In the work of Ament and Rado,[1] the effective magnetic permeability μ_{eff} is calculated for a ferromagnetic metal having an electric conductivity, and its influence on the width, shape, and position of the resonance line is estimated. For ferromagnetic conductors placed in a radio-frequency field, the displacement and polarization currents can be neglected, as was done in Ref. 1. For ferrites, this cannot be done in general, and therefore Eq. (3) takes these currents into account.

Using the original system of equations to solve the boundary problem for a half-space filled with ferrite, in the case of a plane wave normally incident on the boundary, we obtain, in analogy with the procedure given in Ref. 1, the impedance Z $= [\mu_{eff}/\epsilon_{eff}]$, where according to (2) and (3)

$$\varepsilon_{eff} = \varepsilon' - i\varepsilon'' = 1 + 4\pi \sum_i g_i (\omega_j^2 - \omega^2 + i\omega\alpha_j)^{-1}. \quad (4)$$

Then

$$\mu_{eff} = \frac{\eta - \Omega^2 + 2i\,(a\omega^2/2\pi M_s^2 c^2)^{1/2} (\varepsilon' - i\varepsilon'')^{1/2}}{[\eta - \Omega^2 + i\,(a\omega^2/2\pi M_s^2 c^2)^{1/2} (\varepsilon' - i\varepsilon'')^{1/2}]^2}. \quad (5)$$

Here η and Ω stand for the same quantities as in Ref. 1, and the quantity a is denoted in Ref. 1 by A.

It follows from (5) that the electric polarization of the ferrite is accounted for by a shift of the resonant frequency and to a broadening of the resonance-absorption line.

The results by Ament and Rado are contained in Eq. (5). In fact, the conduction current can be taken into account by putting j = 0 in (4), for the case of the free electrons. Then $\omega_0 = 0$, α_0 is the frequency of collision between the electrons and the lattice, and $4\pi g_0$ is the square of the Langmuir frequency. Now $\sigma = g_0/\alpha_0$ is the static value of the electron conductivity. In metals $\sigma/\omega \gg 1$, and $\omega \ll \alpha_0$, and therefore (5) changes into (31b) of Ref. 1.

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CROSS SECTIONS FOR INTERACTION OF PI MESONS WITH CARBON NUCLEI

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In the optical model of the nucleus, scattering and absorption of particles is described as the motion of these particles in the field of a complex potential. A considerable part of the work done on the optical model is devoted to a determination of the complex potential or of uniquely-related parameters which provide best agreement with experimental data on the interaction of fast particles with nuclei. In a number of other analyses[1,2] it was sought to relate these parameters, which characterize the interaction of fast particles with nuclei, to the scattering cross sections and amplitudes for elementary particles. In particular, Frank, Gammel, and Watson[1] have obtained formulas relating the complex potential of the optical model to the amplitude of the forward scattering of mesons against nucleons, averaged over the protons and neutrons of a nucleus,* and have calculated the values of the real and imaginary parts of the potential, and thus of the mean free path λ_t of mesons in nuclear matter at energies of 0 to 350 Mev. This data will be used in the present article to calculate the total effective cross sections for elastic and inelastic interactions of π mesons with carbon nuclei in this energy region. The computations are carried out according to the formulas of the quasi-classical approximation.[4,5]

$$\sigma_{el} = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{l\leqslant R/\lambda\!\!\!^{-}} (2l+1)\,|\,1 - \exp\{[-K + 2ik(n-1)]\,s_l\}\,|^2, \quad (1)$$

$$\sigma_{inel} = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{l\leqslant R/\lambda\!\!\!^{-}} (2l+1)\,[1 - \exp(-2Ks_l)], \quad (2)$$

$$s_l = (R^2 - l^2\lambda\!\!\!^{-2})^{1/2},$$

where $\lambda\!\!\!^{-} = k^{-1}$ is the wavelength of the incident mesons; $K = \lambda_t^{-1}$ is the absorption coefficient and n is the index of refraction of nuclear matter; R is the nuclear radius taken to be $1.4A^{1/3} \times 10^{-13}$ cm, which gives for carbon 3.2×10^{-13} cm. In order to estimate the error of the quasi-classical approximation, the cross sections were also calculated according to the exact quantum mechanical formulas:[5]

$$\sigma_{el} = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{\infty} (2l+1)\,|\,\beta_l - 1\,|^2, \quad (3)$$

$$\sigma_{inel} = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{\infty} (2l+1)\,(1 - |\,\beta_l\,|^2), \qquad \beta_l = e^{2i\delta_l}, \quad (4)$$

