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TIME REVERSAL AND POLARIZATION PHENOMENA IN REACTIONS INVOLVING GAMMA-QUANTA

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Polarization phenomena in the photoproduction and radiative capture of pions as well as in the Compton effect are treated. Consequences of invariance under time reversal are studied. Wolfenstein's theorems are generalized to the case of reactions involving γ-quanta.

1. INTRODUCTION

ADVANCES in experimental technique lead one to hope that in the near future polarization experiments will become possible in such elementary reactions as the Compton effect on nucleons, photoproduction of pions and photodisintegration of the deuteron. In this connection we meet the problem of treating polarization phenomena in reactions involving γ-rays, delineating independent experiments and setting up a complete set of experiments which would be needed for reconstructing the reaction amplitude. In determining the number of independent experiments, in addition to invariance under rotations and space inversions, the consideration of symmetry under time reversal and the unitarity condition are important.

The present paper treats polarization phenomena in the photoproduction of pions and the Compton effect on nucleons. Invariance of the interaction with respect to time reversal leads to relations not only between unpolarized (spin-averaged) cross sections, but also between the polarization phenomena in inverse reactions. Although it has been asserted[1-4] that Wolfenstein's theorems are valid for elastic scattering and nuclear reactions, they require a separate consideration in the phenomenological analysis of reactions involving γ-rays. Wolfenstein's theorems are generalized to such reactions in the present paper.

The condition of unitarity of the S-matrix, which encompasses elastic scattering of pions by nucleons

$$-p \to -p,\ 0n \to 0n;\ +n \to +n,\ 0p \to 0p;$$

exchange scattering

$$-p \rightleftarrows 0n,\ +n \rightleftarrows 0p;$$

photoproduction and radiative capture

$$+n \rightleftarrows \gamma p,\quad -p \rightleftarrows \gamma n,\quad 0n \rightleftarrows \gamma n,\quad 0p \rightleftarrows \gamma p$$

and the Compton effect

$$\gamma p \to \gamma p,\quad \gamma n \to \gamma n,$$

enables us to introduce three real phases and three transformation parameters for each state. This enables us to determine the necessary number of experiments, and this number decreases if we take isotopic invariance into account. The unitarity conditions are treated in the Appendix.

2. PHOTOPRODUCTION OF PIONS AND RADIATIVE CAPTURE

To find the photoproduction amplitude, we first represent it as

$$M = a + \mathbf{b}\boldsymbol{\sigma}. \qquad (1)$$

Since the amplitude M must be a pseudoscalar, the quantities a and **b** must be a pseudoscalar and a vector, constructed from the polarization vector **e**, and the vectors $\mathbf{n}' = [\mathbf{q}\times\mathbf{k}]$, $\boldsymbol{\pi}' = \mathbf{q}+\mathbf{k}$, $\boldsymbol{\Delta}' = \mathbf{q}-\mathbf{k}$, where **q** and **k** are the momenta of the pion and photon, respectively. The construction of the photoproduction amplitude and the investigation of the consequences of invariance under time reversal for the case of photoproduction of pions on nucleons has been done in many papers.[5-7] By using gauge invariance, the expression for M can be represented as

$$M_{\pi\gamma} = A(\sigma\mathbf{e}) + B(\sigma\mathbf{q})(\mathbf{e}\mathbf{q}) + C(\mathbf{e}\mathbf{n}') + D(\sigma\mathbf{k})(\mathbf{q}\mathbf{e}). \qquad (2)$$

For the inverse reaction — radiative capture, we find from the same considerations

$$M_{\gamma\pi} = A'(\sigma\mathbf{e}) + B'(\sigma\mathbf{q})(\mathbf{e}\mathbf{q}) + C'(\mathbf{e}\mathbf{n}') + D'(\sigma\mathbf{k})(\mathbf{q}\mathbf{e}). \qquad (2')$$

