ELECTRICAL RESISTANCE OF IRON, COPPER AND NICKEL — COPPER ALLOYS AT LOW TEMPERATURES

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The electrical resistance of nickel, iron, and copper alloys of nickel, containing up to 25% copper, have been measured in the temperature interval from 2 to 78°K. It has been found that in the region $4° < T < 78°K$, within experimental error, the temperature dependence of the electrical resistance is given by formulas (3) or (4) where $m \approx 3/2$, $l \approx n \approx 5$, where the phonon terms T^l and T^n become important only at temperatures above 20 to 30°K. Assuming a three-halves law for the temperature dependence of the spontaneous magnetization I_S, it is easy to relate the electrical resistance ρ to the ferromagnon concentration $n = 1 - I_S/I_0$, (where I_0 is the magnetization at $T \to 0$) and to obtain a numerical value for the coefficient $(\rho - \rho_0)/n$, where ρ_0 is the residual resistance. A comparison of this coefficient in nickel with the quantity $\Delta\rho/\Delta n$, where $\Delta\rho$ and Δn are the changes in ρ and n due to changes in the true magnetization in strong magnetic fields, shows that these quantities are approximately the same. This may be taken as indirect evidence of the validity of the assumption that the electrical resistance of ferromagnetic metals at low temperatures is related to scattering of electrons by inhomogeneities in the magnetization of the lattice (scattering by ferromagnons).

THE electrical resistance of iron, nickel and nickel-copper alloys has been studied in a number of papers,[1-5] but only in a relatively narrow temperature range. Thus, in our previous paper[4] the electrical resistance of nickel and nickel-copper alloys was studied between 2 and 4.2°K and between 14 and 20.4°K. In the present work the temperature dependence of the electrical resistance of these metals and alloys was studied over the entire temperature region from 2 to 78°K. The data obtained help evaluate the validity of various theoretical expressions[6,7] and establish the relation between changes in electrical resistance and spontaneous magnetization in ferromagnets in the low temperature region.

1. METHOD OF MEASUREMENT. SAMPLES.

The resistance was measured by a potentiometer method (using a PPTN-1 potentiometer). The samples were wires 0.1 to 0.2 mm in diameter and 150 to 160 mm long wound on a copper coil which was placed in a copper container 8 mm high for the measurement. The compositions of the alloy samples are given in the table. The iron samples were made from Armco iron wire. The nickel was chemically pure and its residual resistance was approximately 0.2×10^{-6} ohm-cm rather than 1.54×10^{-6} ohm-cm for the nickel sample used earlier.[4]

All samples were annealed in vacuum at 900°C for an hour and then slowly cooled at the rate of

Specimens	$\rho_0 \cdot 10^6$, $\Omega\cdot$cm	$\rho_{01} \cdot 10^6$, $\Omega\cdot$cm	$\alpha \cdot 10^{10}$, $\Omega\cdot$cm/deg	$\beta \cdot 10^{10}$, $\Omega\cdot$cm/deg^2	$\gamma \cdot 10^{16}$, $\Omega\cdot$cm/deg^5	$a \cdot 10^{10}$, $\Omega\cdot$cm/deg$^{3/2}$	$b \cdot 10^{16}$ $\Omega\cdot$cm/deg^5	m	n	l
Nickel	0.20	—	1	0.4	0.2	2	5	1.51	4.9	5.6
Iron	1.458	1.44	3	0.4	1.5	1.8	7.7	1.50	4.8	5.2
Alloy of Ni-Cu (annealed)										
4.6% Cu	4.95	—	17	0.4	0.8	4.5	8.2	1.49	4.8	5.3
9.9% Cu	9.27	—	30	0.4	—	8.0	—	1.48	—	—
15.1% Cu	12.65	—	40	0.4	0.2	8.4	5	1.47	4.8	5.6
20.0% Cu	16.22	—	49	0.6	—	15.0	—	1.51	—	—
25.1% Cu	20.25	19.85	51	1.1	—	—	—	—	—	—
Alloy of Ni-Cu (hardened)										
9.9% Cu	10.0	9.78	36	0.5	—	—	—	—	—	—
15.1% Cu	14.26	13.74	58	0.6	—	—	—	—	—	—
20 0% Cu	17.80	17.70	70	0.6	—	—	—	—	—	—