*The real part of the forward-scattering amplitude was obtained in Ref. 1 from dispersion relations.[3]

where δ_ℓ is the complex phase shift, obtained from the continuity condition imposed at the nuclear surface on the logarithmic derivative of the radial part of the wave function which satisfies the Klein-Gordon equation. For a square well, β_ℓ is expressed in terms of Bessel functions of half-integer order and arguments kR and κR where k and $\kappa = k + k_1 + iK/2$ denote the wave number of the incident particles outside and inside the well;

$$k + k_1 = nk = \frac{E - \mathrm{Re}\, V_0}{\hbar c}\left[1 - \frac{4\mu^2 c^4}{4(E - \mathrm{Re}\, V_0)^2 + (\hbar c K)^2}\right]^{1/2},$$

E is the total energy of the meson outside the nucleus and μ is the mass of the π meson. The results of the computation are presented in Figs. 1 and 2. The energy dependence calculated by Sternheimer[2] is also shown for purpose of comparison. In addition, the corresponding experimental values obtained from Refs. 6 — 15 are also shown on the figures. It may be seen from the figures that at large energies, the cross section calculated from formulas (1) and (2) is about 20 — 25% higher than that computed from (3) and (4). The curve showing the energy dependence of the inelastic cross section, as calculated by formulas (4), is in satisfactory agreement with experimental data. On the other hand, the elastic cross section which we have computed is considerably higher than the experimental data at meson energies below 100 Mev. Since the elastic scattering cross section increases rapidly with the depth of the potential well $|\mathrm{Re}\, V_0|$ at these energies, the presence of a sharp maximum in the cross section around 65 Mev, which

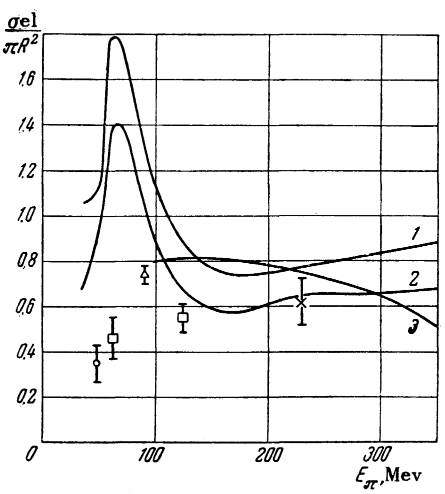

FIG. 1. Cross section for elastic scattering of π mesons by carbon nuclei as a function of energy. Curve 1 was calculated according to formula (1), Curve 2 according to formula (3), Curve 3 is taken from Ref. 2. □ - Refs. 6 and 7; o - Ref. 9; × - Ref. 13; Δ - Ref. 15.

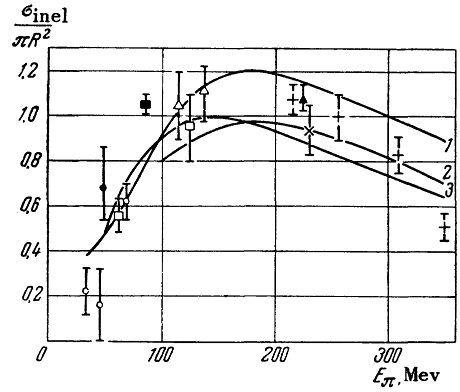

FIG. 2 Cross section for inelastic interaction of π mesons with carbon nuclei as a function of energy. Curve 1 was calculated according to formula (2), Curve 3 according to formula (4), Curve 2 is taken from Ref. 2 o - Ref. 8; ● - Ref. 9; □ - Ref. 6 and 7. ■ - Ref. 10; Δ - Ref. 11; Δ - Ref. 12; × - Ref. 13; + - Ref. 14.

does not agree with experiment, attests to the fact that the potential well depth used in the computations is too large in this energy region. A comparison of empirical estimates of Re V_0 with values computed in Ref. 1 is similarly not opposed to such a conclusion.

