If q does not have an extremum, then $F^{V'T}_{VT'} = 0$. If q passes through zero at $\epsilon = 1$, then $F^{S'V}_{SV'} = 0$. If $F^{V'T}_{VT'} = F^{S'V}_{SV'}$, then q can be drawn as a parabola with its vertex at $\epsilon = 0$.

Let us pause to consider one more kind of polarization — the polarization in the direction perpendicular to the plane of the decay. A polarization in this direction is forbidden by the law of conservation of the combined parity, if such a law exists. Standard calculations give the following expression for the difference of the probabilities of polarizations parallel and antiparallel to the direction $\mathbf{J}_3 = [\mathbf{p}_\pi \times \mathbf{p}_e]/|[\mathbf{p}_\pi \times \mathbf{p}_e]|$:

$$\delta W_4\, dE_e dE_\pi = E_e p_\pi |\sin\theta|[-2\,\mathrm{Im}(g_V g_T^* - g'_V g_T'^*) \times (-2E_e + M - E_\pi)M^{-1} - 2\,\mathrm{Im}(g_S g_V^* - g'_S g_V'^*)]\,dE_e dE_\pi/32M^3\pi^3, \quad (9)$$

or in the more compact form:

$$\delta W_4(\varepsilon) = E_e|\sin\theta|[\mathscr{F}^{VT}_{V'T'}(1-\varepsilon) + \mathscr{F}^{SV}_{S'V'}]. \quad (10)$$

If $\delta W_4(\epsilon) \neq 0$, the law of conservation of the combined parity is not valid. If $\mathscr{F}^{VT}_{V'T'} = 0$, then $\delta W_4/E_e|\sin\theta| = d$ does not depend on $\epsilon$, and conversely. If $\mathscr{F}^{SV}_{S'V'} = -\mathscr{F}^{VT}_{V'T'}$, then $d(\epsilon)$ corresponds to a straight line passing through the point $\epsilon = 0$. If $\mathscr{F}^{SV}_{S'V'} = 0$, then the plot of $d(\epsilon)$ passes through zero at $\epsilon = 1$.

In order to settle the question as to whether one can determine all the interaction constants from experiments on $K_{e3}$ decays, we write out together all the equations that can be obtained from experiments:

$$\mathscr{F}^{VT}_{V'T'}(1-\varepsilon) + \mathscr{F}^{SV}_{S'V'} = A_1, \quad F^{V'T}_{VT'}(\varepsilon^2-\varepsilon) + F^{S'V}_{SV'}\varepsilon = A_2,$$

$$F_{SS'} + F^{S'T}_{ST'}(1-\varepsilon) + F_{V'V}[(\varepsilon-1)^2 - \varepsilon_0^2] + F_{T'T}(\varepsilon-1)^2 = A_3,$$

$$\Phi^{SS}_{S'S'} + \Phi^{VV}_{V'V'}[\varepsilon_0^2 - (1-\varepsilon)^2] + \Phi^{TT}_{T'T'}(1-\varepsilon)^2 + \Phi^{TS}_{T'S'}(1-\varepsilon) = A_4.$$

Here $A_1$, $A_2$, $A_3$, and $A_4$ are functions of the energies of the $\pi$ meson and the electron which can be obtained from experiments. The total number of equations is 10, and the number of unknown functions is 11.

If $|g_V|^2 + |g'_V|^2 = 0$, the number of equations is 6, and the number of unknowns is 7. And only if $\mathrm{Re}(g_V g_V'^*) = 0$ does one have 11 equations for 11 unknowns, if at the same time these constants are not both zero, i.e., if $|g_V|^2 + |g'_V|^2 \neq 0$. In all other cases we are short one equation.

Thus the constants can be completely determined (apart from an unimportant phase factor) only if one of the constants of the vector interaction vanishes while the other does not, or if $g_V$ and $g'_V$ are displaced in phase relative to each other by 90°. With regard to the neutrino it must be noted that if one of the transverse polarizations exists, the neutrino cannot be a two-component particle. One cannot, however, draw the converse conclusion: if there are no transverse polarizations, this still does not mean that the neutrino is a two-component particle.

Let us go on to the second group of experiments: measurements of the spectrum and the longitudinal polarization of the electrons.