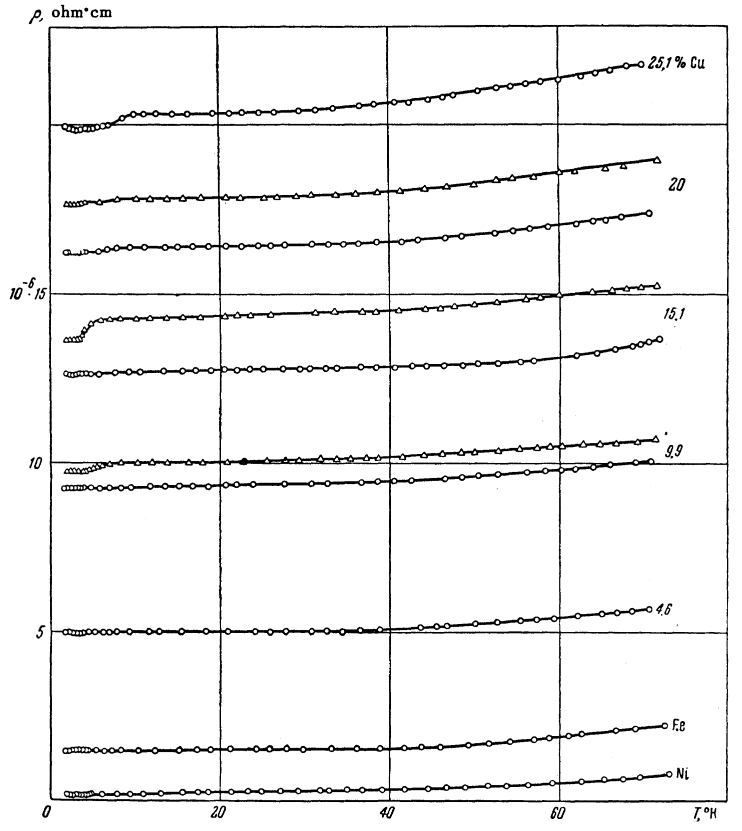

FIG. 1. Electrical resistance of nickel, iron and nickel-copper alloy as a function of temperature: o – after annealing, Δ – after hardening.

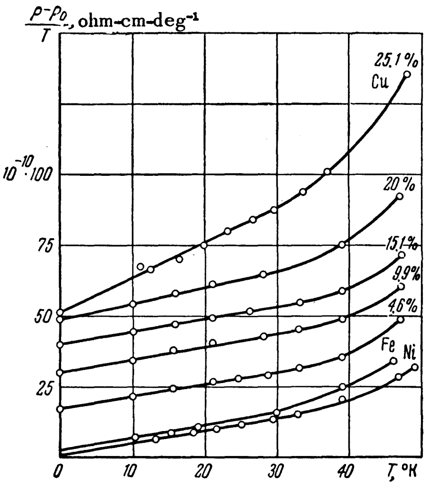

FIG. 2

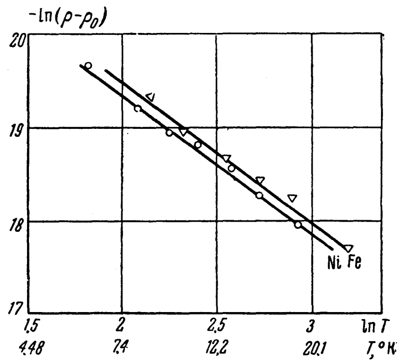

FIG. 3

100 degrees per hour. In addition, three samples were fabricated for which the annealing at 900°C for an hour was followed by rapid cooling (hardening) in air.

The temperature in the intervals 2 to 4.2°K, 14 to 20.4°K and 63.1 to 77.3°K was determined by measuring the pressure. In the measurements in the temperature region 4.2 to 14°K and above 20.4°K the container with the sample was suspended in a Dewar flask above the level of liquid and the temperature measured with a carbon resistance thermometer which had been calibrated against a gas thermometer. The error in the measurements in the 4.2 to 20°K region was 0.1 degree; in the region above 20.4°K the error was 0.5 degrees.