If ω_0 and ω_1 are functions describing one nucleon, and nucleon plus meson, respectively, then, for example,

$$A \equiv (\omega_1, \hat{D}\omega_0), \quad A' \equiv (\omega_0, \hat{D}\omega_1),$$

where the effect of the time reversal operator K on the functions ω_0 and ω_1 reduces to

$$K\omega_0 = \omega_0, \quad K\omega_1 = -\omega_1,$$

and the requirement of invariance under time reversal gives

$$K\hat{D}K^{-1} = \hat{D}^+.$$

Repeating the arguments of Watson,[5] one can show that $A' = -A$. Similarly $B' = -B$, $C' = +C$, $D' = -D$. Thus

$$M_{\pi\gamma} = A(\sigma e) + B(\sigma q)(eq) + C(en') + D(\sigma k)(qe) = M,$$

$$-M_{\gamma\pi} = A(\sigma e) + B(\sigma q)(eq) - C(en') + D(\sigma k)(qe) = M'. \quad (3)$$

From the transversality of γ-quanta, $(\mathbf{ek}) = 0$ and

$$B(\sigma q)(eq) + D(\sigma k)(eq) = \delta'(\sigma\pi')(e\pi') + \gamma'(\sigma\Delta')(e\Delta').$$

If we go over to the orthonormal vectors $\boldsymbol{\pi}$, $\boldsymbol{\Delta}$ and $\mathbf{n}$ and use the relation

$$(\sigma e) = (\sigma n)(en) + (\sigma\pi)(e\pi) + (\sigma\Delta)(e\Delta),$$

we obtain finally

$$M = \alpha(en) + \beta(\sigma n)(en) + \gamma(\sigma\Delta)(e\Delta) + \delta(\sigma\pi)(e\pi); \quad (4)$$

$$M' = -\alpha(en) + \beta(\sigma n)(ne) + \gamma(\sigma\Delta)(e\Delta) + \delta(\sigma\pi)(e\pi). \quad (4')$$

For the cross section for production of mesons by unpolarized γ-quanta on unpolarized protons, we get (omitting the statistical factor, as we shall do throughout the paper)

$$2I_0(\theta) = |\alpha|^2 + |\beta|^2 + |\gamma|^2\cos^2\frac{\theta}{2} + |\delta|^2\sin^2\frac{\theta}{2}. \quad (5)$$

In view of the relation ("semidetailed" balancing) between the cross sections

$$I_0(\gamma n \to -p) = I_0(-p \to \gamma n)$$

it may turn out that it is not entirely hopeless to attempt experimental investigation of photoproduction by studying the inverse process — radiative capture of the π^--meson by a proton. Although such experiments are very difficult, their accomplishment will make it possible to get information concerning photoproduction on the free neutron by monochromatic γ-rays.

The relation between the nucleon polarization $\langle\sigma\rangle_f$ in photoproduction, when the γ-quanta and target are unpolarized, and the additional photoproduction cross section I_p

$$I = I_0 + I_p = \frac{1}{4}\,\mathrm{Sp}\,M^+M + \frac{1}{4}\,\mathrm{Sp}\,M^+M\sigma_1\cdot\mathbf{N}, \quad \mathbf{N} \equiv \langle\sigma\rangle_i,$$

when the target is polarized, i.e., a Wolfenstein theorem of the form

$$I_p = I_0\langle\sigma_1\rangle_f\mathbf{N}$$

follows from representing the amplitude as in (1) and using arguments concerning time reversal.[2] The expression for the polarization of the nucleon can be put in the form

$$2I_0(\theta)\langle\sigma\rangle_f = \mathbf{n}\left\{\alpha^+\beta + \alpha\beta^+ + \frac{i}{2}(\gamma\delta^+ - \gamma^+\delta)\sin\theta\right\}. \quad (6)$$

Let us continue our consideration of photoproduction of mesons by polarized γ-quanta on an unpolarized target.

We know that the state of polarization of a particle of spin 1 can be prescribed by giving the average values of the operators $T_{1,\pm1}$, $T_{1,0}$, $T_{2,\pm1}$, $T_{2,0}$ and $T_{2,\pm2}$, which are constructed from the spin operator.[8] Because of the transversality of γ-quanta, in a completely polarized beam, $\langle T_{1,\pm1}\rangle = \langle T_{2,\pm1}\rangle = 0$,[9] so that the expression for the cross section for photoproduction of mesons by a partially polarized beam of quanta can be written as

$$I(\theta,\varphi) = I_0(\theta) + \langle T_{2,0}\rangle_i\frac{1}{4}\,\mathrm{Sp}\,MT_{2,0}M^+$$

$$+ \langle T_{2,2}\rangle_i\frac{1}{4}\,\mathrm{Sp}\,MT_{2,2}M^+ + \langle T_{2,-2}\rangle_i\frac{1}{4}\,\mathrm{Sp}\,MT_{2,-2}M^+. \quad (7)$$