Expressions for the electron spectrum have been obtained by Furuichi and others[5,8,9] and partially by Matinian,[10] and have been discussed in detail. Expressions for the longitudinal polarization have been obtained by Okun'[1] and Werle,[2] who have also discussed them in detail. Therefore we shall give below a very brief discussion of the problem of what changes are made in these expressions by the use of a neutrino of arbitrary type.

First we note that from the spectrum and the longitudinal polarization one can obtain six equations for the determination of the interaction constants, which are assumed independent of the energy of the $\pi$ meson. In this way one can determine the following combinations

$$\overline{\Phi}^{TT}_{T'T'};\quad \overline{\Phi}^{ST}_{S'T'};\quad \overline{\Phi}^{SS}_{S'S'} - \overline{\Phi}^{VV}_{V'V'};$$

$$\overline{F}_{SS'} + \overline{F}_{VV'};\quad \overline{F}_{TT'};\quad \overline{F}^{S'T}_{ST'}$$

(the bar denotes values averaged over the $\pi$-meson energy).

If the neutrino is a two-component particle, then

$$\overline{\Phi}^{TT}_{T'T'} = \overline{F}_{TT'},\quad \overline{\Phi}^{ST}_{S'T'} = \overline{F}^{S'T}_{ST'}. \quad (11)$$

If we assume the constants to be independent of the $\pi$-meson energy, the equations (11) are equivalent to the following:

$$|g_T|^2 + |g'_T|^2 = 2\mathrm{Re}(g_T g'^{*}_T); \quad (12)$$

$$\mathrm{Re}(g_S g^{*}_T + g'_S g'^{*}_T) = \mathrm{Re}(g'_S g^{*}_T + g_S g'^{*}_T). \quad (13)$$

We note that it follows unambiguously from Eq. (12) that $g_T = g'_T$. Equation (13), however, is a mere consequence of Eq. (12), i.e., a second independent check on the equality of the two tensor constants. For the case in which the equations (11) hold except that the signs of the left members are changed, we also have as the unambiguous result that the tensor constants are equal in magnitude and opposite in phase, which corresponds to the case of the "two-component antineutrino for the tensor-type interaction". Here we have used an expression for the degree of polarization which is the analogue of the equation obtained by Okun',[1] namely:

$$\overline{P} = \frac{-\overline{\Phi}^{SS}_{S'S'}J_3 + \overline{\Phi}^{VV}_{V'V'}J_3 - \overline{\Phi}^{TT}_{T'T'}(\overline{A}-\overline{B}) - \overline{\Phi}^{ST}_{S'T'}(\overline{C}-\overline{D})}{\overline{F}_{SS'}J_3 + \overline{F}_{VV'}J_3 + \overline{F}_{TT'}(\overline{A}-\overline{B}) + \overline{F}^{S'T}_{ST'}(\overline{C}-\overline{D})}.$$

The values of $J_3$, $\overline{C}$, $\overline{A}$, $\overline{B}$, $\overline{D}$ as functions of the electron energy are given in Ref. 1.

In conclusion we write down the probability for the emission of an electron with given energy and polarization in a given direction and the emission of a $\pi$ meson with a given energy:

$$W(\mathbf{J}, E_\pi, E_e)\,dE_\pi\,dE_e = \frac{1}{2}\{\cos(\widehat{\mathbf{JJ}_3})\,\delta W_4 + \cos(\widehat{\mathbf{JJ}_2})\,\delta W_3 + \cos(\widehat{\mathbf{JJ}_1})\,\delta W_2 + 2W_1\}.$$

The values of $\delta W_4$, $\delta W_3$, $\delta W_2$, add $W_1$ and the definitions of the three directions $\mathbf{J}_{1,2,3}$ have been given in the text of the present paper.

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# *INELASTIC SCATTERING OF DEUTERONS*

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The inelastic scattering of deuterons by nuclei is assumed to occur in some cases through direct interaction. The incident deuteron is merely scattered and forms the outgoing particle. An expression is derived for the angular distribution of the scattered deuterons. This expression agrees with the experimental data on the scattering of 14.4 Mev deuterons from the 4.61 Mev level of $Li^7$.

THE experiments of Holt and Young[1] on inelastic scattering of deuterons show that the angular distribution of the scattered deuterons usually has a maximum in the forward direction. This is a characteristic feature of the stripping and direct-interaction theories. Huby and Newns[2] have explained