2. RESULTS

In Fig. 1 are shown curves which indicate the temperature dependence of the electrical resistance ρ in iron, nickel and nickel-copper alloys. In the curves for certain alloys there are "steps" in the temperature region 3 to 10°K. In the annealed samples these "steps" are generally smaller than in those which were hardened. In our earlier work[4] data have been given concerning the effect of magnetic field on the size and position of these "steps." It is possible that these "steps" are due to the presence of inclusions of superconducting materials which are not detected in chemical or spectral analysis; however, it is also possible that what is being observed is an anomaly, similar to that which was found in Refs. 8 and 9 in certain non-ferromagnetic alloys. In the present paper we shall not discuss these "steps" and consider the temperature dependence of the electrical resistance only outside the region in which they appear.

At the outset, we remove from ρ the residual resistance term ρ_0 which is independent of temperature. In many cases ρ_0 can be determined by linear extrapolation of the left-hand part of the curve of $\rho(T)$ to zero; however, this extrapolation procedure is not very accurate. We have determined ρ_0 by expanding $\rho(T)$ in a power series

$$\rho(T) = \rho_0 + \alpha T + \beta T^2 + \ldots \quad (1)$$

In this case, if the first three terms are considered, the error in the determination of ρ_0 does not exceed the error in the measurements. In those cases

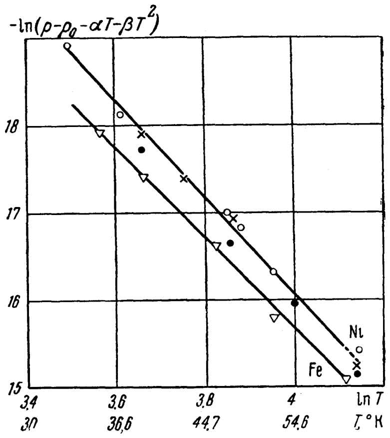

FIG. 4. o – nickel, ∇ – iron, • – alloy with 4.6 % copper, × – alloy with 15.1% copper.

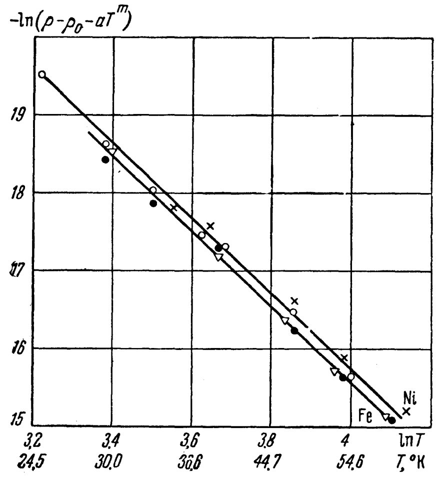

FIG. 5. o – nickel, ∇ – iron, • – alloy with 4.6% copper, × – alloy with 15.1% copper.

in which "steps" are observed on the $\rho(T)$ curve, two values of the residual resistance are taken, namely: ρ_0 for the part of the curve lying to the right of the step, in which case the expression in (1) is used, and ρ_{01} which is obtained by linear extrapolation to zero of the part of the $\rho(T)$ curve lying to the left of the step $(T < 3°K)$. The values of ρ_0 and ρ_{01} and α and β, obtained in this way for the different samples, are shown in the table.

The quantity $(\rho - \rho_0)/T$ as a function of T and the quantity $\ln(\rho - \rho_0)$ as a function of ln T are shown in Figs. 2 and 3 respectively. Departures from a straight line are observed in Fig. 2 starting at $T > 30°K$. In the temperature region $4 < T < 18°K$ the dependence of the electrical resistance on temperature may be described by three terms in the formula in (1) or by the formula*

$$\rho = \rho_0 + aT^m, \tag{2}$$

where a and m are constants. The values of these constants are given in the table. In all samples the exponent m is approximately $3/2$.