We note first that, after averaging over the nucleon spin,

$$\mathrm{Sp}\,MSM^+ = 0 \quad (8)$$

(where $\mathbf{S}$ is the spin of the photon). This result is immediately obvious if we use the formula

$$(fSg) = -i\,[\mathbf{fg}].$$

for functions of the type $f = (\mathbf{fe})$ and $g = (\mathbf{ge})$.

The same result is obtained if we calculate the average value of the spin vector of the photon which is produced in the radiative capture of mesons by unpolarized protons. This result is very general. This is the reason why there is no term proportional to $\cos\varphi$ in the expression for the cross section of a reaction induced by a polarized beam of γ-quanta.

In calculating the remaining terms in (7) and the average values of the tensors $T_{2,\pm 2}$, $T_{2,0}$ after radiative capture, we use the formula[4]

$$(S_iS_k + S_kS_i) M = 2M\delta_{ik} - (M_ie_k + M_ke_i). \quad (9)$$

For the tensors which are different from zero, we have

$$\frac{1}{4}\,\mathrm{Sp}\, MT_{2,\pm 2}M^+ = \frac{\sqrt{3}}{4}\{(|\alpha|^2 + |\beta|^2)(n_y^2 - n_x^2 \mp 2in_xn_y)$$

$$+ |\gamma|^2(\Delta_y^2 - \Delta_x^2 \mp 2i\Delta_x\Delta_y) + |\delta|^2(\pi_y^2 - \pi_x^2 \mp 2i\pi_x\pi_y)\};$$

$$\frac{1}{4}\,\mathrm{Sp}\, MT_{2,0}M^+ = I_0(\theta)/\sqrt{2};$$

$$2T_{2,\pm 2} = \sqrt{3}\{(S_x^2 - S_y^2) \pm i(S_xS_y + S_yS_x)\};$$

$$\sqrt{2}T_{2,0} = 3S_z^2 - 2. \quad (10)$$

If we choose the z axis to be along the momentum of the γ-ray,

$$\frac{1}{4}\,\mathrm{Sp}\, MT_{2,\pm 2}M^+$$

$$= \frac{\sqrt{3}}{4}\left\{|\alpha|^2 + |\beta|^2 - |\gamma|^2\cos^2\frac{\theta}{2} - |\delta|^2\sin^2\frac{\theta}{2}\right\}e^{\pm 2i\varphi}$$

and the cross section for meson production by polarized quanta finally becomes

$$I(\theta, \varphi) = I_0(\theta)\left[1 + \frac{1}{\sqrt{2}}\langle T_{2,0}\rangle_i\right]$$

$$+ \langle T_{2,2}\rangle_i\frac{\sqrt{3}}{2}\left\{|\alpha|^2 + |\beta|^2 - |\gamma|^2\cos^2\frac{\theta}{2} - |\delta|^2\sin^2\frac{\theta}{2}\right\}\cos 2\varphi. \quad (11)$$

We now turn to the polarization of γ-quanta from radiative capture of π^--mesons by unpolarized protons. For the non-zero average values of tensors

$$I_0(\theta)\langle T_{2,m}\rangle = \frac{1}{4}\,\mathrm{Sp}\, M'T_{2,m}M'$$

we obtain expressions which coincide with (10) if we take the z axis along the direction of the emergent γ-rays from the radiative capture. If we write (11) in the form

$$I(\theta, \varphi) = I_0(\theta) + \langle T_{2,0}\rangle I_{2,0} + \langle T_{2,2}\rangle I_{2,2},$$

the result obtained can be written as

$$I_{2,m} = I_0(\theta)\langle T_{2,m}\rangle, \quad (12)$$

which is the statement of Wolfenstein's theorem for this reaction.

This means that the study of polarization of γ-quanta in the $-p \to \gamma n$ reaction gives the same information as the study of photoproduction of mesons by polarized γ-rays. We should add to this the result already mentioned concerning the polarization of the nucleon. Study of the nucleon polarization in radiative capture gives the same information as the investigation of photoproduction on polarized target protons, while the proton polarization in photoproduction is related to the cross section for radiative capture by polarized protons.