Lack of experimental data on elastic scattering does not allow any more definite conclusions on the agreement between experimental and calculated results. In computing the cross sections it was also assumed that the nucleus is a sphere of constant density with a sharp surface. This is a crude approximation, at least for light nuclei. It has been shown[20] that the charge density decreases from the center to the surface. Calculations of the angular distribution of 62-Mev π mesons elastically scattered by carbon nuclei using a diffuse surface model[21] are in satisfactory agreement with

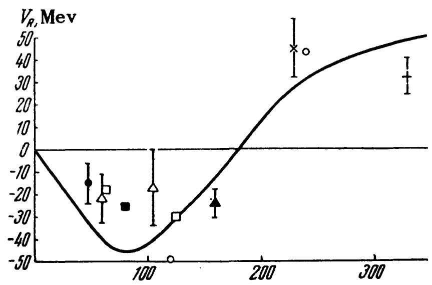

FIG. 3. Energy dependence of the real part Re V_0 of the optical-model potential for interaction of π mesons with complex nuclei according to Ref. 1 (solid curve). The estimates of the potential depth were obtained from experiments with carbon nuclei: □ - Refs. 6 and 7; ● - Ref. 9; × - Ref. 13; with aluminum nuclei: ■ - Ref. 16; with helium nuclei: Δ - Ref. 17; + - Ref. 18; with photoemulsion nuclei: Δ - Ref. 19. The symbol o denotes the maximum and minimum values of the potential obtained in Ref. 2.

experiment.[7] We therefore believe that computations of the total cross section for a diffuse surface nucleus is of definite interest.

In conclusion we take this opportunity to thank I. A. Kropin and B. F. Chumakov for carrying out a large part of the numerical calculations.

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RADIATIVE CORRECTIONS TO BREMSSTRAHLUNG

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FEYNMAN'S method,[1] which is usually used to obtain radiative corrections, cannot be applied in practice to bremsstrahlung because of the much greater calculational difficulties than those encountered, for instance, in the application to the corrections to Compton scattering.[2] In the present work, we use the so-called mass-operator method,[3,4] which has many advantages over the Feynman method.

The expression for the renormalized mass operator to the necessary order in e^2 has been obtained by Newton.[4] Not only the mass operator, but also vacuum polarization contributes to the radiative corrections to bremsstrahlung. In order to avoid the infrared catastrophe, as usual, we make use of the fictitious photon mass λ. We eliminate λ from the final expression by adding to the usual bremsstrahlung cross section the cross section for double bremsstrahlung, when two photons are emitted simultaneously and the energy of one of these photons is less than some quantity ΔE determined by the accuracy of the measurement.

The total cross section is conveniently written in the form

$$d\sigma \doteq d\sigma_0 [1 - (e^2/\pi)(\delta_R + \delta_D)], \quad e^2 = 1/137,$$

where $d\sigma_0$ is the cross section for the basic process and is given by the Bethe-Heitler formula,[5,6] δ_R gives the radiative corrections, and δ_D gives the double bremsstrahlung.

Exact expressions for δ_R and δ_D, together with a description of the method by which they have been calculated, will be presented in a detailed article. Here we shall give only some limiting values of δ_R and δ_D. We choose units in which $\hbar = c = m = 1$, and make use of the following notation: $\mathbf{p}_1$ and ϵ_1 are the initial, and $\mathbf{p}_2$ and ϵ_2 are the final electron momentum and energy; $\mathbf{k}$ and $\omega = \epsilon_1 - \epsilon_2$ are the momentum and energy of the emitted photon;

$$\varkappa = -2\omega(\varepsilon_2 - p_2 \cos\theta_2), \quad \theta_2 = \widehat{\mathbf{kp}_2};$$
$$\tau = 2\omega(\varepsilon_1 - p_1 \cos\theta_1), \quad \theta_1 = \widehat{\mathbf{kp}_1};$$
$$\alpha = \varkappa + \tau;$$