The sharp increase in electrical resistance at temperatures greater than 20 to 30°K indicates that we may be dealing here with a T^5 relation. To verify this assumption the curves shown in Figs. 4 and 5 were plotted; in these curves the quantities $\ln(\rho - \rho_0 - \alpha T - \beta T^2)$ and $\ln(\rho - \rho_0 - aT^m)$ are plotted against ln T. In both figures the points lie on straight lines. Thus the temperature dependence in the region $4°K < T < 77°K$ is given either by

$$\rho = \rho_0 + \alpha T + \beta T^2 + \gamma T^l \tag{3}$$

or

$$\rho = \rho_0 + aT^m + bT^n, \tag{4}$$

where l and n are approximately 5. The values of l, n and the coefficients γ and b are shown in the table.

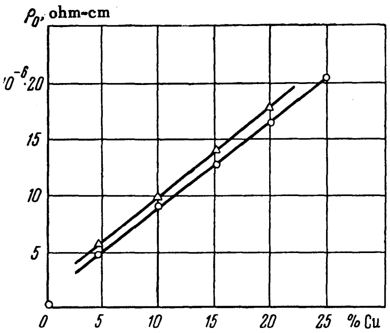

FIG. 6. Residual electrical resistance of a nickel-copper alloy as a function of composition. o – after annealing, Δ – after hardening.

Figure 6 shows the residual resistance ρ_0 as a function of the copper concentration in nickel-copper alloys for hardened samples and annealed samples. This relation is linear up to copper concentrations of 25%. Points corresponding to ρ_{01} in samples in which "steps" were observed, as is apparent from the table, lie below the lines which pass through the ρ_0 points in the "normal" samples. Thus the anomaly is observed as a reduction in the resistance as compared with "normal" behavior; this is the basis for the assumption that the effect is due to superconducting inclusions.

*The fact that there are two formulas for the temperature dependence in a limited low-temperature range is not inconsistent. It is apparent that the formula for ρ can be written in the form of a power series, using any parameter which is small close to absolute zero. Such a parameter could be T or T^m with $m > 0$. The present experimental data indicates that in the region 4 – 20°K the dependence of ρ on T is given by the first three terms in the power series or two terms in the series in powers of $T^{3/2}$.

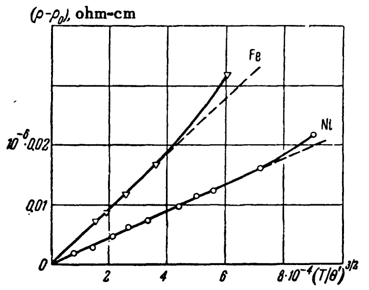

FIG. 7. Increase in electrical resistance with reduction in spontaneous magnetization.

3. DISCUSSION OF RESULTS

The electrical resistance of metals is related to scattering of electrons by thermal vibrations of the lattice (phonon scattering), scattering by inhomogeneities in the magnetization (in ferromagnetic materials — scattering by ferromagnons) and scattering by impurity ions which appear in the lattice. From the results which have been given above it follows that in iron, nickel, and nickel-copper alloys the phonon term, which is proportional to T^5 in the Bloch theory, is important at temperatures above 18 to 30°K. We may note that if the Bloch relation

$$\rho_1 = 497.6\,(\rho_2 / \Theta_d^4 T_2)\, T^5$$

(where Θ_d is the Debye temperature $T_1 \ll \Theta_d \ll T_2$) is used to compute the T^5 coefficient in nickel, a value of 9.5×10^{16} is obtained and this is close to the value of b given in the table.

Scattering by inhomogeneities in the magnetization of the lattice (ferromagnons) has been considered theoretically by Vonsovskii and Turov.[6,7] The quantity $\Delta\rho$, that part of the electrical resistance of a ferromagnetic metal which depends on the inhomogeneity in the magnetization, is given by Turov in the form $\Delta\rho = c_1 T + c_2 T^2$, which yields the same relation between ρ and T as the empirical relation in (1). The theory does not give the values of the coefficients c_1 and c_2 and thus cannot be compared quantitatively with the experimental results. However, using the experimental data it is possible to show indirectly that the temperature dependence of the resistance ρ in the ferromagnetic metals investigated in the region from 4 to 18°K is actually related to inhomogeneities in the magnetization and to make a rough estimate of the interaction energy which characterizes the scattering of electrons on the indicated inhomogeneities. For this purpose we have compared the change in ρ in the indicated temperature region with the isotropic changes in this quantity in strong magnetic fields. The latter are proportional to the true magnetization and are thus directly related to the concentration of ferromagnons. We have also compared both quantities with the change in residual resistance in nickel-copper alloys as the copper concentration is changed.