To conclude the treatment of polarization phenomena in $\gamma N \rightleftarrows \pi N$ reactions, let us compare the expression for the polarization correlation in radiative capture

$$I_0(\theta)\langle(\sigma\mathbf{a})(T_{ik}b_ic_k)\rangle = \frac{1}{4}\,\mathrm{Sp}\, M'^+(\sigma\mathbf{a})(T_{ik}b_ic_k)M'$$

with the additional photoproduction cross section I_{pp}, when the γ-rays and the target are polarized:

$$I_{pp} = \frac{1}{4}\,\mathrm{Sp}\, M(\sigma\mathbf{a})(T_{ik}b_ic_k)M^+.$$

For the correlation $I_0\langle(\sigma\mathbf{a})(\mathbf{Sb})\rangle$, we have

$$-2I_0(\theta)\langle(\sigma\mathbf{a})(\mathbf{Sb})\rangle$$

$$= -i\{(\alpha^+\gamma - \alpha\gamma^+)(\mathbf{a}\Delta)(\pi\mathbf{b}) - (\alpha^+\delta - \alpha\delta^+)(\mathbf{a}\pi)(\Delta\mathbf{b})\}$$

$$+ \{(\beta^+\gamma + \beta\gamma^+)(\mathbf{a}\pi)(\mathbf{b}\pi)$$

$$+ (\beta^+\delta + \beta\delta^+)(\mathbf{a}\Delta)(\mathbf{b}\Delta) + (\gamma^+\delta + \gamma\delta^+)(\mathbf{an})(\mathbf{bn})\}, \quad (13)$$

from which we see, in particular, that

$$\langle(\sigma\mathbf{n})(\mathbf{S}\pi)\rangle = \langle(\sigma\mathbf{n})(\mathbf{S}\Delta)\rangle = \langle(\sigma\pi)(\mathbf{Sn})\rangle = \langle(\sigma\Delta)(\mathbf{Sn})\rangle = 0$$

$$2I_0(\theta)\langle(\sigma\mathbf{n})(\mathbf{Sn})\rangle = -(\gamma^+\delta + \gamma\delta^+), \text{ etc.} \quad (14)$$

For the additional photoproduction cross section we find

$$-\frac{1}{4}\,\mathrm{Sp}\, M(\sigma\mathbf{a})(\mathbf{Sb})M^+$$

$$= \frac{i}{2}\{(\alpha\gamma^+ - \alpha^+\gamma)(\mathbf{a}\Delta)(\pi\mathbf{b}) - (\alpha\delta^+ - \alpha^+\delta)(\mathbf{a}\pi)(\Delta\mathbf{b})\}$$

$$+ \frac{1}{2}(\beta^+\gamma + \beta\gamma^+)(\mathbf{a}\pi)(\mathbf{b}\pi)$$

$$+ (\beta^+\delta + \beta\delta^+)(\mathbf{a}\Delta)(\mathbf{b}\Delta) + (\gamma\delta^+ + \gamma^+\delta)(\mathbf{an})(\mathbf{bn})\}. \quad (15)$$

From a comparison of (15) and (13) we see that, knowing the results of investigation of the polarization correlation in radiative capture, i.e., knowing the combinations $(\alpha\gamma^+ - \alpha^+\gamma)$, $(\alpha\delta^+ - \alpha^+\delta)$ etc., we can predict the results of experiments using polarized beams and target. Consequently, these experiments are not independent.

We also see from (13) and (15) that relations like (12) hold whenever one of the vectors **a** or **b** is along the normal **n**. Similar conclusions follow when S is replaced by $T_{2,m}$ in (13) and (15).

We note in conclusion that, in the expression for the polarization of the recoil nucleon in photoproduction by polarized γ-quanta on an unpolarized nucleon,

$$4I(\theta, \varphi)\langle\sigma\rangle_f = \mathrm{Sp}\, M^+\sigma M + \langle T_{1,0}\rangle_i\, \mathrm{Sp}\, T_{1,0}M^+\sigma M$$
$$+ \langle T_{2,2}\rangle_i\, \mathrm{Sp}\, T_{2,2}M^+\sigma M + \langle T_{2,-2}\rangle_i\, \mathrm{Sp}\, T_{2,-2}M^+\sigma M \quad (16)$$

the first term is the same as (6), while the second term is proportional to

$$-i\,[(\alpha\gamma^+ - \alpha^+\gamma)\,\Delta_z\boldsymbol{\pi} - (\alpha\delta^+ - \alpha^+\delta)\,\pi_z\boldsymbol{\Delta}]$$
$$+ (\beta\gamma^+ + \beta^+\gamma)\,\pi_z\boldsymbol{\pi} + (\beta^+\delta + \beta\delta^+)\,\Delta_z\boldsymbol{\Delta}.$$

From the last expression we see the correspondence with (13) and (15). A similar correspondence exists for the other terms in (16).

3. COMPTON EFFECT

The Compton effect amplitude can be represented in the form of (1), but for elastic scattering M must be a scalar. Using the standard arguments, including symmetry under time reversal, we find that the most general expression for M is

$$M = A(\mathbf{ee'}) + B(\mathbf{n}[\mathbf{ee'}])$$
$$+ C(\sigma\mathbf{n})(\mathbf{ee'}) + D(\sigma\mathbf{n})(\mathbf{n}\cdot\mathbf{e}\times\mathbf{e'}) + F(\sigma[\mathbf{ee'}])$$
$$+ E\{(\boldsymbol{\pi}\mathbf{e'})(\sigma[\Delta\times\mathbf{e}]) + (\boldsymbol{\pi}\mathbf{e})(\sigma[\Delta\times\mathbf{e'}])\}, \quad (17)$$

where **e** and **e′** are the polarization vectors before and after scattering, **n**, **Δ** and **π** are constructed, as before, from the (unit) vectors **k** and **k′** along the direction of the γ-ray momentum before and after scattering. Expression (17) contains six terms, as it should. The expression for M can be given another form*

$$M = R_1(\mathbf{ee'}) + R_2(\mathbf{ss'}) + R_3(\sigma[\mathbf{e'e}]) + R_4(\sigma[\mathbf{s's}])$$
$$+ R_5\{(\sigma\mathbf{k})(\mathbf{s'e}) - (\sigma\mathbf{k'})(\mathbf{se'})\} + R_6\{(\sigma\mathbf{k'})(\mathbf{s'e}) - (\sigma\mathbf{k})(\mathbf{e's})\}, \quad (17')$$

if we introduce the vectors $\mathbf{s} = [\mathbf{k}\times\mathbf{e}]$ and $\mathbf{s'} = [\mathbf{k'}\times\mathbf{e'}]$. Under time reversal, $\mathbf{e} \rightleftarrows -\mathbf{e'}$, $\mathbf{s'} \rightleftarrows \mathbf{s}$.

The amplitude for the Compton effect, up to terms linear in the frequency, has been found in various papers:[10-12]

$$M = \frac{e^2}{m}(\mathbf{ee'}) - \frac{ie}{m}\left(2\mu - \frac{e}{m}\right)k\,(\sigma[\mathbf{e'}\times\mathbf{e}]) -$$
$$-\frac{2\mu^2 i}{k}(\sigma[\mathbf{s}\times\mathbf{s'}]) - i\frac{e}{m}\frac{\mu}{k}[(\mathbf{ek'})(\sigma\mathbf{s'}) - (\mathbf{e'k})(\sigma\mathbf{s})], \quad (18)$$

Here μ is the magnetic moment of the nucleon.

From (17) we find for the cross section for scattering of an unpolarized γ-ray beam by an unpolarized target:

$$2I_0(\theta) = \{|R_1|^2 + |R_2|^2$$
$$+ 4\mathrm{Re}\,[R_3^+(R_5 + R_6) + R_4^+R_5]\}(1 + \cos^2\theta)$$
$$+ (|R_3|^2 + |R_4|^2)(3 - \cos^2\theta) + 2(|R_5|^2 + |R_6|^2)(3 + \cos^2\theta)$$
$$+ 4\mathrm{Re}\,\{(R_1^+R_2) + R_4^+(R_3 - R_6) + R_5^+R_6(2 + \cos^2\theta)\}\cos\theta. \quad (19)$$