In the low temperature region, it follows from the Bloch theory and has been shown experimentally,[10,11] that

$$(I_0 - I_s) / I_0 = (T/\Theta')^{3/2},$$

where I_0 is the magnetization at $T = 0$ and Θ' is a parameter related to the exchange energy. In Fig. 7 the values of $\rho - \rho_0$ are plotted against $(T/\Theta')^{3/2}$. In plotting these curves the values of Θ' for nickel and iron were taken from Refs. 10 and 11. In Fig. 7, as can also be seen directly from Fig. 3 and Eq. (2), in the temperature region 4 to 18°K ρ is a linear function of $(T/\Theta')^{3/2}$, and thus of $I_0 - I_s / I_0 = n$, i.e., the concentration of ferromagnons (in the region 4 to 18°K, $n < 0.001$). According to the present data the quantity $(\rho - \rho_0)/n$ is 2.2×10^{-5} in nickel and 4.4×10^{-5} in iron.

We now compare $(\rho - \rho_0)/n$ with $I_0 \Delta\rho/\Delta I_s = \Delta\rho/\Delta n$ where $\Delta\rho$ is the change in electrical resistance which accompanies a change ΔI_s in the true magnetization in strong magnetic fields. In nickel, in the region from 4,000 to 18,000 oersteds, we have at room temperature $\Delta\rho/\rho\Delta H = 0.25 \times 10^{-6}$ (Ref. 12), $\Delta I_s/\Delta H = 1.3 \times 10^{-4}$, (Ref. 13), $0.7 \times 10^{-4} < \Delta I_s/\Delta H < 1.2 \times 10^{-4}$ (Ref. 14), and $\rho = 11 \times 10^{-6}$ ohm-cm. Assuming $I_0 = 509$ we have $\Delta\rho/\Delta n \approx 10^{-5}$, or $2.0 \times 10^{-5} > \Delta\rho/\Delta n > 1.1 \times 10^{-5}$. These quantities are of the same order of magnitude as $(\rho - \rho_0)/n$, and this may be considered an indirect verification of the fact that the electrical resistance of ferromagnetic metals and alloys in the low-temperature region is related to scattering of electrons by inhomogeneities in magnetization of the lattice.

In those cases in which electron scattering is due to an irregular static potential due to impurities or inhomogeneities in the lattice, this potential, which is usually taken as a small perturbation, can be estimated roughly from the difference in the ionization potentials of the host atoms and the impurity atoms. If, using this very rough estimate, we take the perturbing potential in the lattice of a nickel-copper alloy as the difference in the ionization potentials for copper and nickel, a value of approximately 0.07 ev is obtained. The magnitude of the perturbing potential due to an inhomogeneity in the magnetic moment corresponds, in order-of-magnitude terms, to the exchange integral, i.e., 0.01 to 0.1 ev. Thus, in the first approximation electron scattering on inhomogeneities in the magnetization of the nickel lattice and electron scattering by copper ions in copper-nickel alloys should lead to a change in the electrical resistance which is of the same order of magnitude. On the other hand, as the change in electrical resistance is the same for identical increases in the concentration of ferromagnons in the nickel lattice and copper ions in the alloy lattice it may be concluded that the exchange integral for the s and d electrons is a quantity of the order of 0.01 to 0.1 ev.

The curve shown in Fig. 6 indicates that the residual resistance of nickel-copper alloys increases in proportion to the copper concentration ν for $\nu < 0.25$ and $\Delta\rho_0/\nu = 7.7 \times 10^{-5}$ where $\Delta\rho_0$ is the difference in the values of ρ_0 for an alloy sample with a copper concentration equal to ν and a nickel sample. Comparing these quantities with $(\rho - \rho_0)/n$ for nickel we see that both are approximately the same order of magnitude. From a comparison of $\Delta\rho_0/\nu$ and $(\rho - \rho_0)/n$ it is apparent that electron scattering on impurities in the copper is somewhat stronger than on inhomogeneities in the magnetization.

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225