The condition of unitarity of the S-matrix leads to the optical theorem[13]

$$k\sigma_t = 4\pi\,\mathrm{Im}\,[R_1(0°) + R_2(0°)], \quad (20)$$

where σ_t is the total interaction cross section, including both elastic scattering and inelastic processes. From (20) we get for the forward elastic scattering cross section $\sigma_s(0^0)$ the inequality*

$$\sigma_s(0°) \geqslant (k\sigma_t/4\pi)^2, \quad (21)$$

as a consequence of which the elastic scattering at high energies is pushed forward and compressed into the small solid angle

$$\Delta\omega = \pi\theta^2 \leqslant (4\pi/k\sigma_t)^2\,\sigma_s. \quad (22)$$

The inequalities (21) and (22) show the basic features of elastic scattering at high energies without appealing to an optical model.

The expression for the polarization of the recoil nucleon, when target and beam are unpolarized,

$$2I_0(\theta)\langle\sigma\rangle_f = \mathbf{n}\sin\theta\, 2\mathrm{Re}\,\{(R_1^+R_4)$$
$$+ \left[(R_2^+R_4) - (R_1^+R_3)\right]\cos\theta - R_2^+R_3\cos 2\theta\}, \quad (23)$$

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*Addition in proof (March 18, 1958). The author recently learned that formula (17′) was found earlier by V. I. Ritus, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1264 (1957); Soviet Physics JETP (in press).

*The existence of the inequality (21) for elastic scattering of particles was shown in Ref. 14 and, in less general form, by Rarita[15] as well as by Karplus and Ruderman (cf. Ref. 15). This inequality was first published in a brief note by Wick,[16] which went unnoticed.

coincides with the expression for the azimuthal asymmetry in the cross section for the Compton effect on a polarized nucleon. We note that the quantities R_5 and R_6 do not appear in (23).

When the target and beam are unpolarized, the average value of the photon spin (and the corresponding addition to the cross section) become zero, as in the case of photoproduction. This result is obvious from dimensional considerations. In these circumstances the average value of the spin can be directed only along the one pseudovector — the normal **n**. Since $\langle S_x \rangle = \langle S_y \rangle \equiv 0$ for photons propagating along the z axis, we need only consider $\langle S_z \rangle$, but $\langle S_z \rangle \sim n_z = 0$.

For the non-zero average values of the $T_{2,m}$, we find, using (9),

$$2I_0(\theta)\langle T_{2,\pm 2}\rangle = \sqrt{3}\{|R_4|^2 + \mathrm{Re}\,[R_4^+(R_5 + R_6\cos\theta)]\}\, e^{\pm 2i\varphi};$$

$$\sqrt{2}\, I_0(\theta)\langle T_{2,0}\rangle = I_0(\theta). \tag{24}$$

For the additional Compton effect cross section when the incident quanta are polarized, we get expressions coinciding with (24).

CONCLUSION

The main content of the present paper is the extension of the consequences of invariance under time reversal to reactions involving γ-quanta. We have treated the examples of the Compton effect, photoproduction, and radiative capture of pions. The generalization to the case of any binary reaction (with two particles in the initial and in the final state), is done similarly.

Aside from its theoretical interest, this result is of interest to experiment, for the same reason that in photodisintegration of the deuteron, the study of the polarization of the nucleons gives the same information as the radiative capture by a polarized nucleon, or the study of polarization of γ-quanta from radiative capture of neutrons by protons is equivalent to studying the cross section for photodisintegration of the deuteron by polarized gamma rays.

The relations between polarization phenomena in inverse reactions, as well as the relation between averaged values, are based on invariance of the interaction under time reversal. However, in contrast to detailed balancing, for the Wolfenstein relation it is essential whether or not spatial parity is conserved. For illustration, let us consider the elastic scattering of a neutrino by a spinless particle. If we do not demand invariance under space and time inversion, the most general form for the scattering amplitude is

$$M = a + b(\sigma\mathbf{n}) + c(\sigma\boldsymbol{\pi}) + d(\sigma\Delta). \tag{25}$$

The last two terms, which are pseudoscalar for space inversions, transform differently under time reversal. If there is no conservation of spatial parity, but we keep the invariance under time reversal (combined parity), the last term in (25) is absent. Though in this case detailed balancing remains valid (trivially), there is no Wolfenstein relation. With $d = 0$, the polarization resulting from the scattering of an unpolarized beam is

$$I_0(\theta)\langle\sigma\rangle_f = (a^+b + ab^+)\,\mathbf{n} + (ac^+ + a^+c)\,\boldsymbol{\pi} + i(b^+c - c^+b)\,\Delta, \tag{26}$$

while the additional cross section I_p is

$$(a^+b + ab^+)\,\mathbf{n} + (ac^+ + a^+c)\,\boldsymbol{\pi} - i(b^+c - c^+b)\,\Delta. \tag{27}$$

Although, as before, polarization and azimuthal asymmetry give the same information (since (26) and (27) differ only in the sign of the last term), there is no Wolfenstein relation.

APPENDIX

Unitarity Conditions

The unitarity conditions together with requirements of invariance determine the number of independent parameters necessary for the phenomenological analysis of the experimental data. If we consider the range of γ-ray energies near 300 Mev where, in the interaction of photons with nucleons, only photoproduction of single pions occurs in addition to the elastic scattering, we introduce a 3-by-3 S-matrix to describe the Compton effect, photoproduction and radiative capture, scattering and charge exchange of mesons. To be specific, let us consider the Compton effect on a proton. In this case we can introduce a unitary S-matrix to describe the processes

$$\begin{aligned} &+n \to +n\,(S_{11}); \quad 0p \to +n\,(S_{12}); \quad \gamma p \to +n\,(S_{13}) \\ &+n \to 0p\ \ (S_{21}); \quad 0p \to 0p\ \ (S_{22}); \quad \gamma p \to 0p\ \ (S_{23}) \\ &+n \to \gamma p\ \ (S_{31}); \quad 0p \to \gamma p\ \ (S_{32}); \quad \gamma p \to \gamma p\ \ (S_{33}) \end{aligned} \tag{A.1}$$

(the elements of the S-matrix are given in parentheses, and $S_{ki} = S_{ik}$).

If to start with we do not consider isotopic invariance, we must introduce another S-matrix for the processes which include the Compton effect on the neutron,

$$\begin{aligned} &-p \to -p\ (S'_{11}); \quad 0n \to -p\ (S'_{12}); \quad \gamma n \to -p\ (S'_{13}) \\ &-p \to 0n\ (S'_{21}); \quad 0n \to 0n\ (S'_{22}); \quad \gamma n \to 0n\ (S'_{23}) \\ &-p \to \gamma n\ (S'_{31}); \quad 0n \to \gamma n\ (S'_{32}); \quad \gamma n \to \gamma n\ (S'_{33}). \end{aligned} \qquad \text{(A.2)}$$

The unitarity conditions enable us to express (separately for (A.1) and (A.2)) the six independent complex quantities S_{ik} in terms of six real quantities by introducing, for example, three scattering phases δ_1, δ_2, δ_3 and three transformation parameters φ, ψ and θ:

$$S_{11} = e^{2i\delta_1}(\cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi)^2 + e^{2i\delta_2}(\sin\varphi\cos\psi + \cos\theta\sin\psi\cos\varphi)^2 + e^{2i\delta_3}\sin^2\theta\sin^2\psi;$$

$$S_{12} = e^{2i\delta_1}(\cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi)(\sin\psi\cos\varphi + \cos\theta\sin\varphi\cos\psi)$$
$$+ e^{2i\delta_2}(\sin\varphi\cos\psi + \cos\theta\sin\psi\cos\varphi)(\sin\varphi\sin\psi - \cos\theta\cos\varphi\cos\psi) - e^{2i\delta_3}\sin^2\theta\sin\psi\cos\psi;$$

$$S_{13} = e^{2i\delta_1}\sin\theta\sin\varphi(\cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi) - e^{2i\delta_2}\sin\theta\cos\varphi(\sin\varphi\cos\psi + \cos\theta\sin\psi\cos\varphi) + e^{2i\delta_3}\sin\theta\cos\theta\sin\psi;$$

$$S_{22} = e^{2i\delta_1}(\sin\psi\cos\varphi + \cos\theta\sin\varphi\cos\psi)^2 + e^{2i\delta_2}(\sin\varphi\sin\psi - \cos\theta\cos\varphi\cos\psi)^2 + e^{2i\delta_3}\sin^2\theta\cos^2\psi; \qquad \text{(A.3)}$$

$$S_{23} = e^{2i\delta_1}\sin\theta\sin\varphi(\sin\psi\cos\varphi + \cos\theta\sin\varphi\cos\psi) + e^{2i\delta_2}\sin\theta\cos\varphi(\cos\theta\cos\varphi\cos\psi - \sin\varphi\sin\psi) - e^{2i\delta_3}\sin\theta\cos\theta\cos\psi;$$

$$S_{33} = e^{2i\delta_1}\sin^2\theta\sin^2\varphi + e^{2i\delta_2}\sin^2\theta\cos^2\varphi + e^{2i\delta_3}\cos^2\theta.$$

If we omit reactions with γ-quanta, i.e., set $\delta_3 = \theta = 0$, only S_{11}, S_{12} and S_{22} are different from zero, and we get from (A.3) the familiar expressions for the elements of the 2-by-2 S-matrix, in which the transformation parameter is $\psi + \varphi$. The quantity θ characterizes the effect of radiative processes.

Instead of introducing real scattering phases and transformation coefficients, one can proceed differently and, to determine a complete set of experiments, make use of the unitarity conditions in the form of a generalization of the optical theorem[17,18] analogous to that for the case of elastic scattering alone, as was done in Ref. 19. The unitarity relations then enable one to reduce the number of necessary experiments (from the point of view of completeness of the phenomenological analysis) to the total number of independent terms in the amplitudes of all the reactions included in the unitary S-matrix. If we include radiative processes, additional terms which are not isotopically invariant appear in the amplitudes for meson-nucleon scattering, and in the general case the total number of terms reaches 20. If we take the matrix elements for photoproduction and Compton effect proportional to e and e^2, respectively, and expand (A.3), we find that in processes of the type of meson scattering the inclusion of radiative processes gives rise to small corrections (of order e^2). We shall therefore take the phases δ_1 and δ_2 to be the same as the phases for $\pi - N$ scattering in the states with isotopic spin $T = 3/2$ and $T = 1/2$, respectively (cf. the Appendix to Reference 20). On the other hand, the influence of $\pi - N$ scattering on the Compton effect is large and determines its magnitude.

Various considerations related to the reduction of the number of parameters in the S-matrix have been presented in Refs. 21, 22. We shall therefore not discuss them here.

Expression (18) for the amplitude of the Compton effect is only approximately unitary since, for example, the right side of (20) becomes zero. However, the restriction

$$\mathrm{Im}\,[R_1(0°) + R_2(0°)] \ll \mathrm{Re}\,[R_1(0°) + R_2(0°)],$$

which is necessary for the validity of (18) is satisfied with plenty to spare.

We note in conclusion that when we include considerations related to isotopic invariance, the same $\pi - N$ scattering phases appear in the S-matrix for the Compton effect on a neutron, and this results in various common features of the Compton effect on protons and neutrons.

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POLARIZATION AND ANGULAR DISTRIBUTION OF X-RAYS EMITTED AFTER NUCLEAR CAPTURE OF ELECTRONS AND AFTER CONVERSION TRANSITIONS

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Results obtained previously[1] are extended to K and L capture of any order of forbiddenness, taking parity nonconservation into account. It is shown that observation of the correlation of the polarization of x-rays emitted after K or L capture with nuclear spin directions or with the polarization of subsequent gamma rays may provide important information concerning β interactions. The angular distribution of x-rays is anisotropic when capture occurs from the L_{III} subshell.

Formulas are presented which permit determination of the spins and parities of nuclear levels from the observation of x-ray polarization following conversion transitions.

1. POLARIZATION AND ANGULAR DISTRIBUTION OF X-RAYS EMITTED FOLLOWING ELECTRON CAPTURE

In an earlier paper[1] the author proposed observation of the correlation between the angular polarization of x-rays emitted in the filling of atomic shell vacancies following allowed K capture and the orientation of nuclear spins or the polarization of subsequent gamma radiation. It was shown that information might thus be obtained regarding the relative signs of the scalar and tensor β-interaction constants. A number of recent experiments[2] have confirmed Lee and Yang's[3] hypothesis of par-